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Compressed Time Difference of Arrival Based Emitter Localization

Johannes Schmitz, Rudolf Mathar
Institute for Theorectical Information Technology
RWTH Aachen University
D-52074 Aachen, Germany
Email: {schmitz, mathar}@ti.rwth-aachen.de

Dominik Dorsch
Lehrstuhl C für Mathematik (Analysis)
RWTH Aachen University
D-52062 Aachen, Germany
Email: dorsch@mathc.rwth-aachen.de

***Abstract*—Utilizing the time differences of arrival of a signal, impinging on a distributed sensor network, is a well known approach for the location estimation of a radio wave emitter. The high sampling rate necessary for a precise positioning implies a huge amount of data exchange between the sensor nodes. Contrary, the absence of high data rate enabled backbone links, connecting the nodes, restricts the system performance or may even render it dysfunctional in some cases. In order to tackle the problem we propose a novel method for compressed time difference of arrival based localization. Due to the joint spatial sparsity of the underlying problem the amount of exchanged samples can be reduced by applying the compressed sensing methodology. Furthermore, our algorithm provides a direct way of estimating the location, avoiding the necessity of solving a system of nonlinear hyperbolic equations. The performance of the proposed method and the impact on the estimation accuracy are evaluated based on simulations.**

***Keywords*—TDOA, Positioning, Compressed Sensing, Spatial Sparsity, Joint Sparsity, Block Sparsity**

I. Introduction

Passive localization of a signal source can be achieved by means of different methods and algorithms. Such methods usually rely on time, frequency or signal strength related parameters or a combination thereof. By jointly processing a multitude of received signals in a distributed system, spatial information can be extracted from the signal parameters.

For the case of missing synchronization between transmitters and receivers, methods using the angle or direction of arrival can be used. Well known algorithms from this class include [1] and [2] among others. Another possibility is to apply difference based method such as time-, frequency- [3], or power-(difference)-of-arrival [4].

More recently another step of innovation also for localization methods was enabled by the introduction of compressed sensing (CS). The concept of CS [5], [6] has introduced a new paradigm for the sampling of signals that are sparse in a certain domain. It has been shown that under certain conditions concerning the involved matrices, the so called restricted isometry property [7], it is possible to design a system that subsamples the signal with respect to the Nyquist-rate while still being able to fully reconstruct it with a very high probability. CS has led to new approaches of localization where it is assumed that the location of the object to be observed and the location of the observers are sparsely distributed in the spatial domain as described in [8]–[10]. By finding an appropriate formulation that physically relies on the received signal strength of the signal, the localization problem is then solved using algorithms that are based on $\ell 1$-norm minimization.

In contrast, the authors of [11], [12] make use of the time difference of arrival (TDOA) of the signals assuming sparsity, while in [13] additionally the doppler shift is used to obtain a direct location estimate. However, these papers do not investigate the impact of the number of recorded samples on the estimation performance. This is important due to the fact that systems with TDOA based localization using radio waves are usually distributed, with widely separated receiver nodes. In [14] it is described how a larger distance between observing nodes simplifies the TDOA estimation and decreases the localization error. However, in practice large distances impose some limitations on the data rate between the nodes. For example, it might be necessary to use locally available landline, cellular network or other types of small data rate wireless links to connect the sensor network. On the other hand, TDOA estimation methods require wide bandwidths in order to increase accuracy. This requirement in conjunction with low data rate links may severely restrict the performance of the system or may even render it useless.

To overcome this problem, we proposes a CS based method for localization based on TDOA that particularly aims for a reduction of the number of samples that need to be transmitted between the distributed nodes. More specifically, the reduction can be achieved by the choice of the CS measurement matrix at the cost of estimation performance. As the approach determines the transmitter location directly based on the received signals, it avoids solving nonlinear hyperbolic equations that occur in the problem, as described in [15]–[17]. In [18] a group of receive antenna elements performs direction finding by applying CS. This work describes how the signal at a single antenna element is sampled at Nyquist-rate, yielding a reference signal, while the remaining elements are subsampled following a CS based approach. A distributed (multi-static) active radar system described in [19] performs non-coherent combining of the received signals in order to obtain joint sparsity. Following these ideas, we propose to use one receiver in the distributed system as a reference and derive a novel

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approach for the application of CS to TDOA based location estimation.

