Optimal Synchronization of Local Clocks by GPS 1PPS Signals Using Predictive FIR Filters

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***Abstract*—In this paper, we discuss optimal synchronization of local clocks using Global Positioning System (GPS) one-pulse-per-second (1PPS) timing signals. To eliminate sawtooth errors that are peculiar to the 1PPS signals and optimally steer the clock errors each M seconds, we propose the use of a ramp-predictive finite-impulse-response (FIR) filter that is known to be optimal for clock models on large averaging horizons. A low-pass filter is used to smooth the hold filter output between the optimally predicted points. A GPS-locked crystal clock has been investigated in detail in terms of the time interval error, Allan deviation, and precision time protocol (PTP) variance. A high-efficiency implementation of the proposed synchronization algorithm is experimentally demonstrated.**

***Index Terms*—Allan deviation, Global Positioning System (GPS)-based clock synchronization, precision time protocol (PTP) variance, predictive finite-impulse-response (FIR) filter, time interval error (TIE).**

I. Introduction

THE NEED to synchronize local time scales arises from different allowed uncertainties in digital communications [1], [2], bistatic radars [3], telephone networks [4], networked measurement and control systems [5], space systems [6], [7], computer nets [8], etc. To discipline the clocks, commercially available Global Positioning System (GPS) timing receivers are often used by conveying the reference time to the locked-clock loop via the one-pulse-per-second (1PPS) output. An organization of the loop is provided such that the clock time error ranges over time below an allowed threshold that, for digital communication networks, is specified in [9].

GPS disciplining of local clocks can be organized in two ways.

1) Both the time error and the frequency offset are corrected via the GPS-disciplined oscillator (GPSDO).
2) Only the time error is adjusted in the controlled digital parts of the clocks without touching a local oscillator.

The basic block diagram of a GPS-disciplined clock is shown in Fig. 1. The 1PPS output of the GPS timing receiver is used as a reference, in which the regular time delays caused by signal

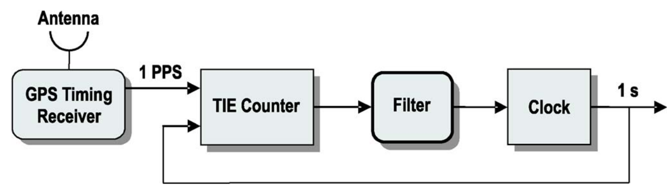

Fig. 1. Generalized structure of GPS disciplining of local clocks by the 1PPS timing signals.

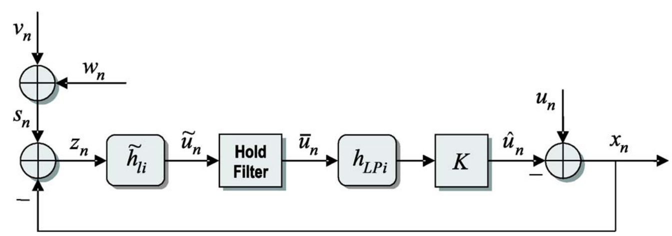

Fig. 2. Loop model of GPS synchronization of a local clock by the 1PPS timing signals.

propagation and other factors are eliminated at the early stage. The time difference between the 1PPS that is accurate but not precise, owing to noise, and the 1-s output of a local clock that is precise but inherently inaccurate is measured each second with a high-resolution time interval error (TIE) counter. The clock TIE is then estimated by a filter to produce a synchronizing signal that is intended to discipline the clock time scale.

In a locked-clock time scale, noise mostly depends on the precision of the local oscillator, and time error departures are limited by the accuracy of the reference time signals. The high accuracy of synchronization is achieved using a filter, the design of which can vary. Averaging and low-order low-pass (LP) filters are typical for commercially manufactured GPSDOs. The use of averaging and integrating filters, along with a phase-locked loop (PLL), was proposed in [10]. In [11], the use of a three-order PLL was proposed for minimizing phase errors. A linear least squares estimator was exploited in [12] to discipline a rubidium standard. Some authors proposed the application of Kalman filters [13]–[15] and even neural networks [16]. Experimental investigations of several GPSDOs utilizing different filters can be found in [17] and [18].

