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Maximum Likelihood TDOA Estimation from Compressed Sensing Samples without Reconstruction

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***Abstract*—One application for Time-Difference-of-Arrival (T-DOA) estimation is in emitter localization. A signal from an emitter reaching a group of sensors, each in a separate location, will have different arrival times. Finding the TDOAs between the output of pairs of sensors will provide the necessary measurements for the hyperbolic localization of the emitter. When the sensors acquire the signal by Compressed Sensing (CS), their outputs are reduced dimension linear transformation of the time samples of the signal. This shuffling of the time samples breaks up their time relation. Thus a cross-correlation of the CS output of two sensors cannot determine the TDOA. To apply cross-correlation, it is necessary to reconstruct the time samples. This paper proposes an alternative that uses only the coefficients of the discrete Fourier Transform (DFT) of the CS samples. It begins with the derivation of the maximum likelihood (ML) equation and the ML estimator. This estimator requires known values of signal and noise powers. Substituting these values by their estimates lead to the approximate ML estimator. The phase of the product of two DFT coefficients from each sensor is proportional to the unknown TDOA. Hence these coefficients can provide an estimation of the TDOA. Simulation results show that although ML is the best, as expected, all these estimators have very close performance.**

***Index Terms*—Time-Difference-of-Arrival, Maximum Likelihood, discrete Fourier Transform, Compressed Sensing.**

I. Introduction

TIME-Difference-of-Arrival (TDOA) estimation refers to determining the difference in arrival times of a signal received at two spatially separated sensors. It has applications in direction finding and source localization [1]–[5]. For example, in the latter, multiplying the TDOA by the signal propagation speed gives the range difference between the source and two receivers. Each range difference defines a hyperbola on which the target must lie in the two-dimensional space, and thus the source position can be obtained from the intersection of at least two hyperbolas.

A standard TDOA estimator cross-correlates the output of the two sensors [1], [2]. The shift from the origin at which the cross-correlation function peaks is the TDOA estimate.

With the introduction of Compressed Sensing (CS) [6], [7], a problem to consider is the estimation of TDOA from CS measurements.

Direct cross-correlation of the CS output of two sensors cannot give a TDOA estimation. To see this, let the $N \times 1$ vector of Nyquist rate samples of the signal $s(t)$ from the sensors be[1], for $i = 1, 2$

$$\mathbf{s}_i = [s_i(0) \quad s_i(1) \quad \cdots \quad s_i(N-1)]^{\mathrm{T}}. \tag{1}$$

Suppose $s_2(n) = s_1(n+4)$, then the cross-correlation function of $\mathbf{s}_1$ and $\mathbf{s}_2$ will peak at a shift of 4 samples, which is the TDOA. Now the CS measurements are

$$\mathbf{y}_i = \mathbf{A}\mathbf{s}_i, \tag{2}$$

where $\mathbf{A}$ is a known $M \times N$ $(N > M)$ sensing matrix [7] (see footnote 2 next page for a discussion on $\mathbf{A}$). The linear transformation by $\mathbf{A}$ on $\mathbf{s}_1$ and $\mathbf{s}_2$ has broken up their time shift relation, i.e., the elements of $\mathbf{y}_1$ and $\mathbf{y}_2$ are not sequential samples of a time sequence. And $\mathbf{y}_1$ is not a time-shifted version of $\mathbf{y}_2$. Hence a cross-correlation of $\mathbf{y}_1$ and $\mathbf{y}_2$ cannot find the TDOA.

If the signal $s(t)$ is sparse [7], a reconstruction of $\mathbf{s}_1$ and $\mathbf{s}_2$, say by ℓ_1 minimization [7], will yield reconstructed samples for a cross-correlation estimation. This is the basis of the estimator in [8]–[13]. The reconstruction algorithms are nonlinear and can have large errors if noise is present.

An alternative [14]–[16] that avoids reconstruction designates a reference sensor to take samples at the Nyquist rate, resulting in $\mathbf{s}_1$ in (1). Working offline, shifting the elements in $\mathbf{s}_1$ by k samples $(-K \le k \le K)$, where K is the largest expected TDOA, produces several $\mathbf{s}_k$ and $\mathbf{y}_k = \mathbf{A}\mathbf{s}_k$. Finding the $\mathbf{y}_{k^\star}$ closest to $\mathbf{y}_2 = \mathbf{A}\mathbf{s}_2$ from the other sensor gives $k^\star$ as the TDOA estimate. The drawback here is that the reference sensor must sample at the Nyquist rate.