The paper is structured as follows. Section II introduces a system model and basic algorithms for TDOA estimation. Sections III explains the application of CS to the problem. In Section IV we provide numerical simulation results demonstrating the impact of the CS method on the estimation performance and conclude the paper in Section V.

II. System Model

A typical TDOA based localization system consists of $R \geq 3$ observing receivers for the estimation of $Q \geq 1$ transmitter locations in a *two*-dimensional space; see Fig. 1 for the case where $R = 4$, $Q = 2$. Each pair of nodes needs to be synchronized by a common clock source and be capable of exchanging measurement data. By cooperatively processing the received signal, a hyperbola curve corresponding to the potential locations of an emitter can be determined for each receiver node pair based on the estimated TDOAs [20]. The intersection point of the curves yields the emitter location.

The TDOA for transmitter location $\mathbf{x}_j$ and receiver locations $\mathbf{z}_k$ and $\mathbf{z}_l$ can be geometrically determined as

$$\Delta(\mathbf{x}_j, \mathbf{z}_k, \mathbf{z}_l) = \frac{1}{c}\|\mathbf{z}_k - \mathbf{x}_j\|_2 - \frac{1}{c}\|\mathbf{z}_l - \mathbf{x}_j\|_2, \quad (1)$$

where c denotes the speed of light. To estimate the TDOA we introduce the following model. We denote the emitted signal with $s_q(t)$ and consider the signals $y_r(t)$ received at different nodes. The received signal at the r-th receiver for Q transmitters with channel coefficients $h_{q,r}$ adds up to

$$y_r(t) = \sum_{q=1}^{Q} h_{q,r} s_q(t - \tau_{q,r}) + w_r(t),$$

where $w_r(t)$ is assumed to be a realization of a white Gaussian noise process and $\tau_{q,r}$ stands for the delay which is related to the free-space propagation distance between transmitter q and receiver r. Further, we assume the transmitted signals to be mutually uncorrelated.

An overview of important symbols used in the description of the system model can be found in Table I.

TABLE I. List of used symbols in system description

Notation	Description
$\mathbf{x}_j$	transmitter location
$\mathbf{z}_k, \mathbf{z}_l$	receiver locations
Q	number of transmitters
R	number of receivers
$s_q(t)$	transmitted signal of transmitter q
$y_r(t)$	received signal at receiver r
$\tau_{q,r}$	time delay between transmitter q and receiver r
Δ	time difference of arrival
$h_{q,r}$	channel coefficients
$w_r(t)$	receiver noise
$\mathbf{y}_r$	sampled signal vector
$\bar{\mathbf{y}}_{ref}$	normalized sampled reference signal vector
N	number of uncompressed samples
K	number of location bins on the grid

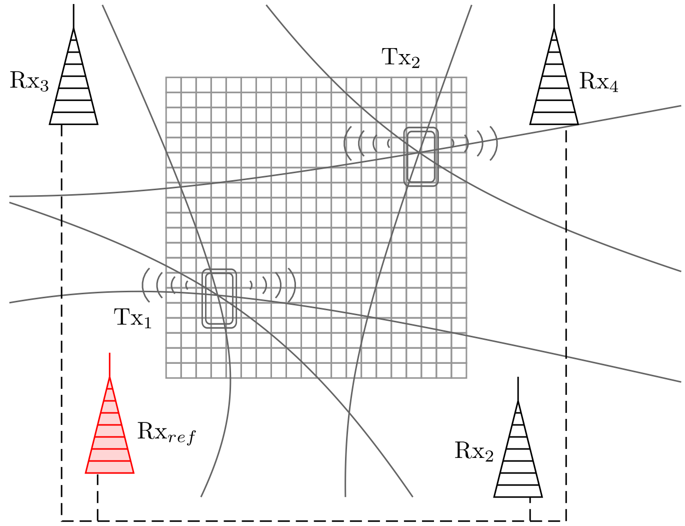

Fig. 1. TDOA based localization system with reference receiver marked in red.