In this paper, we propose a novel approach for the optimal GPS synchronization of local time scales using a predictive ramp finite-impulse-response (FIR) filter. The latter is known to be optimal for clock models [19] on large averaging horizons that are typically used in timekeeping. In our loop (see Fig. 1), we use the Timing SynPaQ III GPS Sensor (Synergy Systems, LLC, San Diego, CA). Frequency Counter SR620 (Stanford

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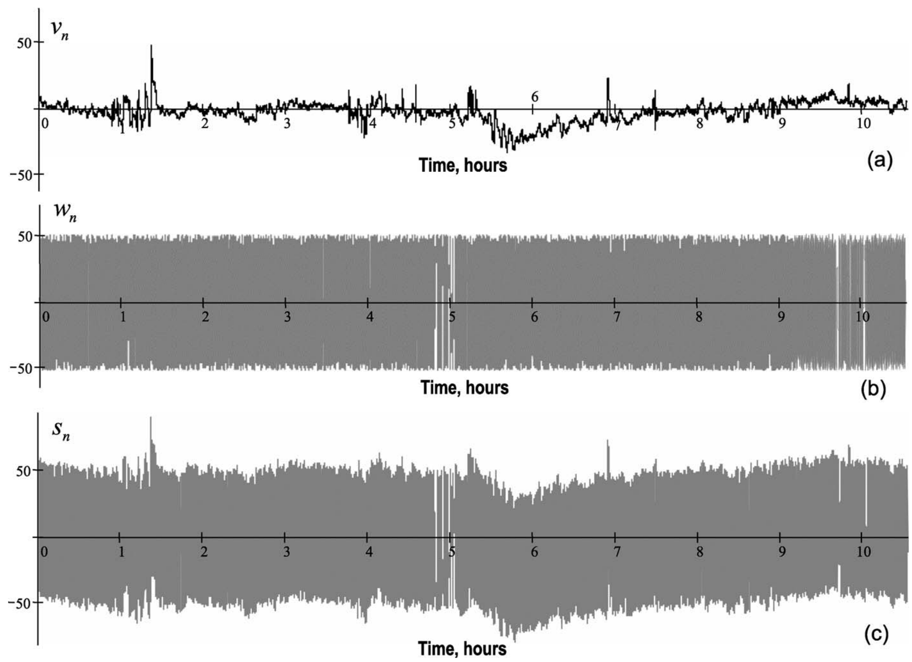

Fig. 3. Typical time errors of a Motorola-based GPS timing receiver. (a) Time uncertainty v_n caused by different satellites in a view and other random factors. (b) Sawtooth noise w_n induced by the receiver. (c) Time error s_n in the 1PPS reference signal.

Research System, Inc., Sunnyvale, CA) is used as the TIE counter. We exploit the oven-controlled crystal oscillator (OCXO) embedded in the SR620 to form the local time scale with a high-resolution divider. For simultaneous measurements of the actual time errors, the reference Cesium Frequency Standard CsIII (Symmetricom, Inc., San Jose, CA) is used.

The rest of this paper is organized as follows. In Section II, we discuss the model of a GPS locked clock. The synchronization algorithm and the general loop equation are described in Section III. Experimental investigations of the GPS locked crystal clock are provided in Section IV in terms of the TIE, Allan deviation, and precision time protocol (PTP) variance. Analysis of the results and some generalizations are given in Section V. Finally, concluding remarks are drawn in Section VI.

II. Synchronization Loop Model

With respect to the clock first state x_n representing the TIE, the synchronization loop (see Fig. 1) can be modeled as shown in Fig. 2. Here, the reference time uncertainty v_n [see Fig. 3(a)] is caused by different satellites in a view (GPS time errors), measurement errors, and other factors [20]. The uniformly distributed sawtooth noise w_n [see Fig. 3(b)] [21] is induced by the receiver to range from -50 to 50 ns, owing to the principle of the 1PPS signal formation [22]. In measurements with sawtooth, reference signal noise s_n comprises an additive sum of v_n and w_n [see Fig. 3(c)]. In some GPS timing receivers, the negative sawtooth correction code is available in the protocol. If sawtooth correction is applied, then one can approximately let $s_n = v_n$. A typical behavior of clock time error u_n, which is caused by a local OCXO, is shown in Fig. 4.

The loop operates as follows. The value x_n is subtracted from s_n by the TIE counter to measure $z_n = s_n - x_n$. It is implied that N such measurements are provided in the nearest past sample history from $n - N$ to $n - 1$. To predict u_n at a current point n, a predictive FIR filter with gain $\tilde{h}_{li}$ of degree l is used. The predicted value $\tilde{u}_n$ is held as $\bar{u}_n$ by a hold filter. Predictions are smoothed by an LP filter, with gain $h_{\mathrm{LP}i}$ and gain K applied to be $\hat{u}_n$. Finally, $\hat{u}_n$ is subtracted from u_n to yield $x_n = u_n - \hat{u}_n$. Having compensated u_n, the clock TIE x_n, in an ideal case, reaches zero.