As a trade-off against the need for a sampler at the Nyquist rate, [17] takes several CS measurements at both sensors, giving rise to several $\mathbf{y}_k = \mathbf{A}\mathbf{s}_k$. As in above, the elements of $\mathbf{s}_k$ are time-shifted version of a reference. Estimation of TDOA is performed by matching the $\mathbf{y}_k$ of one sensor against the other.

It is well known that a time shift in the time domain leads to a phase shift in the frequency domain. Given a time shift D between the elements of $\mathbf{s}_1$ and $\mathbf{s}_2$, there is a corresponding phase shift, proportional to D, between their

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[1] A boldface alphabet (upper- or lower-case, with a subscript of $i = 1, 2$) denotes a vector. An element in that vector has the same alphabet but is in lightface, followed by its position number in bracket. A boldface upper-case alphabet denotes a matrix. An element in that matrix is the same alphabet but in lower-case and lightface. Subscript in the element denotes its position.

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discrete Fourier Transforms (DFT). Furthermore, the DFT of $\mathbf{y}_1$ and $\mathbf{y}_2$ preserve this phase shift, making them applicable for a direct estimation of D.

Building on this shift-to-phase principle, this paper provides two new results for direct TDOA estimation from $\mathbf{y}_1$ and $\mathbf{y}_2$:

1. The derivation of the maximum likelihood (ML) equation and the ML estimator.
2. Under the assumption that the signal is a white noise process, the formulation of a set of frequency estimation equations whose solution is the TDOA parameter. An approximate ML estimation then follows by substituting the true values in the ML equation by their estimates.

In the rest of the paper, Section II contains the theoretical development. The simulation results are in Section III, and the conclusions in Section IV.

II. TDOA Estimation from CS Samples

A. Signal Acquisition

Let a pair of sensors receive, for $i = 1, 2$

$$s_i(t) + \phi_i(t), \tag{3}$$

where

$$s_2(t) = s_1(t + D) \tag{4}$$

with D denoting the TDOA between the emitter signals $s_1(t)$ and $s_2(t)$. Present at the sensors are ambient noise $\phi_i(t)$.

Using, for example, a Random Modulator Pre-Integrator (RMPI) system [18], the sensors have CS measurements[2]

$$\mathbf{y}_i = \mathbf{A}(\mathbf{s}_i + \boldsymbol{\phi}_i) + \mathbf{w}_i, \tag{5}$$

for $i = 1, 2$. Note that (5) contains two noise terms, $\boldsymbol{\phi}_i$ and $\mathbf{w}_i$. This is a more realistic model since in practice, the signal s_i normally comes with additive noise $\boldsymbol{\phi}_i$. In contrast, $\mathbf{w}_i$ is the measurement noise, representing analog-to-digital conversion and other instrumentation errors. Most CS literature assumes $\boldsymbol{\phi}_i = 0$, but [19] draws attention to the need for including $\boldsymbol{\phi}_i$ in CS.

The $N \times 1$ vectors (with T denoting transpose)

$$\mathbf{s}_i = [s_i(0) \quad \cdots \quad s_i(n) \quad \cdots \quad s_i(N-1)]^{\mathrm{T}}, \tag{6}$$

contain the Nyquist rate samples of $s_1(t)$ and $s_2(t)$, respectively, and

$$\boldsymbol{\phi}_i = [\phi_i(0) \quad \cdots \quad \phi_i(n) \quad \cdots \quad \phi_i(N-1)]^{\mathrm{T}}, \tag{7}$$

are vectors of samples of $\boldsymbol{\phi}_1(t)$ and $\boldsymbol{\phi}_2(t)$. The measurement noise vectors

$$\mathbf{w}_i = [w_i(0) \quad \cdots \quad w_i(m) \quad \cdots \quad w_i(M-1)]^{\mathrm{T}}. \tag{8}$$

The CS output vectors are

$$\mathbf{y}_i = [y_i(0) \quad \cdots \quad y_i(m) \quad \cdots \quad y_i(M-1)]^{\mathrm{T}}. \tag{9}$$

[2]The sensing matrix $\mathbf{A}$ in (5) below must meet a set of conditions to ensure recovery of the Nyquist rate samples from the CS samples [7]. The RMPI produces such a matrix. It multiplies an analog signal with a pseudo-random binary sequence, of values +1 or -1, at the Nyquist rate, and integrates the products. The elements of $\mathbf{A}$ are +1 or -1.

