Computers & Industrial Engineering 62 (2012) 870–879

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Inventory management in a two-echelon closed-loop supply chain with correlated demands and returns ☆

Subrata Mitra *

Indian Institute of Management Calcutta, Diamond Harbour Road, Joka, Kolkata 700 104, India

ARTICLE INFO

Article history:
Received 17 February 2011
Received in revised form 8 December 2011
Accepted 10 December 2011
Available online 17 December 2011

Keywords:
Inventory
Multi-echelon
Closed-loop supply chain
Product returns
Correlations

ABSTRACT

Reverse logistics or closed-loop supply chains where product returns are integrated with traditional forward supply chains have been one of the major topics of research since about the last one and a half decades. In this paper, we address the inventory management issue in closed-loop supply chains, and develop deterministic and stochastic models for a two-echelon system with correlated demands and returns under generalized cost structures. In particular, we address the following questions – Do closed-loop supply chains cost more than traditional forward supply chains? Does a higher rate of return always translate into lower demand variability and hence lower expected costs? What is the relationship between expected costs and correlations between demands and returns? Models developed and numerical examples shown in the paper reveal that although a higher rate of return and a higher correlation between demand and return reduce the variability of net demand, it may not necessarily lead to cost savings; rather the movement of costs will depend on the values of system parameters. We also quantify the cost savings in case the actual demand and return information is available at the time of decision-making. We conclude the paper by providing managerial implications and directions for future research.

1. Introduction

Recently, since about the last one and a half decades, a lot of research interest has been shown in reverse logistics (Alamri, 2011; Chan, Yin, & Chan, 2010; Dowlatshahi, 2010a, 2010b; El-Sayed, Afia, & El-Kharbotly, 2010; Fleischmann et al., 1997; Kim, Song, Kim, & Jeong, 2006; Lee, Gen, & Rhee, 2009; Mutha & Pokharel, 2009; Ravi, Shankar, & Tiwari, 2005, 2008; Rubio, Chamorro, & Miranda, 2008; Tsai & Hung, 2009; Weeks, Gao, Alidaeec, & Rana, 2010), closed-loop supply chains (Guide & Van Wassenhove, 2009; Huang, Yan, & Qiu, 2009; Kannan, Haq, & Devika, 2009; Min, Ko, & Ko, 2006; Morana & Seuring, 2007; Neto, Walther, Bloemhof, Van Nunen, & Spengler, 2010), sustainable supply chains (Field & Sroufe, 2007; Geldermann, Treitz, & Rentz, 2007; Linton, Klassen, & Vaidyanathan, 2007; Vachon & Klassen, 2007), and sustainable product design/manufacturing/operations (El Saadany & Jaber, 2010, 2011; Gungor & Gupta, 1999; Jaber & El Saadany, 2009, 2011; Jaber & Rosen, 2008; Jayaraman, 2006; Kleindorfer, Singhal, & Van Wassenhove, 2005; Konstantaras, Skouri, & Jaber, 2010; Nagel & Meyer, 1999; Rubio & Corominas, 2008; Tseng, Divinagracia, & Divinagracia, 2009; Yan, Chen, & Chang, 2009). There has been a recent review of the quantitative models for inventory and production planning in closed-loop supply chains (Akcali & Cetinkaya, 2011). The literature on production, manufacture and waste disposal models assumes that an item can be recovered for an indefinite number of times. This is not true, in general. See, for example, El Saadany, Jaber, and Bonney (in press), who address this limitation. A number of edited books have been published on these subjects (Dekker, Fleischmann, Inderfurth, & Van Wassenhove, 2004; Dyckhoff, Lackes, & Reese, 2003; Flapper, Van Nunen, & Van Wassenhove, 2005; Guide & Van Wassenhove, 2003). Also, many special issues of journals have been devoted to these topics (Interfaces (30 (3), 2000, 33 (6), 2003); California Management Review 46 (2), 2004; Production and Operations Management 15 (3 and 4), 2006; Journal of Operations Management 25 (6), 2007; Computers & Operations Research 34 (2), 2007; International Journal of Production Research 45 (18 and 19), 2007; International Journal of Production Economics 111, 2008). Although known by different names, the basic idea behind all these is to integrate product returns with the traditional forward supply chain, which may involve from collection of returns and design of reverse logistics networks to disposal, product recovery, production scheduling and inventory management with returns, new product design and remarketing of recovered products (Guide & Van Wassenhove, 2002). Handling end-of-use or end-of-life product returns by manufacturers has been made obligatory by many developed countries in North America and Europe to prevent wastage and pollution. Therefore, it has become

☆ This manuscript was processed by Area Editor Mohamad Y. Jaber.

* Tel.: +91 33 24678300; fax: +91 33 24678307.
E-mail address: subrata@iimcal.ac.in

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doi:10.1016/j.cie.2011.12.008

imperative for manufacturers to design products with recyclable components as much as possible so that they can extract the maximum economic value from product returns. The appropriate recovery operation will, however, depend on the quality of returns. Thierry, Salomon, Van Nunen, and Van Wassenhove (1995) classify product recovery operations into five categories – repair, refurbishing, remanufacturing, cannibalization and recycling – based on the quality and degree of disassembly of returns. Among these, the most prominent recovery operation is remanufacturing, which is particularly useful for products with long technological cycles such as automobile engines, machine tools and photocopiers. The size of the remanufacturing industry in the US is estimated between $40 and $53 billion. The cost of remanufacturing is generally 40–60% (sometimes as low as 20%) of the cost of manufacturing a new product. However, a remanufactured product is considered to be 'as good as new' and sold often with the same warranty but at discounted (as low as 50%) prices either through the same channel as a manufactured product or through a separate channel (Souza, 2009; Thierry et al., 1995). This provides manufacturers with an opportunity to turn in profits from returns and simultaneously build corporate image by projecting "green" and environment-friendly supply chains.

Inventory management in closed-loop supply chains is much more complicated than in traditional forward supply chains since returns are more uncertain than demands in terms of quantity, quality and timing (Guide, Jayaraman, Srivastava, & Benton, 2000), and also valuation and setting inventory holding costs of returns are not straightforward (Teunter, Van der Laan, & Inderfurth, 2000). In addition, correlation between demands and returns adds another dimension of complexity to such systems. It may vary from a perfect positive correlation for reparable items to a fair degree of correlation for short life-cycle products such as reusable containers (Kelle & Silver, 1989) and single-use cameras (Toktay, Wein, & Zenios, 2000) to almost no correlation for long life-cycle products such as durables (e.g. electrical and electronic equipment). In the literature, it is usually assumed that demands and returns are independent (Fleischmann & Kuik, 2003; Fleischmann et al., 1997; Mahadevan, Pyke, & Fleischmann, 2003). The extension of single-echelon closed-loop supply chains to multi-echelons involving multiple levels of inventory locations further complicates such systems. Recently, Yuan and Gao (2010) has developed an inventory-control model for a closed-loop supply chain with a retailer, a manufacturer, a supplier and a collector for deterministic demand and return rates allowing no shortages. Only a handful of references that deal with stochastic multi-echelon closed-loop supply chains are available in the literature (DeCroix, 2006; Korugan & Gupta, 1998; Minner, 2001; Muckstadt & Isaac, 1981; Savaskan, Bhattacharya, & Van Wassenhove, 2004). Although these papers make valuable contributions to the literature, they make a number of assumptions, including the independence between demands and returns and the non-existence or non-relevance of some of the costs – set-up, inventory holding and shortage – at some or all of the stocking points, for the purpose of tractability. Mitra (2009) does address the above cost issue; however, independence between demands and returns is assumed in the paper.

The present paper considers a two-echelon closed-loop supply chain with set-up and inventory holding costs at all the stages and shortage costs at the stages stocking serviceable inventory. Also, demands and returns may be correlated (for optimal models using genetic algorithms for two-echelon inventory systems with correlated demands, readers may refer to Xiong and Sun (2010)). We develop deterministic and stochastic models for such a system. For the stochastic model, we assume that the system is under periodic review. In particular, we address the following questions in the paper:

- Do closed-loop supply chains cost more than traditional forward supply chains? In other words, does the incorporation of returns into the forward supply chain increase the cost of the supply chain?
- Does a higher rate of return translate into lower demand variability? Does it mean lower expected costs of the systems under every situation?
- What is the relationship between the expected costs of the system and correlations between demands and returns? Do higher correlations necessarily mean lower expected costs?
- Given that the availability of information reduces expected costs (Ketzenberg, Van der Laan, & Teunter, 2006), how can the savings, in case the actual demand and return information is available, be quantified so that the savings can be traded off against the cost of acquisition of information?