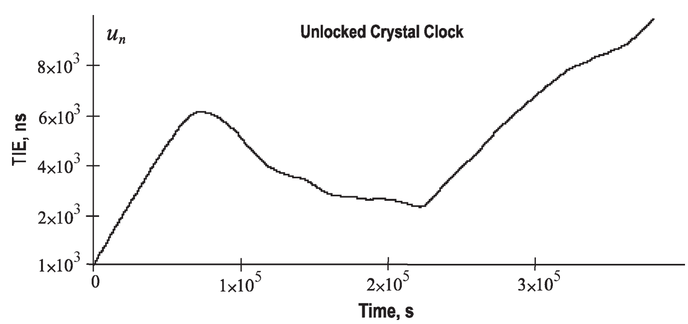

Fig. 4. Typical behavior of the TIE of an unlocked crystal clock.

A. Predictive FIR Filter

It has been shown in [23] and [24] that the unbiased FIR filters produce lower errors than the Kalman filter for GPS-based measurements of the TIE. Contrary to the recursive Kalman filters, such filters are inherently bounded-input/bounded-output stable and have better robustness against uncertainty v_n and roundoff errors [25]. On the other hand, Lepek [26] showed that linear predictors are optimal or close to optimal for the prediction of clock instabilities. We, therefore, infer that the unbiased V ramp FIR filter with the gain

$$h_{1i} = \begin{cases} \frac{2(2N-1)-6i}{N(N+1)}, & 0 \leqslant i \leqslant N-1 \\ 0, & \text{otherwise} \end{cases} \qquad (1)$$

derived in [27] and becoming optimal [19] in the sense of the minimum MSE, by large $N \gg 1$, can be used in the loop (see Fig. 2) after being modified to be a one-step predictive. Such a modification has been provided in [28] and [29] for p steps. Setting $p = 1$, we arrive at the gain of the one-step predictive ramp FIR filter in [28, eq. (60)] and [29, eq. (31)], i.e.,

$$\tilde{h}_{1i} = \begin{cases} \frac{2(2N+1)-6i}{N(N-1)}, & 1 \leqslant i \leqslant N \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

which is originally derived by Heinonen and Neuvo [30]. Contrary to (1), this filter has low efficiency [29], with a small N because of an uncertainty at $N = 1$. However, this does not mean too much for clock applications claiming $N \gg 1$.

By (2), the one-step predictive estimate can be found as

$$\tilde{u}_n = -\sum_{i=1}^{N} \tilde{h}_{1i} z_{n-i} \tag{3}$$

which means that $\tilde{u}_n$ is produced by the discrete convolution via the past values of z_n taken from $n-1$ to $n-N$.

B. Hold Filter

To hold $\tilde{u}_n$ over time for continued steering of x_n between the optimally defined values, a hold filter is used such that

$$\bar{u}_n = \tilde{u}_{\lfloor \frac{n}{M} \rfloor M} \tag{4}$$

where $\lfloor n/M \rfloor$ is an integer part of n/M. With (4), the input and output values of the hold filter become equal when n is a multiple of M. In the gap between two such neighboring points, the output value of the hold filter is constant.

C. LP Filter and Gain K

For multiple steering of clock errors with period τM, where τ is the sampling time, and M is the number of sampling times, the hold filter produces step signal $\bar{u}_n$. Straightforwardly applied to the clock, $\bar{u}_n$ obtains optimal steering and assures that x_n ranges within minimum bounds around zero. On the other hand, an impulsive $\bar{u}_n$ may not be appropriate in obtaining the required noise performance of a locked clock.

To smooth $\bar{u}_n$, an LP filter with impulse response $h_{\mathrm{LP}i}$ is included. Contrary to $\tilde{h}_{1i}$ producing optimal estimates, $h_{\mathrm{LP}i}$ is intended to smooth step signal $\bar{u}_n$ without attenuation. We shall experimentally show that the one-order LP filter with

$$h_{\mathrm{LP}i} = \begin{cases} Ae^{-\frac{\tau}{T}i}, & i \geqslant 0 \\ 0, & i < 0 \end{cases} \tag{5}$$

is able to fit the demands for precise synchronization, where T is a time constant, and $A = 1 - e^{-\tau/T}$.

Finally, if the LP filter does not attenuate $\bar{u}_n$, one may set $K = 1$.

III. Synchronization Algorithm

The GPS synchronization algorithm is shown in Fig. 5. It is implied that measurements of z_n are provided with high resolution and that the clock is designed such that $\hat{u}_n$ directly and linearly steers its errors.

