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A Robust Positioning Technique in DR/GPS using the Receding Horizon Sigma Point Kalman FIR Filter

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Abstract – *This paper describes the receding horizon sigma point Kalman FIR filter for tightly coupled DR/GPS hybrid navigation system. In order to overcome the flaws of the EKF, the SPKF is merged with the receding horizon strategy. This filter has several advantages over the EKF, the SPKF, and the RHKF filter. The advantages include the robustness to the system model uncertainty, the initial estimation error, temporary unknown bias, and etc. The computational burden is reduced. Especially, the RHSPKF filter works well even in the case of exiting the unmodeled random walk of the inertial sensors, which can be occurred in the MEMS inertial sensors by temperature variation. Therefore, the RHSPKF filter can provide the navigation information with good quality in the DR/GPS hybrid navigation system seamlessly.*

Keywords – *DR/GPS, SPKF, RHKF filter, tightly coupled*

I. INTRODUCTION

Commercial navigation technology has been become the core technology in telematics (TELEcommunications and InforMATICS) industry because telematics provides various location based services. The navigation system in telematics system is called CNS (Car Navigation System) because the telematics is utilized based on a car. CNS is comprised of a GPS receiver and a digital map, generally. And CNS is expanded into DR (Dead Reckoning)/GPS hybrid system to calculate the position information even in the urban area seamlessly. DR system for CNS must be implemented as low-cost to extend a market. Therefore, the DR system may include low-cost sensors instead of an IMU. In this paper, it is assumed that the DR system is implemented using an accelerometer and a gyro [1,2].

Currently, the DR/GPS hybrid navigation system has been developed using the extended Kalman filter (EKF). The EKF is the well-known approach in the integration of the nonlinear systems. However, the several flaws of the EKF exist, which may lead to sub-optimal performance and sometimes divergence of the filter. In recent years, various-type filters have been investigated to overcome the flaws. The sigma point kalman filter (SPKF) and the receding horizon Kalman FIR (RHKF) filter are the representative alternative filters [3,4].

If initial estimation error is large in the EKF, this filter may diverge because the Jacobian matrix for implementing the EKF has serious problem. The SPKF does not need to calculate the Jacobian matrix. Therefore, the SPKF is robust to the initial estimation error, unlike the EKF. When system has an unmodeled error or temporary unknown bias, the EKF is under the full influence of the errors. In order to reduce the effect of these kinds of errors, the RHKF filter has been researched. Since the FIR filter utilized finite measurements over the most recent time interval, this filter is known to be robust against temporary modeling uncertainties that may cause a divergence phenomenon in the case of the IIR structure filter [5]. However, the SPKF does not have the merits of the RHKF filter and the RHKF filter also does not have the advantages of the SPKF. In this paper, a novel filter, called the receding horizon sigma point Kalman FIR (RHSPKF) filter, is presented. The RHSPKF is made by fusing the advantages of the two filters.

In this paper, the performance of the EKF, the SPKF, and the RHSPKF filter is compared in the various situations of the DR/GPS hybrid navigation system. The results show that the SPKF and the RHSPKF filter work well even in the case of the initial large azimuth error. Moreover, the performance of the RHSPKF filter is better than the other filters in the cases that the inertial sensors have unmodeled random walk errors or have temporary unknown bias. The result is verified by some simulations.

II. SPKF AND RHKF FILTER

The EKF has various drawbacks in the estimation problem. One of the main drawbacks is that the state distribution is approximated by a Gaussian random variable, which is then propagated through the first-order linearization of the nonlinear system. When the initial estimation error is large, the propagated mean and covariance may have large errors, which may lead to sub-optimal performance and sometimes divergence of the filter. Another weak point is that EKF may have large error in the cases of model uncertainty, unknown time varying bias, etc. because of IIR structure. In this chapter the alternative filters are introduced.

A. Sigma Point Kalman Filter

The main idea of the SPKF: *with a fixed number of parameters it should be easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function* [3]. The fixed number in the SPKF is the minimal set of weighted sample points chosen deterministically, called sigma points. Generally, the number of sigma points is $2L+1$ (state dimension L). The SPKF is constructed as follows [4]:

0) A discrete time nonlinear system

$$x_{k+1} = f(x_k) + Gw_k,\ \ w_k \sim N(0,Q) \tag{1a}$$

$$y_k = h(x_k) + v_k,\ \ v_k \sim N(0,R) \tag{1b}$$

1-1) Initialization: augmented states and covariance

$$\hat{x}_0^a = E\begin{bmatrix} x_0^T & v_0 \end{bmatrix}^T = \begin{bmatrix} \hat{x}_0^T & 0 \end{bmatrix}^T \tag{2}$$

$$P_0^a = E\left[(x_0 - \hat{x}_0^a)(x_0 - \hat{x}_0^a)^T\right] = \begin{bmatrix} P_0 & 0 \\ 0 & Q \end{bmatrix} \tag{3}$$

1-2) Initialization: weights

$$W_0^{(m)} = \lambda/(L+\lambda) \tag{4a}$$

$$W_0^{(c)} = \lambda/(L+\lambda) + (1-\alpha^2+\beta) \tag{4b}$$

$$W_i^{(m)} = W_i^{(c)} = 1/2(L+\lambda),\ i = 1,\cdots,2L \tag{4c}$$

where $\lambda = (\alpha^2 - 1)L$ is a scaling parameter. α means the spread of the sigma points around $\hat{x}_0$ (set to $1 \le \alpha \le 1e^{-3}$) and β is used to incorporate prior knowledge of the distribution of x (2 for Gaussian distribution).

2) Sigma points Calculation

$$\chi_{k-1} = \begin{bmatrix} \hat{x}_{k-1} & \hat{x}_{k-1} \pm \sqrt{(L+\lambda)P_{k-1}} \end{bmatrix} \tag{5}$$

3) Time propagation

$$\chi_{k|k-1}^x = f(\chi_{k-1}^x) + G\chi_{k-1}^v \tag{6}$$

$$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} \chi_{i,k|k-1}^x \tag{7}$$

$$P_k^- = \sum_{i=0}^{2L} W_i^{(c)} \left[\chi_{i,k|k-1}^x - \hat{x}_k^-\right]\left[\chi_{i,k|k-1}^x - \hat{x}_k^-\right]^T \tag{8}$$

$$\hat{y}_k^- = \sum_{i=0}^{2L} W_i^{(m)} h(\chi_{i,k|k-1}^x) \tag{9}$$

4) Measurement update

$$P_{y_k y_k} = \sum_{i=0}^{2L} W_i^{(c)} \left[h(\chi_{i,k|k-1}^x) - \hat{y}_k^-\right]\left[h(\chi_{i,k|k-1}^x) - \hat{y}_k^-\right]^T \tag{10}$$

$$P_{x_k y_k} = \sum_{i=0}^{2L} W_i^{(c)} \left[\chi_{i,k|k-1}^x - \hat{x}_k^-\right]\left[h(\chi_{i,k|k-1}^x) - \hat{y}_k^-\right]^T \tag{11}$$

$$K_k = P_{x_k y_k} P_{y_k y_k}^{-1} \tag{12}$$

$$\hat{x}_k = \hat{x}_k^- + K_k(y_k - \hat{y}_k^-) \tag{13}$$

$$P_k = P_k^- - K_k P_{y_k y_k} K_k^T \tag{14}$$

The equations (2)~(4) are preprocessed before processing the main SPKF. Then the SPKF is processed using the equations (5)~(14), recursively.

It is well-known fact that the SPKF can overcome the flaws of the EKF such as inaccurate Jacobian matrices caused by the linear approximations of nonlinear functions with large initial estimation error. Therefore, it can be expected that the SPKF can drive the DR/GPS hybrid navigation system no mater what the estimated initial heading error is large.

B. Receding Horizon Kalman FIR Filter

If a filter has a model uncertainty or an unknown temporary time-varying bias, the estimation performance is dependent upon the filter property. Unfortunately, the EKF cannot estimate the state variables exactly because the EKF has an IIR structure. In order to enhance the filter performance in the system that has a model uncertainty or a time-varying bias, this paper introduces the RHKF filter.

Figure 1 shows the concept of the RHKF filter. As can be seen in this figure, the current state, x_k, is estimated only using the current measurements on the horizon $[k-N, k]$ (N is a horizon size). The RHKF filter has a fast estimation property and is influenced restrictively by the errors such as model uncertainty, temporary time-varying bias, etc. due to the FIR construction. And it can be also utilized irrespective of singularity problems caused by unknown information about the horizon initial state in the linear systems. However, the research on the RHKF filter for nonlinear systems is insufficient by this time. The linear filters for nonlinear systems need the linearization of the nonlinear functions, which problem has decelerated the studies of the RHKF filter for nonlinear systems [5]. In order to apply the merits of the RHKF filter into the DR/GPS hybrid navigation system, this paper utilizes the concept of the sigma point. And an advanced RHKF filter for nonlinear systems is presented in the next chapter.