The paper is organized as follows. Sections 2 and 3 present the problem description and model formulations, both deterministic and stochastic, respectively. Section 4 provides numerical examples and sensitivity analyses. The case when the actual demand and return information is available is presented in Section 5. Finally, Sections 6 and 7 present the managerial implications and directions for future research, respectively.

2. Problem description

In this paper, we consider a two-echelon inventory system with returns. Returns are remanufactured, which are 'as good as new' after recovery (100% recovery rate is assumed) and are interchangeable with new items that are procured from an outside supplier to meet customer demand from the serviceable stock. It is assumed that a remanufactured item and a new item are of the same value, and as such they have the same inventory holding costs. It is also assumed that a returned item awaiting recovery is of lower value than an item in the serviceable stock, and hence has a lower inventory holding cost. The time to remanufacture a batch of returns is assumed to be insignificant compared to the time to procure new items from the outside supplier at the corresponding stage. As such, the remanufacturing order is initiated and realized at the same instant as replenishments from the outside supplier are realized at the corresponding stage. The simultaneous replenishment of remanufactured and new items is an assumption in the problem, which leads to the same cycle length at the corresponding stages. However, in general, the cycle lengths need not be the same in case of alternate replenishments of remanufactured and new items, which, of course, is beyond the scope of the system under consideration in the paper.

We consider set-up costs and inventory holding costs at all the stages and shortage costs at the stages containing serviceable stock, and develop deterministic and stochastic models for the system. It may be noted here that in a closed-loop supply chain, there exist many other components of cost such as collection, transportation, inspection, sorting, recovery, disposal and remarketing. However, in this paper, we have analyzed the cost structure of the system from the inventory management point of view, and restricted to set-up, inventory holding and shortage costs (for stochastic models only). The objective is to determine the values of the inventory policy variables (order quantities in case of the deterministic model, and review periods and order-up-to levels in case of the stochastic model) at all the stages that minimize the (expected) total costs of the system. In the stochastic model, it is further assumed that while demands and returns – both Normally distributed – in different periods are i.i.d., demand and return in a given period may be correlated. Also, every return is associated with a demand for the item; in addition, there are fresh customer demands. Demands generated against returns are not immediately fulfilled from the serviceable stock. Neither are they backordered. Rather, they are fulfilled in the next cycle when remanufactured and procured items replenish the serviceable stock. Freshly generated demands, on the other hand, are instantaneously fulfilled from the serviceable stock if there is adequate inventory; otherwise, they are backordered and are fulfilled when replenishments arrive. Photocopiers, which are an ideal item for remanufacturing (Thierry et al., 1995), taken on lease, may be cited as an example in this context. Machines that break down are returned to the dealer for remanufacturing. The concerned customers are quoted a service time until which they have to wait for the delivery of their machines. The delivered machines may be the same ones after remanufacturing or new ones procured by the dealer from the manufacturer. It hardly matters to the customers since the machines have been taken on lease anyway and they should not have any issue as long as the machines perform the required functions to their satisfaction (Souza, 2009). However, if there are fresh demands for photocopiers, they have to be met immediately if there is adequate stock. Otherwise, they have to be backordered and fulfilled as soon as replenishments arrive in the next cycle. In this case also, fresh demands can be met by either remanufactured or new machines from the serviceable stock since, as mentioned before, remanufactured machines are 'as good as new'.

More descriptions of the system are provided in the next section on model formulation.

3. Model formulation

In this section, we first develop an analytic model for a deterministic two-echelon inventory system comprising three stages. Stages 1 and 2 belong to the lower echelon (echelon 1) and higher echelon (echelon 2), respectively, and stock serviceable inventory. On the other hand, Stage 3, which remanufactures returns, belongs to echelon 1, supplementing the serviceable stock at the corresponding echelon. We derive the total cost (*TC*) of the system.

Subsequently, we consider the two-echelon inventory system described above under stochastic demands and returns. The system follows a periodic review inventory control policy where at the beginning of every review period, if the inventory position (on hand plus on order minus backorder) is less than the order-up-to level, an order is placed to bring the inventory position to the order-up-to level. We could also consider a continuous review inventory control policy. However, a continuous review policy is more appropriate for fast moving items. Since, in Section 2, we took the example of photocopiers, which are not so fast moving, we felt a periodic review policy would have been more appropriate for modelling the system. However, the application of a continuous review policy for such systems would be an interesting direction for future research. We consider two situations based on whether any shortage at the higher echelon is fulfilled by emergency shipments or allocated to the lower echelon. We develop expected total cost (ETC) models for the system under consideration.

3.1. Deterministic model

This model considers deterministic, stationary and uniform (uniformly occurring over time) demand and return rates. Since demand and return rates are stationary and uniform, we develop infinite-horizon, average-cost inventory models for the systems under consideration. Had demand and return rates been non-stationary and lumpy, and the length of the planning horizon been finite, we would have applied the Wagner–Whitin dynamic programming algorithm to determine the optimal order quantities in different periods. Also, the Wagner–Whitin algorithm assesses holding costs based on residual inventory levels at the ends of the periods, does not allow shortages and is applicable for solving deterministic problems only (Silver, Pyke, & Peterson, 1998, pp. 201–209). However, in this paper, holding costs are assessed based on the time-average inventory levels and shortages are allowed for the stochastic version of the problem. The results of the deterministic model are used to derive the policy parameters for the stochastic models described in the subsequent sections.

The following notations have been used in model formulation.

- A_i Set-up cost at Stage *i* (i = 1, 2, 3)
- h_i Inventory holding cost per unit per period at Stage *i* (i = 1, 2, 3)
- μ Demand per period
- r Fraction of demand returned per period ($0 < r < 1$)
- Q Batch size/order quantity
- T Cycle length
- n Integer multiple

The system comprises three stages. Stage 1 in echelon 1 faces customer demand and Stage 2 in echelon 2 gets supplies from an outside supplier and caters to the demand of Stage 1. Stage 3, which recovers returns, belongs to echelon 1 and supplements the serviceable stock at Stage 1. The system is represented by Fig. 1.

It is apparent from Fig. 1 that if Q is the batch size at Stage 1, the cycle length is Q/μ. The recovered quantity at Stage 3 in Q/μ is rQ. So, the order quantity with Stage 2 is $(1-r)Q$. The order quantity at Stage 2 with the outside supplier is $n(1-r)Q$, where n is an integer. Fig. 2 shows the inventory diagrams. It may be observed from Fig. 2 that Stages 1 and 3 have the same cycle length since it is assumed in Section 2 that replenishments of new items from Stage 2 and remanufactured items from Stage 3 at Stage 1 are synchronized and they arrive at the same point in time. However, it is also mentioned in Section 2 that, in general, had replenishments from Stages 2 and 3 arrived alternately at Stage 1, the cycle lengths at Stages 1 and 3 would not have been necessarily the same.

From Fig. 2, it is apparent that the number of set-ups at Stages 1, 2 and 3 will be μ/Q, μ/nQ and μ/Q, respectively. The average on-hand inventories at Stages 1 and 3 are $Q/2$ and $rQ/2$, respectively.

The expression for the average on-hand inventory at Stage 2 can be derived as follows:

- Area of the triangle in one cycle = $\frac{1}{2}n(1-r)Q \times \frac{nQ}{\mu}$.
- Area of one small triangle = $\frac{1}{2}(1-r)Q \times \frac{Q}{\mu}$. There are n such triangles in one cycle.
- Hence, the area under the on-hand inventory plot in one cycle =

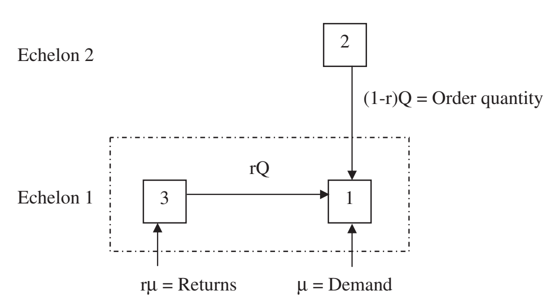

Fig. 1. Set-up of the inventory system.