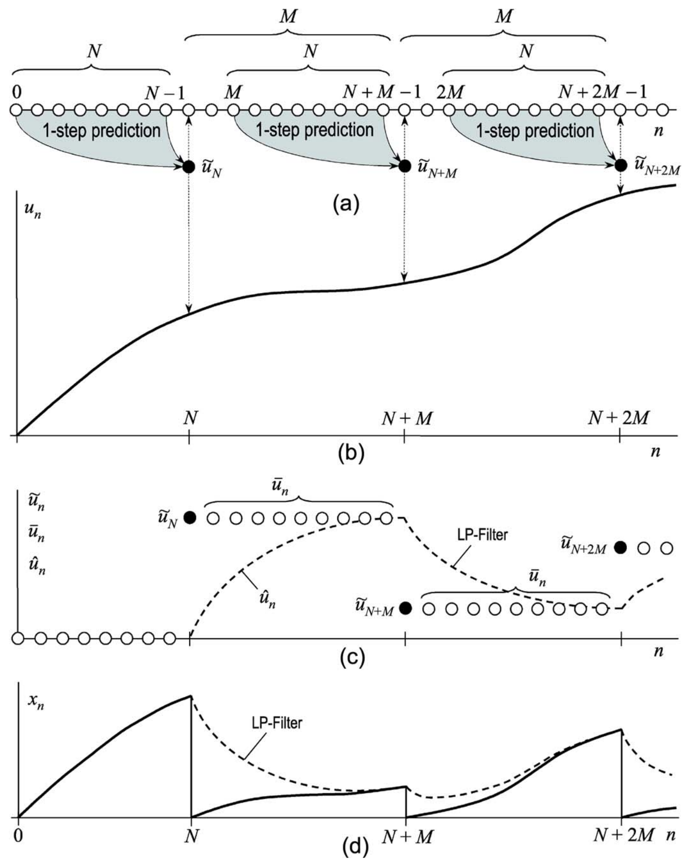

Fig. 5. GPS synchronization algorithm for $M > N$. (a) Time diagram. (b) Time error u_n of an unlocked clock. (c) Optimal predictive estimate $\tilde{u}_n$, hold filter output $\bar{u}_n$, and smoothed estimate $\hat{u}_n$. (d) Time error x_n of a locked clock. (Bold) piecewise and (dashed) smoothed by an LP filter.

The FIR filter processes data from 0 to $N-1$, producing a prediction $\tilde{u}_n$ at N [see Fig. 5(a)]. This value is held by the hold filter (4), starting at $n = N$ [see Fig. 5(c)], and steers x_n [see Fig. 5(b)]. Accordingly, x_n jumps at $n = N$ closely to zero and thereafter behaves as in Fig. 5(d) (bold), starting at $n = N$. Function $\bar{u}_n$ is then smoothed (dashed curve) by an LP filter.

For multiple synchronization with period M, a smoothed value $\hat{u}_n$ can be found, using the discrete convolution and (2)–(5), as

$$\hat{u}_n = K \sum_{l=0}^{L-1} h_{\mathrm{LP}l} \tilde{u}_{\lfloor \frac{n-l}{M} \rfloor M} \tag{6}$$

where L is a reasonable length of impulse response $h_{\mathrm{LP}i}$. Substituting (6) to $x_n = u_n - \hat{u}_n$ and accounting for (3) and $z_n = s_n - x_n$ allow the writing of the general loop equation as

$$x_n = u_n - K \sum_{l=0}^{L-1} \sum_{i=1}^{N} h_{\mathrm{LP}l} \tilde{h}_{1i} x_{\lfloor \frac{n-l}{M} \rfloor M - i} + K \sum_{l=0}^{L-1} \sum_{i=1}^{N} h_{\mathrm{LP}l} \tilde{h}_{1i} s_{\lfloor \frac{n-l}{M} \rfloor M - i} \tag{7}$$

where the difference between u_n and the first block of sums represents the regular error (bias), and the remaining term denotes the noise in x_n.

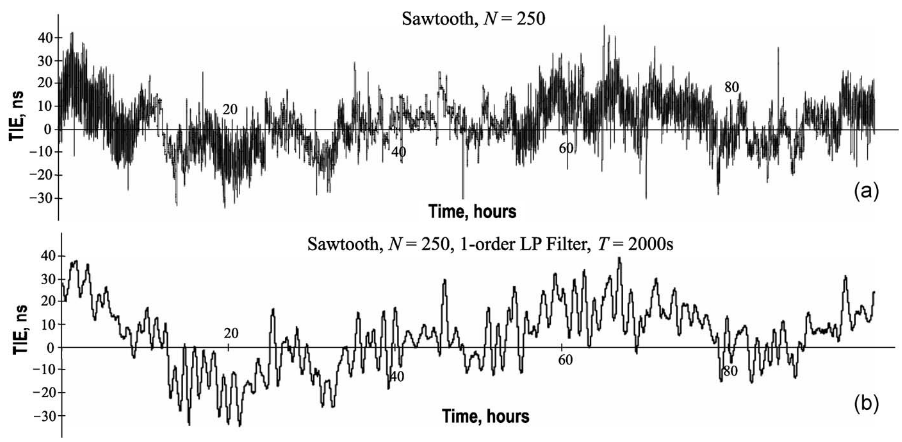

Fig. 6. Time error x_n of the optimally GPS locked crystal clock with sawtooth measurements, i.e., $s_n = v_n + w_n$. (a) Piecewise steering with $N = 250$. (b) Smoothed steering with the one-order LP filter, i.e., $T = 2000$ s.