The signal and noise are stationary, zero mean Gaussian random processes. The noise samples are random variables independent of each other and of the signal samples. The $M \times N$ $(N > M)$ sensing matrix $\mathbf{A}$ is known, while the signal and noise variances are unknown.

The problem is to estimate D, given only $\mathbf{y}_1$ and $\mathbf{y}_2$.

B. The ML Equation

Let $e^{-j\frac{2\pi kn}{N}}$ be an element of the DFT matrix $\mathbf{F} \in \mathbb{C}^{N\times N}$, and its inverse $\mathbf{F}^{-1}$ has an element $\frac{1}{N}e^{j\frac{2\pi kn}{N}}$. Similarly, $\mathbf{G}$ is an $M \times M$ DFT matrix. Taking the DFT of $\mathbf{y}_i$ gives

$$\mathbf{Y}_i = \mathbf{G}\mathbf{y}_i = [Y_i(0) \quad \cdots \quad Y_i(p) \quad \cdots \quad Y_i(M-1)]^{\mathrm{T}}, \tag{10}$$

and the DFT of the other vectors in (5) are

$$\begin{aligned} \mathbf{S}_i = \mathbf{F}\mathbf{s}_i &= [S_i(0) \quad \cdots \quad S_i(k) \quad \cdots \quad S_i(N-1)]^{\mathrm{T}} \quad (11) \\ \boldsymbol{\Psi}_i = \mathbf{F}\boldsymbol{\phi}_i &= [\Psi_i(0) \quad \cdots \quad \Psi_i(k) \quad \cdots \quad \Psi_i(N-1)]^{\mathrm{T}} \\ \mathbf{W}_i = \mathbf{G}\mathbf{w}_i &= [W_i(0) \quad \cdots \quad W_i(p) \quad \cdots \quad W_i(M-1)]^{T}. \end{aligned}$$

It follows from (5) that

$$\mathbf{Y}_i = \mathbf{G}\mathbf{y}_i = \mathbf{G}\mathbf{A}\mathbf{s}_i + \mathbf{G}\mathbf{A}\boldsymbol{\phi}_i + \mathbf{G}\mathbf{w}_i \tag{12}$$

or

$$\mathbf{Y}_i = \mathbf{G}\mathbf{A}\mathbf{F}^{-1}\mathbf{S}_i + \mathbf{G}\mathbf{A}\mathbf{F}^{-1}\boldsymbol{\Psi}_i + \mathbf{W}_i. \tag{13}$$

Let

$$\mathbf{G}\mathbf{A}\mathbf{F}^{-1} = \mathbf{H} \in \mathbb{C}^{M\times N}, \tag{14}$$

and has an element h_{pk}. Then

$$Y_1(p) = \sum_k h_{pk}(S_1(k) + \Psi_1(k)) + W_1(p), \tag{15}$$

$$Y_2(p) = \sum_k h_{pk}(S_2(k) + \Psi_2(k)) + W_2(p), \tag{16}$$

with $\sum_k$ indicating the summation from $k = 0$ to $N - 1$, and in the equations below, $\sum_p$ is from $p = 1$ to $M - 1$.

In most estimation cases, there is the requirement that the data length is larger than the correlation time of a random process, i.e., the time-bandwidth product is large. Then the DFT coefficients of the samples of the random process are independent random variables [20]–[22]. This is the basis for obtaining the probability density function (PDF) of (17).