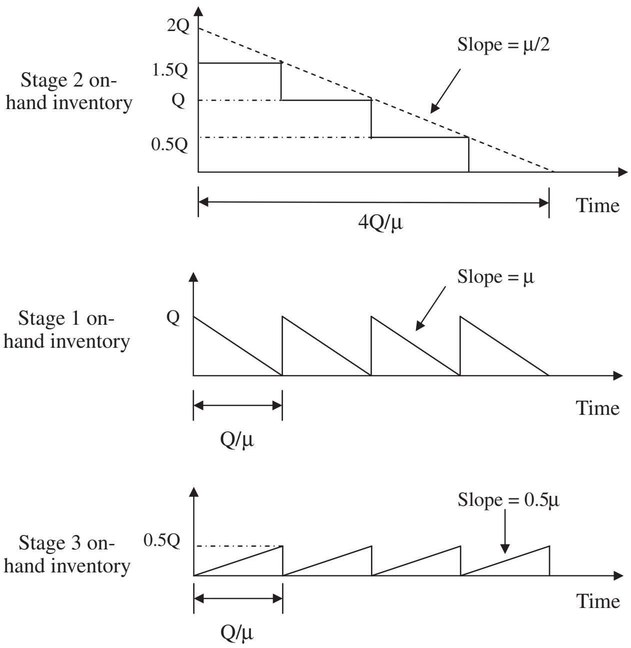

Fig. 2. Echelon and on-hand inventories at Stages 1, 2 and 3 ($n = 4$, $r = 0.5$).

$$\frac{1}{2}n(1-r)Q \times \frac{nQ}{\mu} - n \times \frac{1}{2}(1-r)Q \times \frac{Q}{\mu}$$
$$= \frac{1}{2}n(n-1)(1-r)\frac{Q^2}{\mu}$$

- There are μ/nQ cycles in one time period. Hence, Stage 2 average inventory =

$$\frac{1}{2}n(n-1)(1-r)\frac{Q^2}{\mu} \times \frac{\mu}{nQ} = \frac{1}{2}(n-1)(1-r)Q.$$

Following are the expressions for $TC(n,Q)$, and optimal TC, Q, T and n.

$$TC(n,Q) = \frac{A_1\mu}{Q} + \frac{A_2\mu}{nQ} + \frac{A_3\mu}{Q} + \frac{1}{2}Qh_1 + \frac{1}{2}(n-1)(1-r)Qh_2 + \frac{1}{2}rQh_3$$
$$= \frac{\left(A_1 + \frac{A_2}{n} + A_3\right)\mu}{Q} + \frac{1}{2}Q[h_1 + (n-1)(1-r)h_2 + rh_3]$$

From the Economic Order Quantity (EOQ) formula

$$Q^*(n) = \sqrt{\frac{2\left(A_1 + \frac{A_2}{n} + A_3\right)\mu}{h_1 + (n-1)(1-r)h_2 + rh_3}} \quad \text{and}$$

$$T^*(n) = \sqrt{\frac{2\left(A_1 + \frac{A_2}{n} + A_3\right)}{\mu[h_1 + (n-1)(1-r)h_2 + rh_3]}}$$

$$TC^*(n) = \sqrt{2\left(A_1 + \frac{A_2}{n} + A_3\right)\mu[h_1 + (n-1)(1-r)h_2 + rh_3]}$$

Also,

$$\frac{dTC^*(n)}{dn} = 0 \Rightarrow n^* = \sqrt{\frac{A_2[h_1 - (1-r)h_2 + rh_3]}{(A_1 + A_3)(1-r)h_2}} \qquad (1)$$

We observe from the above deterministic model (1) that when $r = 0$, $A_3 = 0$ and $h_3 = 0$, the closed-form expressions for n^*, Q^* and T^* converge with the respective expressions for a traditional two-stage inventory system (Silver et al., 1998, p. 480). It may also be noted that in the above model, $TC^*(n)$ has been differentiated with respect to n, relaxing the constraint that n has to be an integer. If n turns out to be a non-integer, the values of $TC^*(n)$ will have to be computed for the immediately higher and lower integers surrounding n, and the integer for which $TC^*(n)$ gives the lower value will have to be taken as the optimal value of n. This will ensure the optimality of n since $TC^*(n)$ is a convex function in n (Silver et al., 1998, pp. 517–518).

3.2. Stochastic model

These models consider stochastic, stationary and uniform demand and return rates.

The following notations have been used in model formulation in addition to those described earlier.

S_i	Order-up-to level at Stage i ($i = 1, 2$)
k_i	Safety factor at Stage i (i = 1, 2)
l_i	Lead time at Stage i ($i = 1, 2$)
p_i	Shortage cost per unit at Stage i (i = 1, 2)
f_i	Density function of demand at Stage i (i = 1, 2)
μ	Mean demand per period
σ	Standard deviation of demand per period
γ	Standard deviation of random error per period
ϕ	PDF of the standard normal distribution
Φ	CDF of the standard normal distribution
COV	Covariance
COR	Correlation

Fig. 1 represents the set-up of the system. Suppose D ($\sim N(\mu,\sigma^2)$) and R are the random variables representing demand and return, respectively, per period where demands and returns in different periods are i.i.d. but for any given period, demand and return are correlated given by the relationship: $R = rD + \varepsilon$, ε (random error) $\sim N(0,\gamma^2)$. Therefore, $\mathrm{COV}(D,R) = \mathrm{COV}(D, rD + \varepsilon) = r\sigma^2$ assuming $\mathrm{COV}(D,\varepsilon) = 0$. Also, $\mathrm{COR}(D,R) = \mathrm{COV}(D,R)/(\sigma^2(r^2\sigma^2 + \gamma^2))^{1/2} = r\sigma^2/(\sigma^2(r^2\sigma^2 + \gamma^2))^{1/2}$. Substituting $\gamma^2 = mr^2\sigma^2$, where $m = \gamma^2/r^2\sigma^2$, we can write $\mathrm{COR}(D,R) = 1/(1 + m)^{1/2}$.

Stages 1 and 2 follow periodic review policies and there are n cycles at Stage 1 for every cycle at Stage 2, i.e. if review is done every T periods at Stage 1, at Stage 2, review is done every nT periods. Since the order quantity at Stage 1, i.e. the demand faced by Stage 2, in any period reflects the demand faced by Stage 1 in the immediately preceding period, ignoring the short supply from Stage 2, if any, in the previous period, the net demand per period faced by Stage 1, and Stage 2, after factoring in returns at Stage 1 can be expressed as D–R or $(1 - r)D - \varepsilon$ and it can be shown that D–$R \sim N((1 - r)\mu, (1 - r)^2\sigma^2 + \gamma^2)$. As mentioned in the section on problem description, demands generated against returns are not immediately satisfied from the serviceable stock. Neither are they backordered. The concerned customers are rather quoted a service time equal to the cycle time at Stage 1, T, and the demands are satisfied when the recovered returns replenish the serviceable stock in the next cycle. However, fresh demands are immediately satisfied from the serviceable stock, if available; otherwise, they are backordered and are satisfied as soon as replenishments arrive.

Therefore, the following expressions for the order-up-to levels can be written.

$$S_1 = (1-r)\mu(T+l_1) + k_1\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}$$

$$S_2 = (1-r)\mu nT + k_2\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{nT}$$

It may be noted that since demand, and return, occur uniformly at Stage 1, protection against uncertainty is required over both the review period and the lead time, which is reflected in the expression for the order-up-to level at Stage 1. However, since orders placed by Stage 1 are at discrete time intervals, demand faced by Stage 2 is lumpy and therefore coverage required for Stage 2 is only for the review period, which is why the lead time is missing from the expression for the order-up-to level at Stage 2 (Mitra & Chatterjee, 2004).

3.2.1. Emergency shipment

In case there is any short supply from Stage 2, the shortfall is made up by expediting deliveries to Stage 1 on an emergency basis. The premium for the expedited delivery reflects the shortage cost at Stage 2. In other words, there is no backordering at Stage 2, and it is equivalent to the lost sales case. To determine ETC per period for the system, the following expressions need to be derived.