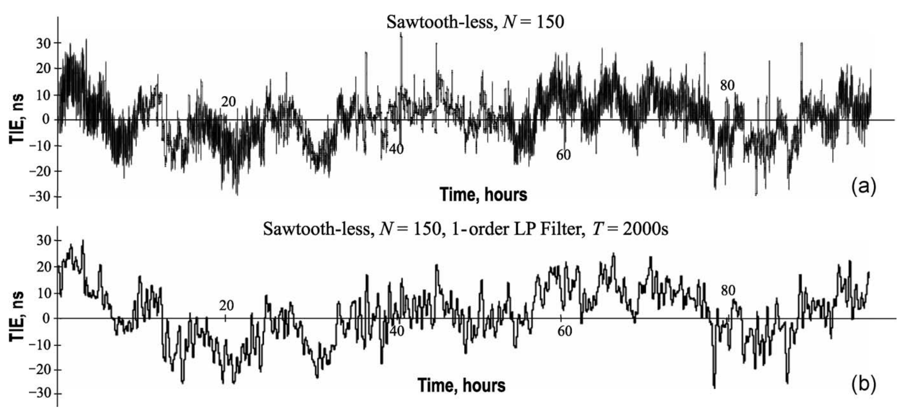

Fig. 7. Time error x_n of the optimally GPS locked crystal clock with sawtoothless measurements, i.e., $s_n = v_n$. (a) Piecewise steering with $N = 150$. (b) Smoothed steering with the one-order LP filter, i.e., $T = 2000$ s.

IV. Experimental Investigation

In this section, the loop (see Fig. 2) modeled by (7), in accordance with the algorithm (see Fig. 5), is examined in terms of the TIE, Allan deviation, and PTP variance. Following [31], the optimum value of τ is allowed to be $\tau_{\text{opt}} = 1$ s. We let $M = N$ and $K = 1$, and experimentally determine N_{opt} and T_{opt} in the sense of the minimum synchronization MSE for the crystal clock TIE measured with and without the sawtooth.

A. Time Interval Error

In the first experiment, we investigate the TIE x_n of a clock locked with and without the sawtooth correction.

For measurements with sawtooth errors, i.e., $s_n = v_n + w_n$, the optimum N has experimentally been found, following [31], to be $N_{\text{opt}} \cong 250$. With this value and without smoothing, i.e., $T = 0$, the TIE behaves as in Fig. 6(a). With $T_{\text{opt}} \cong 2000$, the LP filter minimizes the MSE in the TIE, as shown in Fig. 6(b).

For sawtoothless measurements,[1] i.e., $s_n = v_n$, the optimal N has appeared to be $N_{\text{opt}} \cong 150$, leading to the results shown in Fig. 7(a) and (b). One infers that sawtooth correction does not cardinally change the waveform, although it slightly draws together the TIE x_n bounds.

We finally assume uncertaintyless steering, i.e., $s_n = w_n$, find $N_{\text{opt}} \cong 250$, and arrive at the results shown in Fig. 8. An instant conclusion is that, with $v_n = 0$, the TIE very closely lies to zero at almost each of the points $n = N + mM$, in contrast to those shown in Figs. 6(a) and 7(a). However, this does not lead to substantial narrowing of the TIE range. The latter still fundamentally depends on the time error behavior of the unlocked clock (see Fig. 4), irrespective of the noise in the reference signal. This is further supported by a comparison of Figs. 6(b), 7(b), and 8(b).

B. Allan Deviation

Setting the experimentally determined values of $N_{\text{opt}} = 250$ and $N_{\text{opt}} = 150$ for sawtooth and sawtoothless measurements, respectively, we now investigate the Allan deviation of a locked clock for different values of T of the LP filter.

Inherently, the Allan deviation of the 1PPS with sawtooth, i.e., $s_n = v_n + w_n$ [see Fig. 9(a)], traces higher than that without sawtooth, i.e., $s_n = v_n$ [see Fig. 9(b)]. Using the optimal FIR filter, the former is substantially lowered [see Fig. 9(a)], whereas the latter remains almost unaltered [see Fig. 9(b)]. In both cases, an LP filter forms the final waveforms shown in Fig. 9. In particular, with small averaging times, the

[1]The term "sawtoothless measurement" is used in the sense of the GPS-based sawtooth measurement corrected with the prediction negative sawtooth code provided by the receiver protocol.