III. Location Estimation

For the sake of simplicity we omit the derivation of an equivalent baseband model and assume the vector $\mathbf{y}_r(\gamma)$ to contain an ideally low pass filtered, Nyquist sampled and delayed version of the received signal:

$$\mathbf{y}_r(\gamma) = (y_r(t_0 - \gamma), \ldots, y_r(t_{N-1} - \gamma))^{\mathrm{T}}$$

Inspired by [18], we select one receiver, e.g., $r = 1$ as a reference and denote it's output as $\mathbf{y}_{ref}(\gamma)$. We set

$$\bar{\mathbf{y}}_{ref}(\gamma) = \mathbf{y}_{ref}(\gamma)/\|\mathbf{y}_{ref}(\gamma)\|_2$$

to denote the normalized version of $\mathbf{y}_{ref}(\gamma)$. Since at the remaining receivers $r = 2, \ldots, R$ only a fraction of the samples is processed (see Section III-B), we term them *CS receivers*.

We define an estimator $\tilde{\mathbf{y}}_r$ for the received signal $\mathbf{y}_r$ at a CS receiver:

$$\tilde{\mathbf{y}}_r = \mathbf{\Psi}_r \mathbf{b}_r\,. \quad (2)$$

Here, the $N \times K$ matrix $\mathbf{\Psi}_r$ contains time shifted versions of the normalized reference signal vector with time shifts $\gamma_{r,1}, \ldots, \gamma_{r,K}$, i.e.,

$$\mathbf{\Psi}_r = [\bar{\mathbf{y}}_{ref}(\gamma_{r,1}), \ldots, \bar{\mathbf{y}}_{ref}(\gamma_{r,K})], \quad (3)$$

and $\mathbf{b}_r$ is a vector with sparse support corresponding to the TDOAs of the transmitters' signals between receiver r and the reference receiver. If only one transmitter is active and no noise is present in the system, then the estimator $\tilde{\mathbf{y}}_r$ matches $\mathbf{y}_r$ if the nonzero entry of $\mathbf{b}_r$ contains the quotient of channel coefficients $h_{1,r}/h_{1,ref}$. In general this does not hold due to noise and the different combinations of time shifts between the transmitter and receiver locations. Nevertheless, given the received signal $\mathbf{y}_r$, one can try to reconstruct $\mathbf{b}_r$ and thus, in particular the nonzero entries in $\mathbf{b}_r$ yielding TDOAs corresponding to locations.

It should be emphasized that a single TDOA does not correspond to a unique transmitter location. In order to resolve this ambiguity and be able to directly determine the location, further restructuring of the problem is necessary in order to reveal the joint sparsity between the different receiver pairs.

A. Joint Sparsity

To localize the transmitters we introduce a discrete grid $\mathcal{G}$ in the two-dimensional plane containing potential location bins:

$$\mathcal{G} = \{\mu \cdot (u, v) \mid u \in \{1, \dots, L_1\},\, v \in \{1, \dots, L_2\}\},$$

where μ is a resolution parameter. This leads to a total number of $K = L_1 L_2$ bins. Assuming that $\mathbf{x}_1, \dots, \mathbf{x}_K$ is an enumeration of all bins in $\mathcal{G}$ and the locations of the reference receiver and the CS receivers are denoted as $\mathbf{z}_r$ and $\mathbf{z}_{ref}$, we can define the time shifts in (3) using (1) as

$$\gamma_{r,k} = \Delta(\mathbf{x}_k, \mathbf{z}_r, \mathbf{z}_{ref}), \quad k = 1, \dots, K. \tag{4}$$

This guarantees, that for all CS receivers the mapping from columns of $\mathbf{\Psi}_r$ to location bins is the same. We then observe that the presence of transmitters in certain location bins yields a joint sparsity pattern for all vectors $\mathbf{b}_r$. Such type of structured sparsity has also been studied in [21].