- Expected total set-up cost per period at Stages 1, 2 and 3

$$= \frac{A_1}{T} + \frac{A_2}{nT} + \frac{A_3}{T}$$

- Expected total cycle stock cost per period at Stages 1, 2 and 3

$$= \frac{1}{2}\mu T h_1 + \frac{1}{2}(n-1)(1-r)\mu T h_2 + \frac{1}{2} r\mu T h_3$$

- Expected safety stock cost per period at Stage1

$$= h_1 \int_{-\infty}^{S_1} (S_1 - x_{T+l_1}) f_1(x_{T+l_1}) dx_{T+l_1}$$

where x_{T+l_1} represents net demand during $T+l_1$ periods

$$\approx h_1 k_1 \sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1} \quad \text{ignoring negative net demand and shortage at Stage 1.}$$

- Expected safety stock cost per period at Stage 2

$$= h_2 \int_{-\infty}^{S_2} (S_2 - x_{nT}) f_2(x_{nT}) dx_{nT} \quad \text{where } x_{nT} \text{ represents net demand during } nT \text{ periods}$$

$$\approx h_2\left[k_2\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{nT} + \sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{nT}\{\phi(k_2) - k_2 + k_2\Phi(k_2)\}\right] \quad \text{ignoring negative net demand at Stage 2.}$$

- Expected shortage cost per period at Stage 1

$$= \frac{p_1}{T}\int_{S_1}^{\infty} (x_{T+l_1} - S_1) f_1(x_{T+l_1}) dx_{T+l_1}$$

$$= \frac{p_1}{T}\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}\{\phi(k_1) - k_1 + k_1\Phi(k_1)\}$$

- Expected shortage cost per period at Stage 2

$$= \frac{p_2}{nT}\int_{S_2}^{\infty} (x_{nT} - S_2) f_2(x_{nT}) dx_{nT}$$

$$= \frac{p_2}{nT}\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{nT}\{\phi(k_2) - k_2 + k_2\Phi(k_2)\}$$

While the rest of the derivations are straightforward, for the last three derivations, readers are referred to Hadley and Whitin (1979, pp. 168–169) and Silver et al. (1998, pp. 720–723).

Therefore, the following expression for ETC per period can be written.

$$\begin{aligned} \text{ETC} = {} & \frac{A_1}{T} + \frac{A_2}{nT} + \frac{A_3}{T} + \frac{1}{2}\mu T h_1 + \frac{1}{2}(n-1)(1-r)\mu T h_2 \\ & + \frac{1}{2} r\mu T h_3 + h_1 k_1 \sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1} \\ & + h_2 k_2 \sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{nT} + \frac{p_1}{T}\sqrt{(1-r)^2\sigma^2+\gamma^2} \\ & \times \sqrt{T+l_1}\{\phi(k_1) - k_1 + k_1\Phi(k_1)\} + \left(\frac{p_2}{nT} + h_2\right) \\ & \times \sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{nT}\{\phi(k_2) - k_2 + k_2\Phi(k_2)\} \end{aligned} \quad (2)$$

The optimal values of T, n, k_1 and k_2 may be obtained from (2) by differentiating and iteratively solving four simultaneous equations. However, for simplicity, in this paper, we take the optimal values of T and n from the deterministic model (1) and solve for k_1 and k_2 from (2), which is known to work well in models with forward flows only. The extent of error introduced by this approximation is checked by simulation in the section on numerical examples. It can be shown that the optimal values of k_1 and k_2 may be obtained by solving the following two equations:

$$1 - \Phi(k_1) = \frac{h_1 T}{p_1} \quad \text{and} \quad 1 - \Phi(k_2) = \frac{h_2 nT}{p_2 + h_2 nT} \quad (3)$$

3.2.2. Allocation

In this case, if there is any shortage at Stage 2, the shortage quantity is allocated to Stage 1 and is also backordered at Stage 2. As such, there is no separate shortage cost at Stage 2; the effect of any shortage at Stage 2 is reflected in the possibility of added shortage costs at Stage 1. In the expression for ETC, the expected total set-up cost and cycle stock cost per period for Stages 1–3, and the expected safety stock cost per period at Stage 2 will remain the same as in (2). However, the expected safety stock cost and shortage cost per period at Stage 1 are derived as follows.

As Stage 1 will have n cycles for every cycle at Stage 2, it is assumed that the first $n-1$ cycles will face no short supply from Stage 2 whereas if there is any shortage at Stage 2, the shortage quantity will be reflected in the order-up-to level at Stage 1 in the last cycle. Therefore, for the first $n-1$ cycles, the expected safety stock and shortage quantity per period at Stage 1 will remain the same as in (2). However, in the last cycle, depending on whether there is a shortage at Stage 2 or not, there will be two expressions for the safety stock at Stage 1. Let x_{T+l_1} and y_{nT} represent net demands faced by Stage 1 in $T+l_1$ periods and Stage 2 in nT periods, respectively. The shortage quantity at Stage 2, provided that there is a shortage at that stage, is given by $y_{nT} - S_2$. Since any shortage at Stage 2 will get reflected in the order-up-to level at Stage 1, the effective order-up-to level at Stage 1 in case of a shortage at Stage 2 will be set at $S_1 - (y_{nT} - S_2)$ or $S_1 + S_2 - y_{nT}$. Therefore, for the last cycle, the following expression for the safety stock at Stage 1 can be written:

Safety stock at Stage 1 for the last cycle =

$$\int_{y_{nT}=-\infty}^{y_{nT}=S_2}\left[\int_{x_{T+l_1}=-\infty}^{x_{T+l_1}=S_1}(S_1 - x_{T+l_1}) f_1(x_{T+l_1}) dx_{T+l_1}\right] f_2(y_{nT}) dy_{nT}$$
$$+ \int_{y_{nT}=S_2}^{y_{nT}=\infty}\left[\int_{x_{T+l_1}=-\infty}^{x_{T+l_1}=S_1+S_2-y_{nT}}(S_1 + S_2 - x_{T+l_1} - y_{nT}) f_1(x_{T+l_1}) dx_{T+l_1}\right] f_2(y_{nT}) dy_{nT}$$

The above expression cannot be expressed in a closed form. Therefore, we make an approximation by substituting the actual shortage quantity at Stage 2, $y_{nT}-S_2$, by the expected shortage quantity, given that there is a shortage at Stage 2, i.e.

$$\frac{\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{nT}\{\phi(k_2)-k_2+k_2\Phi(k_2)\}}{1-\Phi(k_2)}$$

Then the above expression simplifies to the following.

Safety stock at Stage 1 for the last cycle =

$$\Phi(k_2)\left[k_1\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}\right] + [1-\Phi(k_2)]\left[k\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}\right]$$

where $k=k_1-\sqrt{\frac{nT}{T+l_1}}\frac{\phi(k_2)-k_2+k_2\Phi(k_2)}{1-\Phi(k_2)}$.

It was shown by Mitra and Chatterjee (2004) that the approximation introduced negligible errors in determining the actual ETC, especially for high shortage cost to inventory holding cost ratios.

The following expression for the expected shortage quantity at Stage 1 in the last cycle can be written in a similar way by making the same approximation.

Expected shortage quantity at Stage 1 for the last cycle =

$$\begin{aligned}&\int_{y_{nT}=-\infty}^{y_{nT}=S_2}\left[\int_{x_{T+l_1}=S_1}^{x_{T+l_1}=\infty}(x_{T+l_1}-S_1)f_1(x_{T+l_1})dx_{T+l_1}\right]f_2(y_{nT})dy_{nT}\\ &+\int_{y_{nT}=S_2}^{y_{nT}=\infty}\left[\int_{x_{T+l_1}=S_1+S_2-y_{nT}}^{x_{T+l_1}=\infty}(x_{T+l_1}+y_{nT}-S_1-S_2)f_1(x_{T+l_1})dx_{T+l_1}\right]f_2(y_{nT})dy_{nT}\\ &\approx\Phi(k_2)\left[\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}\{\phi(k_1)-k_1+k_1\Phi(k_1)\}\right]\\ &+[1-\Phi(k_2)]\left[\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}\{\phi(k)-k+k\Phi(k)\}\right]\end{aligned}$$

where k is given as above.