The derivation of the likelihood function of (27) below begins with the introduction of the TDOA parameter, τ, such that $s_2(n) = s_1(n + \tau)$. This parameter τ is different from D in (4) in that D is a specific constant, while τ is a variable that denotes any TDOA. Given τ, the joint conditional Gaussian PDF of $\mathbf{Y}_1$ and $\mathbf{Y}_2$ is [20]

$$f\{(\mathbf{Y}_1; \mathbf{Y}_2)|\tau\} = \left[\pi^{2M}\prod_{p=1}^{M-1}|\mathbf{V}(p)|\right]^{-1}\exp\left[-\sum_p \mathbf{R}(p)\right] \tag{17}$$

where (with $*$ denoting complex conjugate)

$$\mathbf{R}(p) = [Y_1^*(p) \quad Y_2^*(p)]\,\mathbf{V}^{-1}(p)\begin{bmatrix} Y_1(p) \\ Y_2(p) \end{bmatrix} \tag{18}$$

and

$$\mathbf{V}(p) = \mathbb{E}\left\{\begin{bmatrix} Y_1(p) \\ Y_2(p) \end{bmatrix}[Y_1^*(p) \quad Y_2^*(p)]\right\}, \tag{19}$$

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Eq. (17) excludes the elements at $p = 0$ because they do not contain any phase information.

Now the DFT coefficients are independent, so that

$$\mathbb{E}\left\{S_1(k)S_1^*(l)\right\} = \mathbb{E}\left\{S_2(k)S_2^*(l)\right\} = \begin{cases}\sigma^2(k), & k = l\\ 0, & k \neq l\end{cases} \quad (20)$$

where $\sigma^2(k)$ is the power spectral density of the signal at the kth bin. Also, for simplicity, assume that the ambient noise powers are the same at both sensors, so that

$$\mathbb{E}\left\{\Psi_1(k)\Psi_1^*(l)\right\} = \mathbb{E}\left\{\Psi_2(k)\Psi_2^*(l)\right\} = \begin{cases}\sigma_\phi^2, & k = l\\ 0, & k \neq l\end{cases} \quad (21)$$

and similarly for the noise samples of (8),

$$\mathbb{E}\left\{W_1(p)W_1^*(q)\right\} = \mathbb{E}\left\{W_2(p)W_2^*(q)\right\} = \begin{cases}\sigma_w^2, & p = q\\ 0, & p \neq q\end{cases}. \quad (22)$$

Since $s_2(n) = s_1(n+\tau)$, it follows that

$$S_2(k) = \begin{cases} S_1(k)e^{j\frac{2\pi k\tau}{N}}, & 0 \le k \le \frac{N}{2} - 1\\ S_1(k)e^{j\frac{2\pi k\tau}{N}}e^{-j2\pi\tau}, & \frac{N}{2} \le k \le N-1\end{cases}. \quad (23)$$

In (23), the factor $e^{-j2\pi\tau}$ is needed to maintain the DFT relationship of $S_2(k) = S_2^*(N-k)$, when τ is not an integer. Putting this two-part representation of $S_2(k)$ into (16) then gives rise to the two separate sums in (26) and (32).

Taking the expectation in (19) and then its inverse yields

$$\mathbf{V}^{-1}(p) = \frac{1}{|\mathbf{V}(p)|}\begin{bmatrix} a(p) & b^*(p)\\ b(p) & a(p)\end{bmatrix} \quad (24)$$

where, from using (15),

$$a(p) = \sum_k |h_{pk}|^2(\sigma^2(k) + \sigma_\phi^2) + \sigma_w^2 = \mathbb{E}\{Y_1(p)Y_1^*(p)\} \quad (25)$$

and from (16) and (23),

$$\begin{aligned}-b(p) &= \sum_{k=0}^{\frac{N}{2}-1} |h_{pk}|^2\sigma^2(k)e^{j\frac{2\pi k\tau}{N}} + \sum_{k=\frac{N}{2}}^{N-1} |h_{pk}|^2\sigma^2(k)e^{j\frac{2\pi k\tau}{N}}e^{-j2\pi\tau}\\ &= \mathbb{E}\{Y_1^*(p)Y_2(p)\}.\end{aligned} \quad (26)$$

Note that $|\mathbf{V}(p)|$, the determinant of the covariance matrix $\mathbf{V}(p)$, is a function of τ. This makes it rather cumbersome to derive the Cramer-Rao-Lower-Bound (CRLB) [23] from (17).

Taking the natural logarithm of (17) and neglecting the terms independent of τ gives the likelihood function

$$L(\tau) = -\left\{\sum_p \ln|\mathbf{V}(p)| + \sum_p \mathbf{R}(p)\right\}. \quad (27)$$

The ML estimation of D is the τ that maximizes $L(\tau)$.