B. Compressed Sensing

The joint sparsity makes it feasible to apply ideas from CS and therefore estimate all vectors $\mathbf{b}_r$ recording only a subsampled signal $\hat{\mathbf{y}}_r$. We consider reduced versions of the received signal $\mathbf{y}_r$ and the estimator $\tilde{\mathbf{y}}_r$ by introducing

$$\hat{\mathbf{y}}_r = \mathbf{\Phi}_r \mathbf{y}_r, \tag{5}$$

$$\check{\mathbf{y}}_r = \mathbf{\Phi}_r \tilde{\mathbf{y}}_r = \underbrace{\mathbf{\Phi}_r \mathbf{\Psi}_r}_{\mathbf{A}_r} \mathbf{b}_r = \mathbf{A}_r \mathbf{b}_r, \tag{6}$$

with the $m \times N$ matrix $\mathbf{\Phi}_r$ which is obtained from the $N \times N$ identity matrix by randomly selecting m rows. This means that each CS receiver only transmits $m < N$ samples to the reference receiver instead of N samples taken at Nyquist-rate. Nevertheless, CS theory suggests that the sparsity ensures that $\mathbf{b}_r$ can be recovered. In fact, for the case of one active transmitter the recovery of $\mathbf{b}_r$ from $\hat{\mathbf{y}}_r$ reduces to a classical CS problem. The latter can be solved by ℓ_0-minimization searching for a solution with a low number of non zero elements which, however, is known to be NP-hard. It has been shown that under additional assumptions on the matrix $\mathbf{A}_r$ the ℓ_0-minimization can be relaxed by ℓ_1-minimization [22]. In our case we expect that ℓ_1-minimization similarly promotes sparse solutions and consider the relaxation

$$\text{minimize } \|\mathbf{b}_r\|_1 \quad \text{subject to} \quad \|\check{\mathbf{y}}_r - \hat{\mathbf{y}}_r\|_2^2 \leq \varepsilon_r, \tag{7}$$

where ϵ_r has to be adjusted to the level of noise being present. This relaxation can be solved using well known linear and convex optimization methods. For fast numerical implementations, a greedy heuristic called orthogonal matching pursuit (OMP) [23] can be used (see Section III-C).

In case that the time shifts in (3) are multiples of the sampling rate, the resulting matrices $\mathbf{A}_r$ are closely related to so-called *partial random circulant matrices*. There exists a comprehensive theory for these matrices showing that they are feasible for the CS approach [24] and, indeed, the necessary number of measurements for sparse recovery basically grows linearly with the level of sparsity.