Now the corresponding expressions for the safety stock and expected shortage quantity at Stage 1 have to be weighted in the ratio $(n-1)$:1 and the following expression for ETC per period can be written.

$$\begin{aligned}\mathrm{ETC}=&\frac{A_1}{T}+\frac{A_2}{nT}+\frac{A_3}{T}+\frac{1}{2}\mu Th_1+\frac{1}{2}(n-1)(1-r)\mu Th_2+\frac{1}{2}r\mu Th_3\\ &+\frac{n-1}{n}\left[h_1k_1\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}+\frac{p_1}{T}\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}\{\phi(k_1)-k_1+k_1\Phi(k_1)\}\right]\\ &+\frac{1}{n}\left[\begin{array}{l}\Phi(k_2)\left[h_1k_1\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}+\frac{p_1}{T}\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}\{\phi(k_1)-k_1+k_1\Phi(k_1)\}\right]\\ +[1-\Phi(k_2)]\left[h_1k\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}+\frac{p_1}{T}\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{T+l_1}\{\phi(k)-k+k\Phi(k)\}\right]\end{array}\right]+h_2k_2\sqrt{(1-r)^2\sigma^2+\gamma^2}\sqrt{nT}\end{aligned} \quad (4)$$

where k is given as above.

As before, the optimal values of T and n can be obtained from the deterministic model (1). However, it may be difficult to determine the optimal values of k_1 and k_2 from (4) in a straightforward manner. We may, therefore, conduct an exhaustive search on the Normal table to obtain the values of k_1 and k_2 that minimize ETC.

4. Numerical examples and sensitivity analysis

We consider the following numerical examples to estimate the extent of errors introduced by the approximations made in the models.

$$\begin{aligned}&\mu=100,\quad \sigma=1,5,10,\quad \gamma=1,2,3,\quad r=0.1,0.15,0.2,0.3,0.5\\ &A_1=25,\quad A_2=100,\quad A_3=50\\ &h_1=2,\quad h_2=1,\quad h_3=0.5\\ &p_1=10,20,50,\quad p_2=10\\ &l_1=0.25,\quad l_2=0.50\end{aligned}$$

The maximum coefficient of variation for D, R and D–R is 0.32, which means a very insignificant probability of D, R and D–R being negative. For our convenience, let us denote the models by the following: Model I: Emergency shipment, Model II: Allocation.

For each of the 135 problem instances, for Models I and II the values of the policy variables (n, T, k_1 and k_2) are obtained from the formulas given in the corresponding sections. For example, for $\sigma = 1$, $\gamma = 1$, $r = 0.1$ and $p_1 = 10$, the values of n, T, k_1 and k_2 are 1, 1.31, 0.64 and 1.20, respectively, for Model I, and 1, 1.31, 1.10 and 0, respectively, for Model II. Once the values of the policy variables are determined for each problem instance, the models are simulated using SLAM II (Simulation Language for Alternative Modelling) for 1000 periods while data collection is started after the first 100 periods to eliminate the initial bias. Thus we calculate the ETC per period for each model and for each problem instance. Tables A.1 and A.2 in Appendix A show the ETC per period for Models I and II, respectively. Next, we perform an exhaustive search on the possible ranges of the policy variables, and using SLAM II, determine the combinations of values of the policy variables that result in the best possible ETC per period, denoted by $\mathrm{ETC_{best}}$, for each model and for each problem instance. $\mathrm{ETC_{best}}$ may not represent the true optimal solution to the problem, but it should be very close to optimality since it is obtained by an exhaustive search on the possible values of the policy variables. Thereafter we compute the per cent error for each model and for each problem instance by the following formula: $((\mathrm{ETC}-\mathrm{ETC_{best}})/\mathrm{ETC_{best}})\times 100$. We note that the per cent errors increase with σ, and hence reproduce in Table 1 the average and maximum per cent errors only when $\sigma = 10$, i.e. when σ is the maximum.

It may be observed from Table 1 that the average error is less than 1% and the maximum error is less than 2%, and the data are for $\sigma = 10$ when the per cent errors are the maximum. Had we included all the problem instances, both the average errors and the maximum errors would have been significantly lower. Therefore, we may conclude that the models developed in the paper very closely represent the behaviour of the actual system and may be used to derive the values of the policy variables for inventory control.

We also compute the average ETC for different values of r for the models, which are plotted in Fig. 3.

We observe from Tables A.1 and A.2 and Fig. 3 that the average ETC first increases and then decreases with r. For $r = 0.1$ and 0.15, the value of n is 1, and for $r = 0.2$, 0.3 and 0.5, the value of n is 2. From the deterministic model (1), we may verify that when $n = 1$, the effective inventory holding cost is $h_1 + rh_3$, which will increase with r. Therefore, ETC may increase with r as long as $n = 1$, and the

Table 1
Average and maximum per cent errors for different models ($\sigma = 10$).

Model	Average error (%)	Maximum error (%)
I	0.57	0.87
II	0.87	1.93

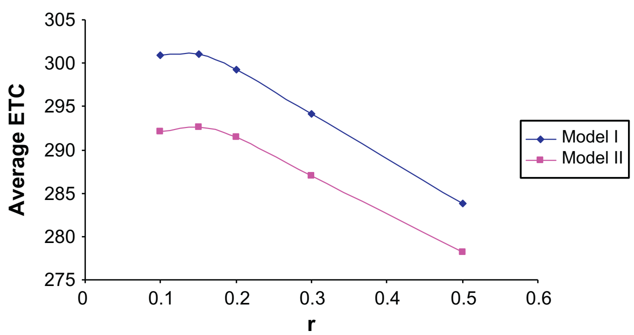

Fig. 3. Average ETC for different values of r for the models.

average ETC does increase when r is increased from 0.1 to 0.15. However, when $n = 2$, the effective inventory holding cost is $h_1 + h_2 - r(h_2 - h_3)$, which will decrease when r is increased as long as $n = 2$ since $h_2 > h_3$ based on our assumption that returned items awaiting recovery are of lower value than items in the serviceable stock. Therefore, ETC is expected to decrease with increase in r as long as $n = 2$, and as a result we observe that the average ETC decreases when r is increased from 0.2 to 0.5. When n also increases with r (from $r = 0.15$ to $r = 0.2$ in this case), which can easily be verified from the expression of n^* in the deterministic model (1), it will not be possible to make a general comment on the effective inventory holding cost because it will depend on the parameter values. The implication of this observation is as follows. From the discussions in Section 3.2 it is clear that COR(D,R) increases with r, and as such the variability of net demand, D–R decreases with increasing r, which intuitively should produce lower expected costs for higher values of r. However, that may not be always true as we have just observed. We can also make out from Section 3.2 that COR(D,R) increases, and the variability of D–R decreases, with decreasing γ, and as expected, Tables A.1 and A.2 in Appendix A show that ETC indeed decreases with decreasing γ.

One final observation from Tables A.1 and A.2 is that when we compare between Model I and Model II, we see that ETC is always lower for allocation than for emergency shipment for a given set of values of the parameters, r, σ, γ and p_1. This, of course, need not be always true and will depend on the parameters. For example, if the shortage cost at Stage 1 is extremely high compared to the cost of emergency shipment in case of a shortage at Stage 2, perhaps emergency shipment will be a more economically viable option than allocation.