Now $L(\tau)$ contains the terms $\sigma^2(k)$, σ_ϕ^2, and σ_w^2. If they are not known, it will be necessary to replace them by their estimates, resulting in an approximate ML (AML) estimation.

C. Product of DFT Coefficients (PDC)

Consider next the estimation of D from products of the DFT coefficients $Y_1(p)$ and $Y_2(p)$. Let

$$P(p) = Y_1(p)Y_2^*(p), \quad (28)$$

so that from (15), (16), and (26)

$$\mathbb{E}\{P(p)\} = -b^*(p). \quad (29)$$

Imposing the condition that the signal is a white noise process, then $\sigma^2(k) = \sigma^2$, independent of k. Minimizing the differences between $P(p)$ and their means in (29) leads to a heuristic estimation of D as

$$\hat{D} = \arg\min_{\sigma^2}\left(\min_\tau \sum_p \|P(p) - \sigma^2 g(p,\tau)\|^2\right). \quad (30)$$

In (30), let

$$P(p) = P_R(p) + jP_I(p) \quad (31)$$

and

$$\begin{aligned}g(p,\tau) &= \sum_{k=1}^{\frac{N}{2}-1} |h_{pk}|^2 e^{-j\frac{2\pi k\tau}{N}} + \sum_{k=\frac{N}{2}+1}^{N-1} |h_{pk}|^2 e^{-j\frac{2\pi k\tau}{N}}e^{j2\pi\tau}\\ &= g_R(p,\tau) + jg_I(p,\tau)\end{aligned} \quad (32)$$

where the subscripts R and I denote respectively the real and imaginary parts of a complex number.

The cost function in (30) is multi-modal. In its inner minimization, setting the derivative of the sum with respect to σ^2 to zero gives

$$\sigma^2 = \frac{\sum_p\left[P_R(p)g_R(p,\tau) + P_I(p)g_I(p,\tau)\right]}{\sum_p |g(p,\tau)|^2}. \quad (33)$$

Putting (33) into (30) reduces it to a function of τ only. The minimization procedure is a grid search of τ in the range $\tau \le |D_M|$, where $|D_M|$ is the largest expected TDOA. Letting $\tau = \hat{D}$ in (33) gives an estimate of σ^2, which the AML uses to replace the true σ^2.

Surprisingly, as the simulation results in Section III show, the PDC and ML estimates are very close.

D. The AML

The ML estimator of (27) requires known signal and noise powers $\sigma^2(k)$, $\sigma^2(k) + \sigma_\phi^2 = \sigma_T^2$, and σ_w^2. Normally σ_w^2 is available from an instrumentation calibration, and since $\sigma_w^2 \ll \sigma_T^2$, the following derivation assumes $\sigma_w^2 = 0$ for convenience.

To estimate σ_T^2, let $\sigma^2(k) = \sigma^2$ as in Section II-C, and consider an approximation of (25) as

$$Y_1(p)Y_1^*(p) \approx a(p) \quad (34)$$

or

$$\sum_p |Y_1(p)|^2 \approx \sum_p |a(p)| = \sum_p |Y_2(p)|^2. \quad (35)$$

Then the estimate of σ_T^2 is

$$\hat{\sigma}_T^2 = \frac{\sum_p |Y_1(p)|^2 + \sum_p |Y_2(p)|^2}{2\sum_p\sum_k |h_{pk}|^2}. \quad (36)$$

Using σ^2 from (33) and $\hat{\sigma}_T^2$ from (36) as the true values in (27) gives the AML. This approach is similar to the approximation taken for the ML estimator in [21], where estimates of the signal and noise spectra substitute for the true values.

Similar to the PDC in Section II-C, the AML also conducts a grid search for maximization of (27) in the range $\tau \le |D_M|$.

III. Simulation Results

This section contains the simulation results of TDOA estimation, for the estimators PDC, AML, and ML.

The CS matrix $\mathbf{A}$ is the $M \times N$ upper part of a 512×512 Hadamard matrix, whose elements represent the pseudo-random binary bits of the RMPI [18].

