TDOA Matrices: Algebraic Properties and Their Application to Robust Denoising With Missing Data

Abstract—Measuring the time delay of arrival (TDOA) between a set of sensors is the basic setup for many applications, such as localization or signal beamforming. This paper presents the set of TDOA matrices, which are built from noise-free TDOA measure-ments, not requiring knowledge of the sensor array geometry. We prove that TDOA matrices are rank-two and have a special sin-gular value decomposition decomposition that leads to a compact linear parametric representation. Properties of TDOA matrices are applied in this paper to perform denoising, by finding the TDOA matrix closest to the matrix composed with noisy measurements. This paper shows that this problem admits a closed-form solution for TDOA measurements contaminated with Gaussian noise that extends to the case of having missing data. This paper also proposes a novel robust denoising method resistant to outliers, missing data and inspired in recent advances in robust low-rank estimation. Experiments in synthetic and real datasets show significant im-provements of the proposed denoising algorithms in TDOA-based localization, both in terms of TDOA accuracy estimation and lo-calization error.

Index Terms—TDOA estimation, TDOA denoising, skew-symmetric matrices, matrix completion, missing data.

I. INTRODUCTION

TIME delay of arrival (TDOA) estimation is an essential pre-processing step for multiple applications in the context of sensor array processing, such as multi-channel source localization [1], self-calibration [2] and beamforming [3]. In

all cases, performance is directly related to the accuracy of the estimated TDOAs [4]. Estimating TDOA in noisy environments has been subject of study during the last two decades [5]–[7], and is still an active area of research, benefiting from current advances in signal processing and optimization strategies [8]–[11].

Typically, the TDOA between a single pair of sensors is obtained by measuring the peak of the generalized cross-correlation (GCC) of the received signals on each sensor [12], which are assumed to be generated from a single source. Many factors, such as the spectral content of the signal, multipath propagation, and noise contribute to errors in the estimation of the TDOA.

Given a set of sensors, TDOA measurements can be obtained for every possible pair of sensors. This is commonly known as the full TDOA set or spherical set [13]. This paper studies how to reduce noise and errors from the full TDOA set. The intuition behind this denoising is to exploit redundancy of the full TDOA set. For n sensors, the full set of $n(n-1)/2$ measurements can be represented by $n-1$ values, which are referred to as the non-redundant set. This problem has been studied in the past, show-ing that one can optimally obtain the non-redundant set when TDOA measurements are contaminated with additive Gaussian noise. This is known as the Gauss–Markov estimator [14]. How-ever, in more realistic scenarios errors are not Gaussian and some of the TDOA measurements may contain outliers. In these cases the Gauss–Markov estimator performs poorly.

This paper presents the TDOA matrix, which is created by the arrangement of the full TDOA set inside a skew-symmetric matrix, and studies the algebraic properties of this matrix, show-ing that it has rank 2 and a singular value decomposition (SVD) decomposition with $n-1$ degrees of freedom. Such matrices have been previously defined in the literature [15], but their properties and applications have not been studied until now.

These algebraic properties are used in this paper to perform denoising under different scenarios, which include the presence of missing TDOA measurements and outliers. These denoising algorithms are tested in the context of speaker localization with microphone arrays, using synthetic and publicly available real datasets. Our denoising algorithms are able to recover accurate TDOA values for high rates of missing data and outliers, sig-nificantly outperforming the Gauss–Markov estimator in those cases. All the proposed methods don’t require knowledge of the sensor positions, so that they can also be used for calibration [2].

The main contributions of this work are threefold: 1) Defini-tion of the algebraic properties of TDOA matrices. 2) A closed-form solution for TDOA denoising for Gaussian noise and the presence of missing data. 3) Novel robust-denoising methods for handling additive correlated noise, outliers and missing data.

XI. CONCLUSION

This paper has studied the properties of TDOA matrices, showing that they can be effectively used for solving TDOA de-noising problems. In particular, the paper has investigated challenging scenarios where the TDOA matrix is contaminated with Gaussian noise, outliers and where a percentage of the measure-ments are missing. The paper shows that denoising in the presence of Gaussian noise and missing data can be solved in closed-form. This result is important, as it is the basis of an iterative algorithm that can also cope with outliers. The paper has tested the proposed algorithms in the context of acoustic localization using microphone arrays. The experimental results, both on real and synthetic data have shown that our algorithms successfully perform denoising (up to 30% of improvement in localization accuracy) with a high rate of missing data (up to 50%) and out-liers, without knowing the sensor positions. This is important as it opens its application to tasks where the sensors geometry is unknown. Interestingly, in real datasets our robust denois-ing algorithm is systematically better than the Gauss–Markov estimator even when there is no missing data. This is also an important result as it proves that the assumption of Gaussian noise does not hold in real cases, while our robust model is capable of automatically discard erroneous measurements. The proposed robust denoising method has also been compared with other methods in the literature on a localization task. Our results are very similar to the state of the art, even though we do not require knowing the array geometry in the denoising stage. Furthermore, our proposal is significantly less computationally demanding.

As for future work, we plan to further test our denoising algorithms in applications where the position of the sensors is unknown in advance, such in self-localization and beamforming.