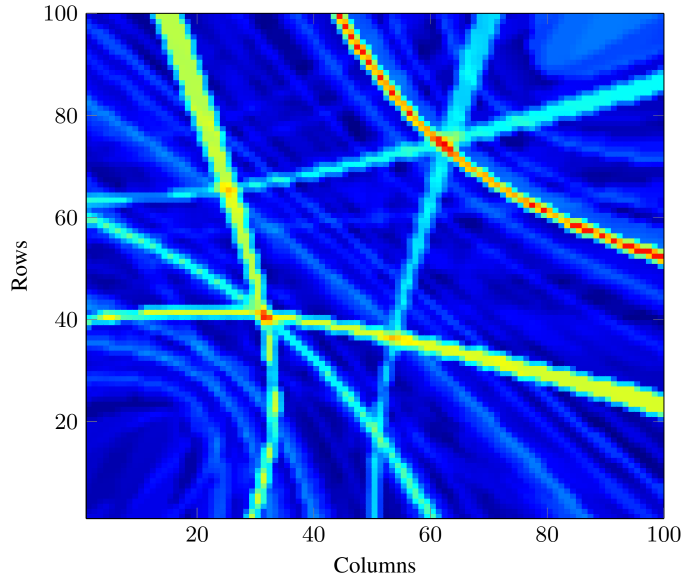

Fig. 2. Example of matrix form of the combined cross correlations $\boldsymbol{\beta}^*$, blue colors denote low and red colors denote high values

In our case, the matrices $\mathbf{A}_r$ are determined by (4) and are less favorable for CS. This is due to the fact that different location bins might result in very close TDOAs and, hence, the columns of $\mathbf{A}_r$ cannot be distinguished, anymore. Nevertheless, by exploiting the joint sparsity in a modified OMP algorithm, this drawback can be overcome.

C. Algorithm

The standard OMP algorithm iteratively builds up an estimate for the unknown support set P. If P has been estimated at the $(i-1)$-th iteration, one uses the residual $\hat{\mathbf{r}}_{r,i}$ of the least squares approximation of $\hat{\mathbf{y}}_r$ by $\mathbf{A}_r \mathbf{b}_r$, where $\text{supp}(\mathbf{b}_r) \subset P$, to choose an update for P. This is achieved by selecting the index of an entry of the correlation vector $\mathbf{A}_r^H \hat{\mathbf{r}}_{r,i-1}$ with maximal norm.

For the jointly sparse case, the basic idea is to perform

Algorithm 1 Localization Algorithm

// Initialize:
$\hat{\mathbf{r}}_{r,0} \leftarrow \hat{\mathbf{y}}_r$ ▷ (5)
$P \leftarrow \emptyset$
// for i-th iteration (beginning with $i = 1$):
while stopping criterion is not met **do**
  // Calculate correlations $\boldsymbol{\beta}_r$ and combine in $\boldsymbol{\beta}^*$:
  $\boldsymbol{\beta}_r \leftarrow \mathbf{A}_r^H \hat{\mathbf{r}}_{r,i-1}$
  $\boldsymbol{\beta}^*[k] \leftarrow \sum_{r=2}^{R} |\boldsymbol{\beta}_r[k]|^2$
  // Estimate a new location bin and add it to the set:
  $p_i \leftarrow \text{argmax}_{k \in \{1,\dots,K\}} \boldsymbol{\beta}^*[k]$
  $P \leftarrow P \cup \{p_i\}$
  // Calculate new residuals:
  $\mathbf{b}_r^* \leftarrow \text{argmin}_{\text{supp}(\mathbf{b}_r) \subset P} \|\hat{\mathbf{y}}_r - \mathbf{A}_r \mathbf{b}_r\|_2^2$ ▷ (7)
  $\hat{\mathbf{r}}_{r,i} \leftarrow \hat{\mathbf{y}}_r - \mathbf{A}_r \mathbf{b}_r^*$
end while

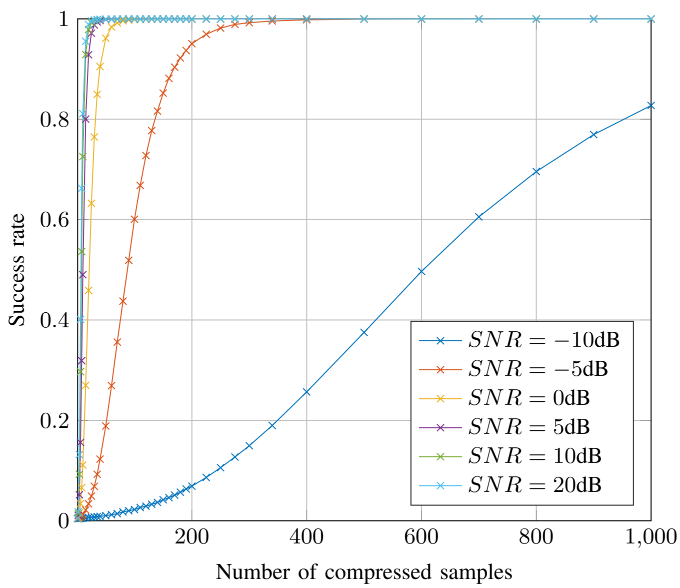

Fig. 3. Success rate of localization for different number of compressed samples with 1 transmitter and 4 receivers