5. Special case: Utilization of actual demand and return information

The model for this special case is developed for the system described in Section 3.2.1. Here it is assumed that demand and return information as they occur at Stage 1 is available at Stage 2. Stage 2 may utilize the actual demand and return information for the last $T - l_2$ periods of the last cycle at Stage 1 at the time of placing orders with the outside supplier. If $\overline{D}_{T-l_2}$ and $\overline{R}_{T-l_2}$ represent the actual demand and return, respectively, at Stage 1 during the last $T - l_2$ periods of the last cycle, the actual net demand faced by Stage 2 during this period is $\overline{D}_{T-l_2} - \overline{R}_{T-l_2}$. Therefore, while setting the order-up-to level at Stage 2, protection is required only for $(n-1)T + l_2$ periods. Hence, although the order-up-to level at Stage 1 remains the same as shown in Section 3.2, the order-up-to level at Stage 2 has to be modified as follows:

$$S_2 = s + \overline{D}_{T-l_2} - \overline{R}_{T-l_2} + (1-r)\mu[(n-1)T + l_2] + k_2 \times \sqrt{(1-r)^2\sigma^2 + \gamma^2}\sqrt{(n-1)T + l_2}$$

where s is the shortfall in supply, if any, to Stage 1 in the previous cycle

It may be noted that the order-up-to level, S_2 at Stage 2 is dynamic even though demand and return distributions are stationary. The following expression for the expected total cost (ETC′) per period can be derived. For a detailed description of the system and model formulations for forward flows only, one may refer to Mitra and Chatterjee (2004).

$$\begin{aligned}\text{ETC}' = {} & \frac{A_1}{T} + \frac{A_2}{nT} + \frac{A_3}{T} + \frac{1}{2}\mu T h_1 + \frac{1}{2}(n-1)(1-r)\mu T h_2 \\ & + \frac{1}{2} r\mu T h_3 + h_1 k_1 \sqrt{(1-r)^2\sigma^2 + \gamma^2}\sqrt{T + l_1} \\ & + h_2 k_2 \sqrt{(1-r)^2\sigma^2 + \gamma^2}\sqrt{(n-1)T + l_2} \\ & + \frac{p_1}{T}\sqrt{(1-r)^2\sigma^2 + \gamma^2}\sqrt{T + l_1}\{\phi(k_1) - k_1 + k_1\Phi(k_1)\} \\ & + \frac{p_2}{nT}\sqrt{(1-r)^2\sigma^2 + \gamma^2}\sqrt{(n-1)T + l_2}\{\phi(k_2) - k_2 + k_2\Phi(k_2)\} \quad (5)\end{aligned}$$

It may be observed that the optimal values of T and n will remain the same as in (2) if the deterministic model (1) is used. Moreover, the expressions for optimal k_1 and k_2 will also be the same as (3). Therefore, the expressions for ETC in Section 3.2.1 and ETC′ can be directly compared to measure the savings in the event of utilization of actual demand and return information.

$$\text{ETC} - \text{ETC}' = \sqrt{(1-r)^2\sigma^2 + \gamma^2}\left[\sqrt{nT} - \sqrt{(n-1)T + l_2}\right]\Big[h_2 k_2 + \frac{p_2}{nT}\{\phi(k_2) - k_2 + k_2\Phi(k_2)\}\Big]$$

It may be observed from the above expression that savings may be realized due to information availability only when the lead time at Stage 2, l_2 is less than the review period at Stage 1, T.

In addition to the above, suppose the actual return information in a period is available to Stage 1 before demand for that period occurs. Given $R = \overline{R}$ (a constant) for a period, the conditional distribution of D for that period is given by $N\Big[\mu + \frac{\text{COR}(D,R)\times\sigma}{\sqrt{r^2\sigma^2+\gamma^2}}(\overline{R} - r\mu), \sigma^2(1 - \text{COR}^2(D,R))\Big]$ and the conditional distribution of net demand D–R for that period is given by $N\Big[\mu + \frac{\text{COR}(D,R)\times\sigma}{\sqrt{r^2\sigma^2+\gamma^2}}(\overline{R} - r\mu) - \overline{R}, \sigma^2(1 - \text{COR}^2(D,R))\Big]$.

Proposition 1. *The variance of net demand at Stage 1 is lower when the actual return information for a period is available before demand for that period occurs.*

Proof. Had the proposition been true, the following would have held.

$$(1-r)^2\sigma^2 + \gamma^2 > \sigma^2(1 - \text{COR}^2(D,R))$$

$$\text{or} \quad (1-r)^2\sigma^2 + mr^2\sigma^2 > \sigma^2\Big(1 - \frac{1}{1+m}\Big)$$

or

$$(1-r)^2 + mr^2 > \frac{m}{1+m}$$

which simplifies to

$$(1 - r - mr)^2 > 0,$$

which completes the proof. □

Proposition 1 implies that if the actual return information for a period is available at Stage 1 before demand for that period occurs, the average inventory holding and shortage costs of the system will further reduce. It may be noted that in this case Stage 1 will also experience dynamic order-up-to levels.

6. Managerial implications

The derivations in the paper and observations from the numerical examples lead to the following managerial implications.

- Incorporating returns in the traditional forward supply chains may or may not increase the cost of the inventory system under generalized cost structures. We may observe from the deterministic model (1) that when $n = 1$, the effective set-up and inventory holding costs for a supply chain with forward flows only are $A_1 + A_2$ and h_1, respectively, while for a closed-loop supply chain, the respective costs are $A_1 + A_2 + A_3$ and $h_1 + rh_3$. Therefore, when $n = 1$, a closed-loop supply chain is always costlier than a supply chain with forward flows only. When $n > 1$, the effective set-up and inventory holding costs for a traditional supply chain with forward flows are $A_1 + \frac{A_2}{n}$ and $h_1 + (n - 1)h_2$, respectively, while the corresponding costs for a closed-loop supply chain are $A_1 + \frac{A_2}{n} + A_3$ and $h_1 + (n - 1)(1 - r)h_2 + rh_3$. The effective inventory holding cost for a closed-loop supply chain can be rewritten as $h_1 + (n - 1)h_2 - r[(n - 1)h_2 - h_3]$. Since $h_2 > h_3$ as mentioned before, the effective inventory holding cost for a closed-loop supply chain is lower than that for a traditional supply chain while the effective set-up cost of a traditional supply chain is lower than that of a closed-loop supply chain. Therefore, when $n > 1$, whether incorporating returns into the forward supply chain will increase or decrease the cost of the system will depend on the values of the parameters. For example, if we consider the following parameter values: $\mu = 100$, $r = 0.2$, 0.5, $A_1 = 25$, $A_2 = 100$, $A_3 = 5$, $h_1 = 2$, $h_2 = 1$ and $h_3 = 0.5$, for a traditional supply chain without returns, $n = 2$ and $TC = 212.13$. On the other hand, for a closed-loop supply chain, when $r = 0.2$, $n = 2$ and $TC = 215.41$, and when $r = 0.5$, $n = 3$ and $TC = 202.90$. Therefore, we observe that when $n > 1$, whether a closed-loop supply chain will cost lower than a traditional forward supply chain will depend on the parameter values. Following the above arguments, it may be concluded that in reverse supply chains, the focus of managers should be on recovering the economic value of returns as far as possible to boost profitability, adhering to rules and regulations imposed by appropriate authorities in connection with returns management and building a socially responsible corporate image.
- If demand and return in a period are correlated, the variability of net demand may decrease. However, implementation of the same especially in the presence of set-up costs is not straightforward. It requires that demands generated against returns are not instantaneously met even if there are serviceable units in the stock; rather the concerned customers are quoted a service time equal to the cycle time of the stage. In other words, demands generated against returns are never backordered and they are satisfied in the next cycle when returns are recovered and replenish the serviceable stock. On the other hand, freshly generated demands are met from the serviceable stock, if possible; otherwise, they are backordered and are satisfied when fresh stocks arrive. If one wishes to do away with all these constraints on implementation of correlated demands and returns, one has to model the system assuming that demands and returns are independent random variables, which is the assumption made by the majority of the papers dealing with inventory systems with returns.
- When the rate of return increases, the correlation between demand and return increases and the variability of net demand decreases. However, in contrast to our intuition, the expected cost of the system may actually not decrease. We have shown in the paper that the expected cost of the system decreases with the rate of return only when $n > 1$. Managers need to realize that correlations between demands and returns may not always lead to cost reduction.
- We have also compared between emergency shipment and allocation in case of a shortage in the higher echelon. In our experimental set-up, it is observed that allocation always results in lower expected costs than emergency shipment. However, we note that this is not true in general and the outcome will depend on the parameter values such as the holding cost and shortage cost ratios. For example, if the system deals with critical components with high service levels, probably emergency shipment will be preferred to allocation. On the other hand, for not-so-critical components, maybe allocating the shortage at the higher echelon to the lower echelon would suffice.
- It is also observed that utilization of actual demand and return information in deriving the values of the policy variables does lead to cost savings. Moreover, if the return information is available to Stage 1 before demand for that period occurs, the variability of net demand further reduces for a given rate of return leading to further reduction in holding and shortage costs. The implications for managers would be capturing the relevant information and making use of it, and also dealing with dynamic order-up-to levels at the respective stages despite facing stationary demand and return distributions.