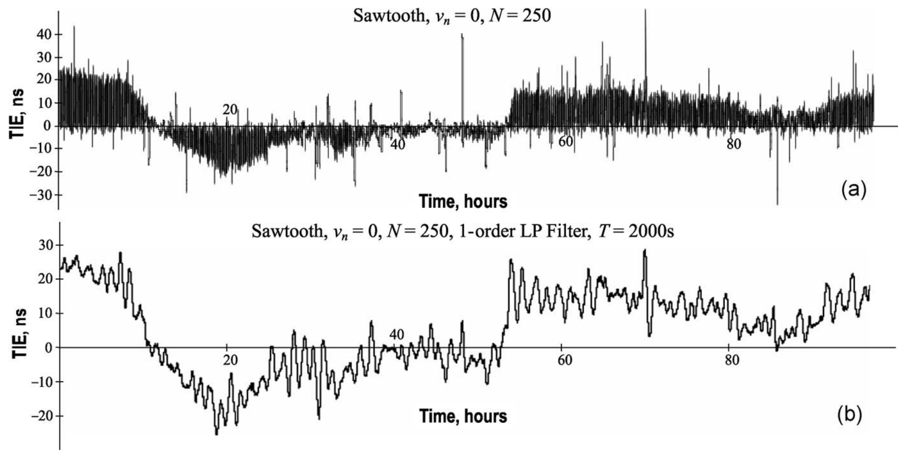

Fig. 8. Limiting time error x_n of the optimally GPS locked crystal clock with measurements without uncertainty, i.e., $s_n = w_n$. (a) Piecewise steering with $N = 250$. (b) Smoothed steering with the one-order LP filter, i.e., $T = 2000$ s.

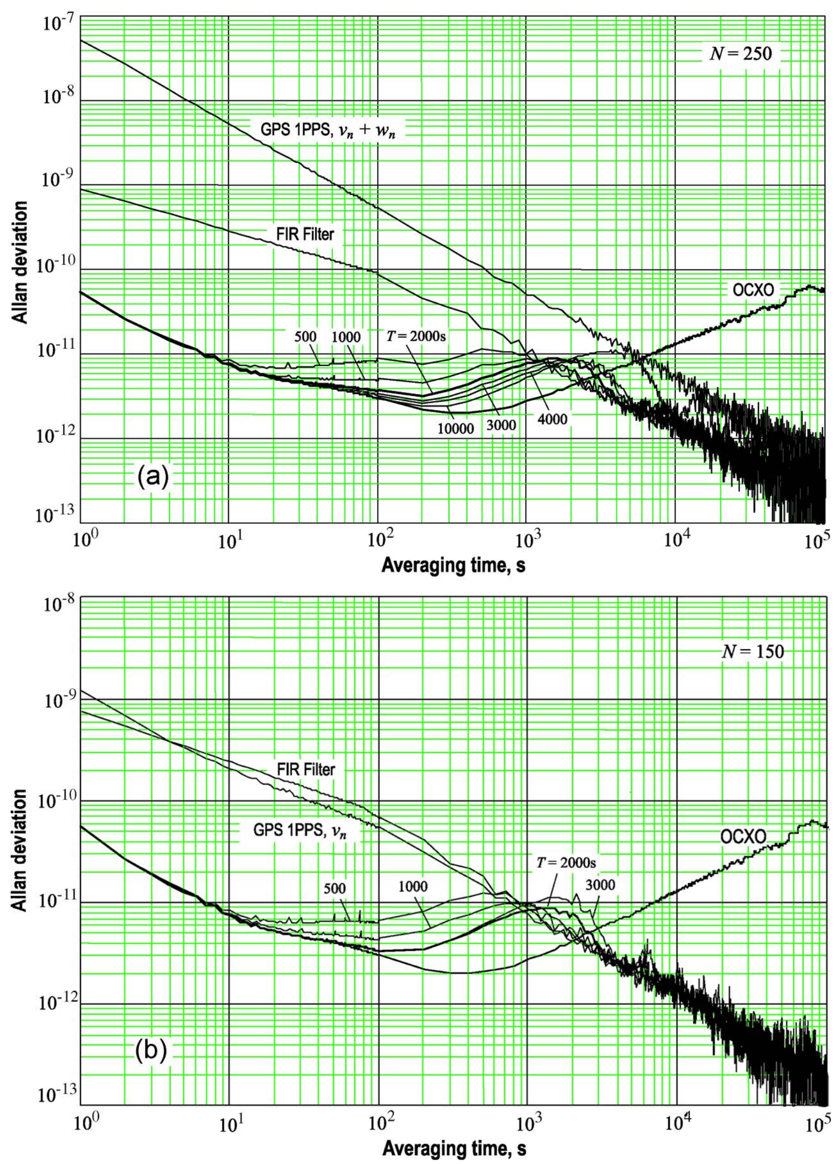

Fig. 9. Allan deviation of the GPS locked crystal clock for different T's of the one-order LP filter. (a) Sawtooth measurement $s_n = v_n + w_n$ with $N_{\text{opt}} = 250$. (b) Sawtoothless measurement $s_n = v_n$ with $N_{\text{opt}} = 150$.