The signal samples $s_1(n)$ and $s_2(n)$ are independent zero mean Gaussian random variables, as are the noise samples $\phi_1(n)$ and $\phi_2(n)$. The signal variance is unity, so that the signal-to-noise ratio (SNR) equals the inverse of the noise variance. The original data length N is 512, and the CS sample length M is $\frac{N}{2}$, or $\frac{N}{4}$. Each estimator first performs a coarse grid search of $-3 \le \tau \le 3$ at steps of 0.2 unit. After finding the best $\tau = \tau^\star$, a fine grid search follows with $\tau^\star - 1 \le \tau \le \tau^\star + 1$, at steps of 0.005 unit to obtain the final answer.

Figure 1 plots the mean square error (MSE) in dB from 100 independent trials, against SNR, for the three estimators. As expected, ML has the lowest MSE, followed by PDC and AML. A probable reason for AML being inferior to PDC is that AML uses estimates of the signal and noise powers. Their inaccuracy could cause more errors for AML. Both AML and PDC exhibit the threshold phenomenon in nonlinear estimation [1] at SNR = 1.

The MSE of Figure 1, as well as those in Table I, are for $D = 0$. For $D = 0.6$ and $D = 1$, the results are similar and not shown due to space limitations.

When estimating directly from the original samples, the CRLB is [1]

$$\mathrm{CRLB} = \frac{3(1 + 2(\mathrm{SNR}))}{N\pi^2(\mathrm{SNR})^2}. \tag{37}$$

There are two ways to quantify the performance degradation due to CS.

One is the SNR loss. From Table I, at SNR = 5 and $M = 256$, the MSE for ML is 2.3010×10^{-3}. Putting this value for CRLB in (37), with $N = 256$, and solving for SNR gives SNR = 2.8. The SNR loss is $5/2.8$.

The other is data length loss. Putting CRLB= 2.3010×10^{-3} and SNR = 5 in (37) and solving for N gives $N = 38$. The data length loss is $512/58$. This loss is greater than N/M because $\mathbf{A}\phi_i$ causes noise folding [19].

Table I shows the MSE for different SNR and M. Table II lists the loss due to CS.

In Table I, the MSE increases by a factor of 2 from $M = \frac{N}{2}$ to $M = \frac{N}{4}$. This indicates that MSE$\propto \frac{1}{M}$.

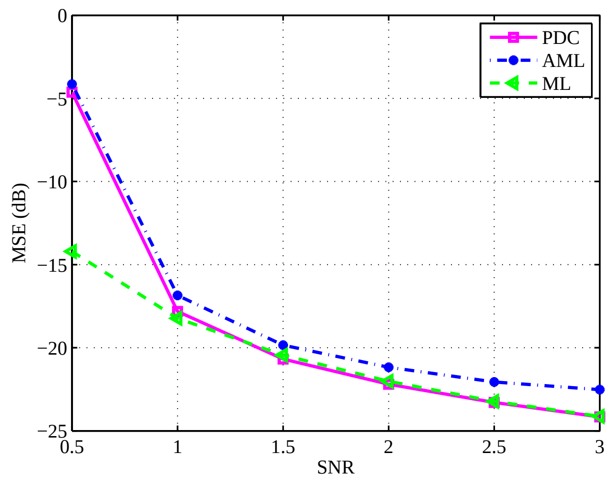

Fig. 1: MSE versus SNR with $M = N/2$, $D = 0$.

TABLE I: MSE, $D = 0$

SNR	Estimator	$M = N/2$	$M = N/4$
$\frac{1}{0.2}$	PDC	2.6535e-03	5.3613e-03
	AML	4.7083e-03	1.0518e-02
	ML	2.3010e-03	4.4738e-03
$\frac{1}{0.3}$	PDC	4.2483e-03	8.6745e-03
	AML	6.4383e-03	1.3826e-02
	ML	3.6233e-03	7.0523e-03

TABLE II: CS loss

M/N	Length loss	SNR loss
1/2	512/58	5/2.8=1.78
1/4	512/30	5/2.84=1.76

IV. Conclusions

It is possible to estimate TDOA directly from the CS output of two sensors, avoiding the need to reconstruct the signal samples. Thus this method is applicable even to non-sparse signals. This paper derived the ML function via the frequency domain and provided two estimators that have close performance to the optimal ML estimator.

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