7. Directions for future research

In this paper, we developed deterministic and stochastic models for a two-echelon closed-loop supply chain with correlated demands and returns under generalized cost structures. The integration of returns with traditional forward supply chains complicates the analysis of such systems to a great extent. Therefore, for tractability, a number of assumptions are made in the literature on closed-loop supply chains. Nevertheless, efforts should be made to develop models for more general multi-echelon inventory systems with returns that provide near-optimal, if not optimal, solutions to such problems. Simulation, in this context, may come in handy when developing models for systems comprising multiple levels with practical considerations turns out to be extremely difficult. Simulation also helps in scenario analysis. We assume in the paper that demands and returns in different periods are independent; however, demand and return in a given period may be correlated. It may be noted that in a set-up with returns, lagged correlations between demands and returns are very much relevant, and we expect to address the issue in future research. Also, it is assumed that demands and returns are Normally distributed. Models should be developed for other continuous (Gamma, Log-Normal, etc.) and discrete (Poisson, etc.) distributional assumptions. Locating recovery operations in the higher echelon would be another interesting case worth considering (Mitra, 2009). Finally, a holistic approach by integrating inventory management with revenue management and vehicle routing with backhauling, for example, might be a relevant direction for future research in closed-loop supply chains.

Table A.1
ETC for Model I for different values of r, σ, γ and p_1.

r	p_1	$\sigma = 1$			$\sigma = 5$			$\sigma = 10$		
		$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
0.1	10	274.63	278.90	283.62	291.06	292.64	295.08	313.43	314.26	315.60
	20	275.91	280.98	286.60	295.44	297.32	300.21	322.03	323.01	324.61
	50	277.32	283.28	289.88	300.27	302.49	305.89	331.53	332.68	334.56
0.15	10	276.09	280.42	285.18	291.45	293.12	295.66	312.54	313.41	314.83
	20	277.33	282.48	288.13	295.58	297.56	300.58	320.64	321.68	323.36
	50	278.70	284.74	291.38	300.14	302.46	306.01	329.56	330.78	332.76
0.2	10	275.94	280.50	285.46	290.77	292.59	295.35	311.32	312.28	313.84
	20	276.91	282.13	287.81	293.90	295.98	299.14	317.43	318.53	320.31
	50	278.01	283.98	290.47	297.43	299.81	303.42	324.34	325.60	327.63
0.3	10	273.30	277.99	283.01	285.94	287.98	291.00	303.85	304.94	306.70
	20	274.23	279.61	285.37	288.71	291.06	294.52	309.25	310.50	312.51
	50	275.29	281.44	288.02	291.85	294.53	298.49	315.34	316.77	319.07
0.5	10	268.05	272.98	278.10	276.28	278.94	282.61	288.86	290.35	292.68
	20	268.91	274.56	280.44	278.35	281.40	285.62	292.77	294.49	297.16
	50	269.88	276.35	283.08	280.68	284.18	289.00	297.20	299.16	302.22

Table A.2
ETC for Model II for different values of r, σ, γ and p_1.

r	p_1	$\sigma = 1$			$\sigma = 5$			$\sigma = 10$		
		$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
0.1	10	272.38	275.23	278.38	283.35	284.40	286.03	298.28	298.83	299.73
	20	273.75	277.47	281.58	288.05	289.43	291.55	307.52	308.24	309.41
	50	275.26	279.92	285.08	293.21	294.94	297.60	317.65	318.55	320.02
0.15	10	273.90	276.79	279.97	284.16	285.28	286.97	298.25	298.84	299.78
	20	275.23	279.00	283.13	288.60	290.04	292.25	306.94	307.70	308.93
	50	276.69	281.42	286.61	293.46	295.27	298.04	316.47	317.42	318.97
0.2	10	273.75	276.82	280.16	283.73	284.96	286.81	297.56	298.21	299.26
	20	274.92	278.79	282.99	287.50	289.04	291.38	304.93	305.74	307.06
	50	276.19	280.92	286.07	291.58	293.47	296.33	312.91	313.90	315.51
0.3	10	271.21	274.35	277.72	279.68	281.05	283.08	291.70	292.44	293.61
	20	272.34	276.32	280.58	283.05	284.79	287.35	298.25	299.18	300.67
	50	273.55	278.43	283.64	286.68	288.80	291.94	305.30	306.43	308.25
0.5	10	266.11	269.41	272.84	271.62	273.40	275.86	280.03	281.03	282.59
	20	267.16	271.35	275.70	274.15	276.41	279.53	284.82	286.09	288.07
	50	268.28	273.42	278.74	276.85	279.62	283.44	289.94	291.49	293.92

Acknowledgements

The author is grateful to the Area Editor and the anonymous reviewers for their valuable comments and suggestions that led to a considerable improvement of the paper over its original version. The author also expresses deep gratitude to Prof. Rahul Mukerjee of Indian Institute of Management Calcutta for his kind advice and guidance.

Appendix A

See Tables A.1 and A.2.

References

Akcali, E., & Cetinkaya, S. (2011). Quantitative models for inventory and production planning in closed-loop supply chains. *International Journal of Production Research, 49*(8), 2373–2407.

Alamri, A. A. (2011). Theory and methodology on the global optimal solution to a general reverse logistics inventory model for deteriorating items and time-varying rates. *Computers & Industrial Engineering, 60*(2), 236–247.

Chan, H. K., Yin, S., & Chan, F. T. S. (2010). Implementing just-in-time philosophy to reverse logistics systems: A review. *International Journal of Production Research, 48*(21), 6293–6313.

DeCroix, G. A. (2006). Optimal policy for a multiechelon inventory system with remanufacturing. *Operations Research., 54*(3), 532–543.

Dekker, R., Fleischmann, M., Inderfurth, K., & Van Wassenhove, L. N. (Eds.). (2004). *Reverse logistics – Quantitative models for closed-loop supply chains*. New York: Springer.

Dowlatshahi, S. (2010a). A cost-benefit analysis for the design and implementation of reverse logistics systems: Case studies approach. *International Journal of Production Research, 48*(5), 1361–1380.

Dowlatshahi, S. (2010b). The role of transportation in the design and implementation of reverse logistics systems. *International Journal of Production Research, 48*(14), 4199–4215.

Dyckhoff, H., Lackes, R., & Reese, H. (Eds.). (2003). *Supply chain management and reverse logistics*. New York: Springer.

El Saadany, A. M. A, Jaber, M. Y., Bonney, M. (in press). How many times to remanufacture? *International Journal of Production Economics*. doi: 10.1016/j.ijpe.2011.11.017.

Flapper, S. D. P., Van Nunen, J., & Van Wassenhove, L. N. (Eds.). (2005). *Managing closed-loop supply chains*. Berlin: Springer.

El Saadany, A. M. A., & Jaber, M. Y. (2010). A production/remanufacturing inventory model with price and quality dependant return rate. *Computers & Industrial Engineering, 58*(3), 352–362.

El Saadany, A. M. A., & Jaber, M. Y. (2011). A production/remanufacture model with returns' subassemblies managed differently. *International Journal of Production Economics, 133*(1), 119–126.

El-Sayed, M., Afia, N., & El-Kharbotly, A. (2010). A stochastic model for forward–reverse logistics network design under risk. *Computers & Industrial Engineering, 58*(3), 423–431.

Field, J. M., & Sroufe, R. P. (2007). The use of recycled materials in manufacturing: Implications for supply chain management and operations strategy. *International Journal of Production Research, 45*(18&19), 4439–4463.