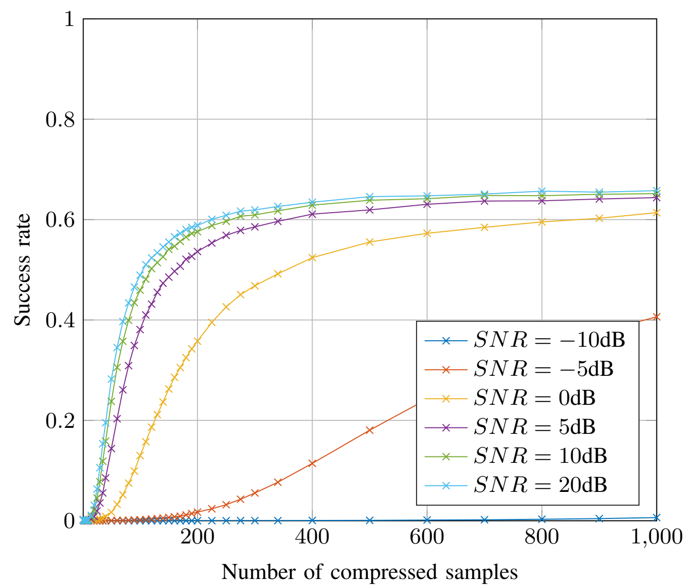

Fig. 4. Success rate of localization for different number of compressed samples with 2 transmitters and 4 receivers

a non-coherent combining of the entries of the correlation at each receiver to obtain a measure of how likely a transmitter is present at a given location bin. This leads to a modified version of the OMP algorithm. Such modifications, in the presence of structured sparsity ("block" or "joint" sparsity), have already been studied by different authors [21], [25]. The resulting algorithm is depicted in Algorithm 1.

After termination, the set P contains estimated location bins. A stopping criterion could either be to stop if the iteration count has reached the number of transmitters (if previously known) or if the norm of the current residual is smaller than a given threshold.

Figure 2 shows a matrix version of the combined cross correlation vector $\boldsymbol{\beta}^*$ for the case of $R = 4$ receivers and $Q = 2$ transmitters. Here, the matrix entry (u, v) is equal to the entry $\boldsymbol{\beta}^*[k]$, with k chosen such that $\mathbf{x}_k = \mu \cdot (u, v)$.

IV. Results

To confirm the theoretical considerations and to evaluate the performance of the proposed algorithm, numerical simulations of a two dimensional free-space radio wave propagation environment have been conducted as described in the following. A random BPSK modulated baseband signal with a sample rate of 100 MHz is generated for each transmitter location. In order to determine the received sum signals, the transmit signals are time-shifted and attenuated according to their free-space path loss between each pair of transmitter and receiver. The sum of signals at each receiver is then added with white Gaussian noise, while for simplicity it is assumed that the SNR of each receiver's output signal is identical.

The scenario is a $100\,\text{m} \times 100\,\text{m}$ square and the receivers are located around the border at $\mathbf{z}_{ref} = (0, 0), \mathbf{z}_2 = (0, 100), \mathbf{z}_3 = (100, 0), \mathbf{z}_4 = (100, 100), \mathbf{z}_5 = (0, 50), \mathbf{z}_6 = (50, 0), \mathbf{z}_7 = (50, 100), \mathbf{z}_8 = (100, 50)$. The transmitters are randomly placed in the region of interest, for each Monte Carlo iteration.