III. RHSPKF FILTER

In this chapter, a novel filter, called receding horizon sigma point Kalman FIR filter, is proposed by merging the two filters introduced in the previous chapter. And a tightly coupled DR/GPS hybrid navigation system is designed using the RHSPKF filter.

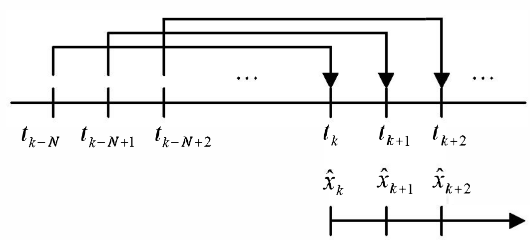


Figure 1. Concept of the RHKF filter

A. RHSPKF Filter

The RHKF filter is designed using the inverse covariance form of the Kalman filter because it is assumed that the initial information of the states is unknown in the linear system. So, the initial value of the inverse covariance matrix is set by 0. However, the initial information must be obtained with small error in the nonlinear system because of the linear approximations of nonlinear functions. So, the RHKF filter has a restriction in the linearization process. In this paper, the RHSPKF filter is designed to weaken the restriction of the RHKF filter. As mentioned previously, the SPKF works well even in case of large initial estimation errors. The RHSPKF filter merges the merits of the RHKF filter and the SPKF to guarantee the robustness in the state estimation.

The concept of the RHSPKF filter is shown in Figure 2. In this figure, k_N means the receding interval. The SPKF driven from time t_k provides the estimated solution in the interval $\left[t_{k+k_N}, t_{k+2k_N}\right]$. Simultaneously, the SPKF for the posterior horizon is processed from time t_{k+k_N}. And the estimated solution is provided by the SKPF for the posterior horizon in the interval $\left[t_{k+2k_N}, t_{k+3k_N}\right]$.

The RHSPKF filter has three advantages over the EKF, RHKF filter, and SPKF. First, the RHSPKF filter has a robust estimation property by the FIR characteristics of the RHKF filter. Secondly, the RHSPKF filter also has robustness to the horizon initial condition due to the strong point of the SPKF. Finally, the RHSPKF filter solved the heavy computational burden of the RHKF filter by extending the receding interval from 1 to k_N.

B. Tightly Coupled DR/GPS using the RHSPKF Filter

It is well-known factor that the visibility of the satellites is low in the urban areas. In these ill-conditioned environments, tightly coupled method is better than loosely coupled method in the DR/GPS implementation. Figure 3 shows the block diagram of the tightly coupled DR/GPS hybrid navigation system using the RHSPKF filter.

It is assumed that the DR system is constructed by an accelerometer and a gyro. The accelerometer measures the forward acceleration of the vehicle and the gyro measures the z-axis angular velocity.

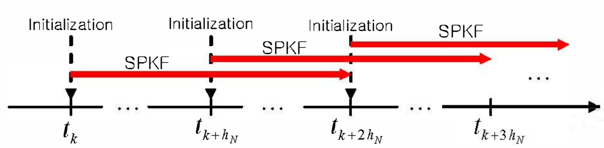


Figure 2. Concept of the RHSPKF filter

The states to be estimated are set by 2D position on the navigation frame (P_N, P_E), velocity on the body frame (V^b), azimuth (θ_z), accelerometer bias (∇), gyro bias (ε), and receiver clock bias in meters (c).

First, the sigma points are generated using (5). Then the time propagation is processed as follows:

for j=1 : 2L+1

$$\chi(4,j)=\chi(4,j)+\left(g_z-\chi(6,j)\right)dt \quad (15)$$

$$\chi(3,j)=\chi(3,j)+\left(a_x-\chi(5,j)\right)dt \quad (16)$$

$$\chi(1,j)=\chi(1,j)+\chi(3,j)\cos\left(\chi(4,j)\right)dt \quad (17)$$

$$\chi(2,j)=\chi(2,j)+\chi(3,j)\sin\left(\chi(4,j)\right)dt \quad (18)$$

end

where a_x denotes the accelerometer output and g_z means the gyro output.

In the EKF, the relations between states are denoted clearly in the Jacobian matrix. In the RHSPKF filter, the relations are shown in the time propagation of the sigma points as equations (15)~(18).