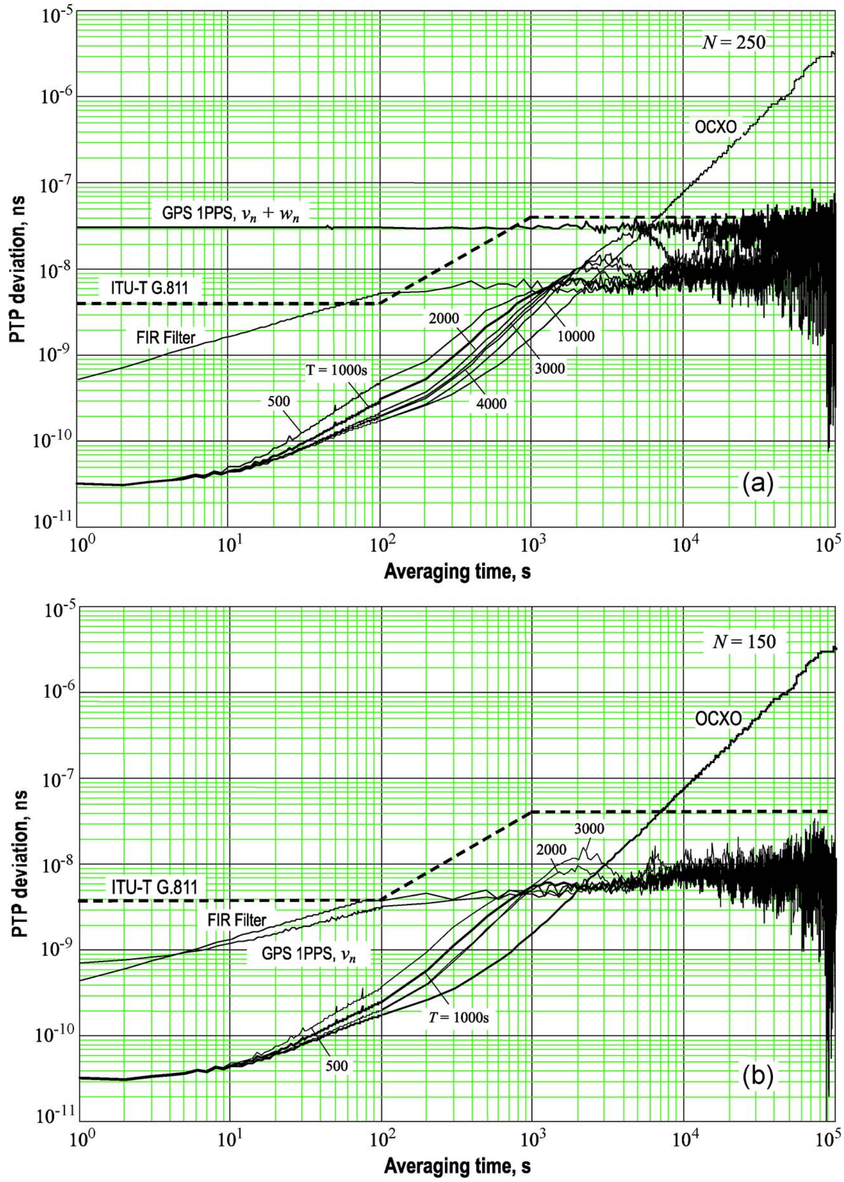

Fig. 10. PTP deviation of the GPS locked crystal clock for different T's of the one-order LP filter. (a) Sawtooth measurement $s_n = v_n + w_n$ with $N_{\text{opt}} = 250$. (b) Sawtoothless measurement $s_n = v_n$ with $N_{\text{opt}} = 150$.

Allan deviation is determined by the unlocked clock and, with large averaging times, by the optimal FIR filter (or sawtoothless measurements). In addition, the Allan deviation exhibits an excursion, the peak value of which reaches a minimum when $T_{\text{opt}} \cong 2000$ s. This local nonuniformity displaces to the right by $T > T_{\text{opt}}$ and to the left by $T < T_{\text{opt}}$.

C. PTP Variance

Because the PTP variance[2] is stated in [5] to be the main measure of errors in locked clocks, relevant investigations were provided. The results are shown in Fig. 10. Here, we selected $T \cong 1000$ s to be optimum once it provides a better placement of the locked-clock PTP deviation (square root of the PTP variance) below the time deviation (TDEV) mask specified by [9] for digital communication networks. This value can differ for another mask.

V. Analysis

Some generalizations regarding the proposed optimal GPS synchronization algorithm and the performance of a locked clock can now be provided.