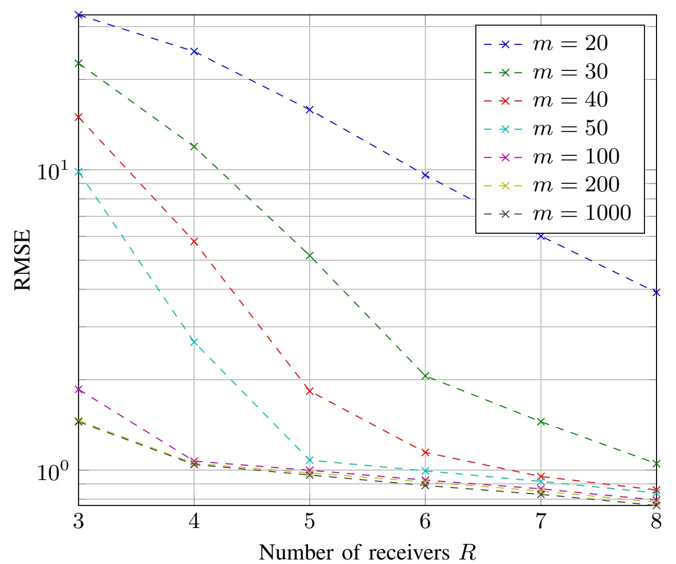

Fig. 5. Root mean squared error of true and estimated location for a single transmitter and 0 dB SNR

Further, the estimator grid dimension is set to cover the scenario with a resolution of one bin per square meter. In order to estimate the transmitter location, the reference receiver takes 1000 samples at Nyquist-rate and obtains compressed samples from the CS receivers. In the first experiment 1 transmitter and 4 receivers are active. The transmitter location is determined using a discrete uniform distribution placing it on the estimator grid in order to simplify the evaluation of the success rate. We perform the estimation algorithm for different numbers of compressed samples and determine the success rate as shown in Fig. 3. A location is estimated successfully if all TDOAs corresponding to the true and estimated bins are equal within a tolerance of one sample duration. We notice that for these parameters the estimator produces acceptable results up to a compression ratio of about 10.

A second experiment is performed with 4 receivers and 2 transmitters. As mentioned in Section III-B, for a single transmitter we may apply classical CS solutions to our problem. For the case of multiple transmitters at different locations we expect worse results. The corresponding success rate is depicted in Fig. 4, where a performance degradation can be observed. This is due to interference between the two transmitted signals that are added with a different relative delay at each receiver. Therefore, when using the signal of the reference receiver in the CS reconstruction, it remains inherently inaccurate. More specifically, we accumulate errors in the calculation of the residual. However, unless the transmitted signal waveforms are known at the reference receiver this situation is difficult to resolve.

The third experiment considers the number of participating receivers. Placement of the single transmitter is now performed using a uniform distribution to allow arbitrary locations. Varying the number of receivers, including the reference receiver, between 3 and 8 we determine the root-mean-square error (RMSE) between the true and estimated transmitter location as shown in Fig. 5. It is clearly visible how the error decreases for a higher number of CS receivers.

V. Conclusion

We have introduced a novel TDOA based method that exploits spatial sparsity of transmitter locations in order to apply compressed sensing to the location estimation. The method, using a reference receiver that samples at the Nyquist-rate and two or more compressed sensing receivers, helps to significantly reduce the amount of samples that have to be exchanged between the nodes of a TDOA based emitter localization system. Furthermore, it avoids the need of solving hyperbolic equations to obtain a location estimate. Simulation results indicate how the number of exchanged samples influences the success rate of the algorithm and demonstrate a high potential for savings in data exchange between the receivers. Due to the structure of the algorithm, additional compressed sensing receivers can be added in a straight forward way and are able to lower the estimation error of the transmitter locations.

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