Fleischmann, M., Bloemhof-Ruwaard, J. M., Dekker, R., Van der Laan, E., Van Nunen, J., & Van Wassenhove, L. N. (1997). Quantitative models for reverse logistics: A review. *European Journal of Operational Research, 103*, 1–17.
Fleischmann, M., & Kuik, R. (2003). On optimal inventory control with independent stochastic item returns. *European Journal of Operational Research, 151*, 25–37.
Geldermann, J., Treitz, M., & Rentz, O. (2007). Towards sustainable production networks. *International Journal of Production Research, 45*(18 and 19), 4207–4224.
Guide, V. D. R., Jr., Jayaraman, V., Srivastava, R., & Benton, W. C. (2000). Supply chain management for recoverable manufacturing systems. *Interfaces, 30*, 125–142.
Guide, V. D. R., Jr., & Van Wassenhove, L. N. (2002). The reverse supply chain. *Harvard Business Review, 80*(2), 25–26.
Guide, V. D. R., Jr., & Van Wassenhove, L. N. (2003). *Business aspects of closed-loop supply chains*. Pittsburgh: Carnegie Mellon University Press.
Guide, V. D. R., Jr., & Van Wassenhove, L. N. (2009). The evolution of closed-loop supply chain research. *Operations Research, 57*(1), 10–18.
Gungor, A., & Gupta, S. M. (1999). Issues in environmentally conscious manufacturing and product recovery: A survey. *Computers & Industrial Engineering, 36*(4), 811–853.
Hadley, G., & Whitin, T. M. (1979). *Analysis of inventory systems*. New Delhi: Prentice-Hall.
Huang, X. Y., Yan, N. N., & Qiu, R. Z. (2009). Dynamic models of closed-loop supply chain and robust H_∞ control strategies. *International Journal of Production Research, 47*(9), 2279–2300.
Jaber, M. Y., & El Saadany, A. M. A. (2009). The production, remanufacture and waste disposal model with lost sales. *International Journal of Production Economics, 120*(1), 115–124.
Jaber, M. Y., & El Saadany, A. M. A. (2011). An economic and production remanufacturing model with learning effects. *International Journal of Production Economics, 131*(1), 115–127.
Jaber, M. Y., & Rosen, M. A. (2008). The economic order quantity repair and waste disposal model with entropy cost. *European Journal of Operational Research, 188*(1), 109–120.
Jayaraman, V. (2006). Production planning for closed-loop supply chains with product recovery and reuse: an analytical approach. *International Journal of Production Research, 44*(5), 981–998.
Kannan, G., Haq, A. N., & Devika, M. (2009). Analysis of closed-loop supply chain using genetic algorithm and particle swarm optimization. *International Journal of Production Research, 47*(5), 1175–1200.
Kelle, P., & Silver, E. A. (1989). Forecasting the returns of reusable containers. *Journal of Operations Management, 89*(1), 17–35.
Ketzenberg, M. E., Van der Laan, E., & Teunter, R. (2006). Value of information in closed loop supply chains. *Production and Operations Management, 15*(3), 393–406.
Kim, K., Song, I., Kim, J., & Jeong, B. (2006). Supply planning model for remanufacturing system in reverse logistics environment. *Computers & Industrial Engineering, 51*(2), 279–287.
Kleindorfer, P. R., Singhal, K., & Van Wassenhove, L. N. (2005). Sustainable operations management. *Production and Operations Management, 14*(4), 482–492.
Konstantaras, I., Skouri, K., & Jaber, M. Y. (2010). Lot sizing for a recoverable product with inspection and sorting. *Computers & Industrial Engineering, 58*(3), 452–462.
Korugan, A., & Gupta, S. M. (1998). A multi-echelon inventory system with returns. *Computers and Industrial Engineering, 35*, 145–148.
Lee, J-E., Gen, M., & Rhee, K-G. (2009). Network model and optimization of reverse logistics by hybrid genetic algorithm. *Computers & Industrial Engineering, 56*(3), 951–964.
Linton, J. D., Klassen, R., & Vaidyanathan, J. (2007). Sustainable supply chains: an introduction. *Journal of Operations Management, 25*, 1075–1082.
Mahadevan, B., Pyke, D. F., & Fleischmann, M. (2003). Periodic review push inventory policies for remanufacturing. *European Journal of Operational Research, 151*, 536–551.
Min, H., Ko, C. S., & Ko, H. J. (2006). The spatial and temporal consolidation of returned products in a closed-loop supply chain network. *Computers & Industrial Engineering, 51*(2), 309–320.
Minner, S. (2001). Strategic safety stocks in reverse logistics supply chains. *International Journal of Production Economics, 71*, 417–428.
Mitra, S. (2009). Analysis of a two-echelon inventory system with returns. *Omega, 37*, 106–115.
Mitra, S., & Chatterjee, A. K. (2004). Leveraging information in multi-echelon inventory systems. *European Journal of Operational Research, 152*, 263–280.
Morana, R., & Seuring, S. (2007). End-of-life returns of long-lived products from end customer – Insights from an ideally set up closed-loop supply chain. *International Journal of Production Research, 45*(18 and 19), 4423–4437.
Muckstadt, J. A., & Isaac, M. H. (1981). An analysis of single item inventory systems with returns. *Naval Research Logistics Quarterly, 28*, 237–254.
Mutha, A., & Pokharel, S. (2009). Strategic network design for reverse logistics and remanufacturing using new and old product modules. *Computers & Industrial Engineering, 56*(1), 334–346.
Nagel, C., & Meyer, P. (1999). Caught between ecology and economy: End-of-life aspects of environmentally conscious manufacturing. *Computers & Industrial Engineering, 36*(4), 781–792.
Neto, J. Q. F., Walther, G., Bloemhof, J., Van Nunen, J. A. E. E., & Spengler, T. (2010). From closed-loop to sustainable supply chains: The WEEE case. *International Journal of Production Research, 48*(15), 4463–4481.
Ravi, V., Shankar, Ravi, & Tiwari, M. K. (2005). Analyzing alternatives in reverse logistics for end-of-life computers: ANP and balanced scorecard approach. *Computers & Industrial Engineering, 48*(2), 327–356.
Ravi, V., Shankar, Ravi, & Tiwari, M. K. (2008). Selection of a reverse logistics project for end-of-life computers: ANP and goal programming approach. *International Journal of Production Research, 46*(17), 4849–4870.
Rubio, S., Chamorro, A., & Miranda, F. J. (2008). Characteristics of the research on reverse logistics (1995–2005). *International Journal of Production Research, 46*(4), 1099–1120.
Rubio, S., & Corominas, A. (2008). Optimal manufacturing–remanufacturing policies in a lean production environment. *Computers & Industrial Engineering, 55*(1), 234–242.
Savaskan, R. C., Bhattacharya, S., & Van Wassenhove, L. N. (2004). Closed-loop supply chain models with product remanufacturing. *Management Science, 50*(2), 239–252.
Silver, E. A., Pyke, D. F., & Peterson, R. (1998). *Inventory management and production planning and scheduling*. New York: Wiley.
Souza, G. C. (2009). Remanufacturing in closed-loop supply chains. *Production and Inventory management Journal, 45*(1), 56–65.
Teunter, R., Van der Laan, E., & Inderfurth, K. (2000). How to set the holding cost rates in average cost inventory models with reverse logistics. *Omega, 28*, 409–415.
Thierry, M., Salomon, M., Van Nunen, J., & Van Wassenhove, L. N. (1995). Strategic issues in product recovery management. *California Management Review, 37*(2), 114–128.
Toktay, L. B., Wein, L. M., & Zenios, S. A. (2000). Inventory management of remanufacturable products. *Management Science, 46*(11), 1412–1426.
Tsai, W. H., & Hung, S. J. (2009). Treatment and recycling system optimization with activity-based costing in WEEE reverse logistics management: An environmental supply chain perspective. *International Journal of Production Research, 47*(19), 5391–5420.
Tseng, M-L., Divinagracia, L., & Divinagracia, R. (2009). Evaluating firm's sustainable production indicators in uncertainty. *Computers & Industrial Engineering, 57*(4), 1393–1403.
Vachon, S., & Klassen, R. D. (2007). Supply chain management and environmental technologies: The role of integration. *International Journal of Production Research, 45*(2), 401–423.
Weeks, K., Gao, H., Alidaeec, B., & Rana, B. S. (2010). An empirical study of impacts of production mix, product route efficiencies on operations performance and profitability: A reverse logistics approach. *International Journal of Production Research, 48*(4), 1087–1104.
Xiong, H., Sun, Y. (2010). Optimal model and genetic algorithm of two-echelon inventory system with correlated demands. In *International conference on management and service science*, Mass.
Yan, W., Chen, C-H., & Chang, W. (2009). An investigation into sustainable product conceptualization using a design knowledge hierarchy and Hopfield network. *Computers & Industrial Engineering, 56*(4), 1617–1626.
Yuan, K. F., & Gao, Y. (2010). Inventory decision-making models for a closed-loop supply chain system. *International Journal of Production Research, 48*(20), 6155–6187.