1) Effect of the Optimal FIR Filter: It follows, by comparing "GPS 1PPS, v_n" and "FIR Filter" in Figs. 9(b) and 10(b), that the optimal FIR filter eliminates sawtooth error w_n similarly to sawtooth correction and does not substantially affect uncertainty v_n. This means that the proposed loop would operate with almost equal efficiency for commercially available GPS timing receivers with and without sawtooth correction.

[2]For $M = N$, the square root of the PTP variance $\sigma^2_{\text{PTP}}(\bar{\tau})$, where $\bar{\tau}$ is the averaging time, is equal to the TDEV specified in [9]. The PTP variance relates to the Allan variance $\sigma^2_y(\bar{\tau})$ as $\sigma^2_{\text{PTP}} = \bar{\tau}^2\sigma^2_y/3$.

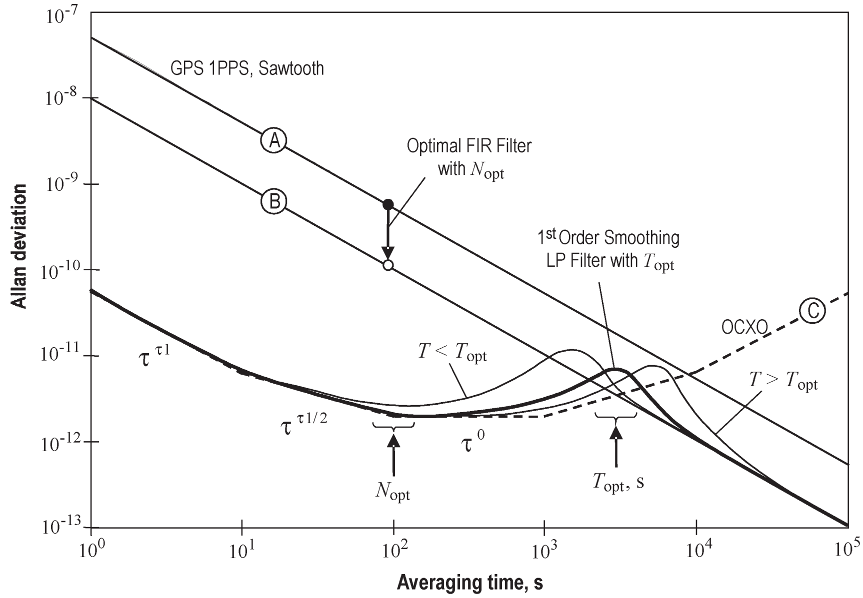

Fig. 11. Allan deviation of a local crystal clock synchronized by GPS 1PPS timing signals.

2) Optimum Parameters: The minimum MSE in x_n can be reached with four optimized parameters.

a) *Sampling time:* $\tau_{\text{opt}} = 1$ s [31].

b) *Averaging horizon:* From Fig. 9(a), it follows that N_{opt} lies about the cross point of the Allan deviation parts with slopes $\tau^{-1/2}$ and τ^0.

c) *Period of synchronization:* Although M can be arbitrary, the filter horizon, with $M < N$, overlaps both free and steered points (see Fig. 5), causing uncertainty errors. On the other hand, with $M > N$, synchronization is inefficient because the points from $n = N$ to $n = M - N$ are not processed. Thus, $M_{\text{opt}} = N_{\text{opt}}$.

d) *Time constant of the LP filter:* It follows from Fig. 9(a) that T_{opt} lies about the cross point between "FIR Filter" and "OCXO."

In summary, Fig. 11 shows a generalized image for the Allan deviation of a locked crystal clock. To obtain the "bold" curve, a designer must first determine N_{opt} about the cross point between $\tau^{-1/2}$ and τ^0. With N_{opt}, function "A" is lowered to "B," corresponding to sawtoothless measurements. Then, T_{opt} is specified about the cross point between "B" and "C," and the goal is reached. Notice that an excursion in the Allan deviation cannot be avoided from the standpoint of control. It can only be minimized, and its position can only be optimized by T_{opt}.

VI. Conclusion

In this paper, we have proposed and examined a novel optimal synchronization loop, in which a local clock is disciplined by GPS timing 1PPS signals using an optimal predictive ramp FIR filter and a smoothing one-order LP filter. Some generalizations of the studies can be found in Section V. The main overall conclusion is that the optimal FIR filter eliminates the sawtooth similarly to sawtooth correction and does not substantially affect uncertainty v_n. The proposed loop can, therefore, operate with almost equal efficiency for different kinds of commercially available GPS timing receivers, in which 1PPS signals are formed both with and without the sawtooth. Finally, we have noticed that, in our experiments, high-order smoothing LP filters did not contribute with lower errors to the loop. Even so, possible optimization of the LP filter is currently under investigation.

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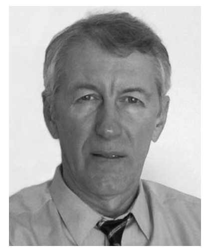

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