After time propagation, the pseudorange domain information is generated to process the measurement update. In this paper, only pesudorange is used in the filter measurement. The pseudorange is calculated as follows:

for j=1 : 2L+1

$$\hat{\rho}_i(j)=\sqrt{\left(x_i-x_u\right)^2+\left(y_i-y_u\right)^2+\left(z_i-z_u\right)^2}+\chi(7,j) \quad (19)$$

end

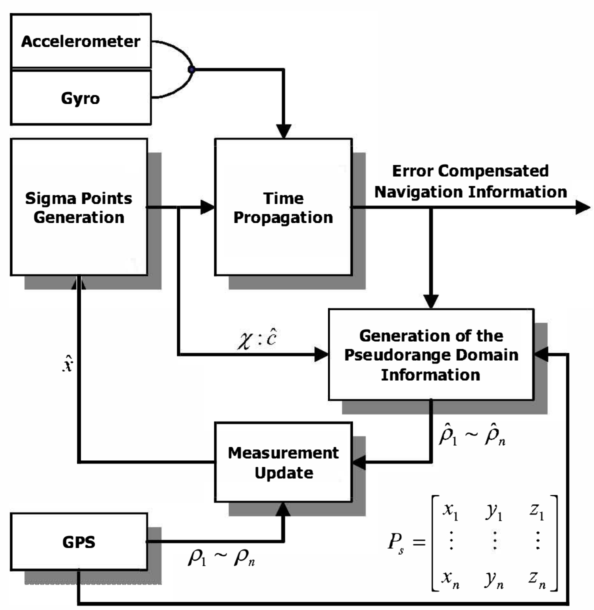


Figure 3. Block diagram of the tightly coupled DR/GPS using the RHSPKF filter

where $[x_i \quad y_i \quad z_i]$ is the i^{th} satellite position and can be obtained from the GPS receiver. $[x_u \quad y_u \quad z_u]$ is the user position on the ECEF frame and can be calculated as follows:

$$[L_u \quad l_u \quad h_u] = [L_0 + P_N / R_e \quad l_0 + P_E / R_e \cos L \quad h_0] \quad (20)$$

$$\begin{bmatrix} x_u \\ y_u \\ z_u \end{bmatrix} = \begin{bmatrix} (R_e + h_u)\cos L_u \cos l_u \\ (R_e + h_u)\cos L_u \sin l_u \\ (R_e + h_u)\sin L_u \end{bmatrix} \quad (21)$$

where $[L_u \quad l_u \quad h_u]$ means the user latitude, longitude and height. $[L_0 \quad l_0 \quad h_0]$ denotes the user's initial position, and R_e is the earth radius.

Finally, the measurement update is carried out by equations (10)~(14).

As can be seen in this chapter, the RHSPKF filter does not have any complex Jacobian matrixes even in the tightly couple DR/GPS hybrid navigation system. Moreover, this filter does not have complex equations required in the RHKF filter. Therefore, the proposed filter can be easily utilized to implement the DR/GPS hybrid navigation system.

IV. SIMULATION AND RESULTS

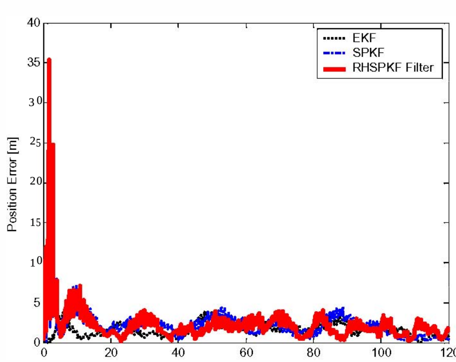
In order to verify the performance of the proposed filter, some simulations are carried. The four situations are made and the EKF, the SPKF, and the RHSPKF filter are driven in these situations. Then the performance of these filters is compared. The simulation results are summarized in Table 1.

A. Situation I

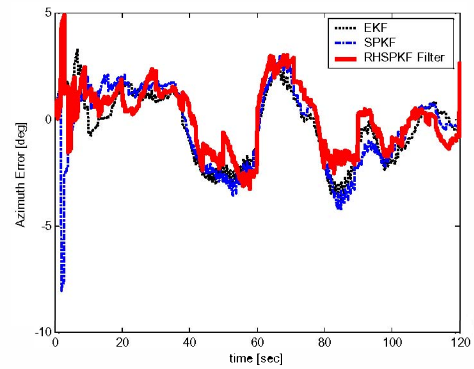
- Sensor bias is random walk.
- The filters consider the sensor bias as random walk.

Usually, the biases of low-cost inertial sensors show non-zero mean and non-stationary behavior, the errors are modeled as random walk as follows:

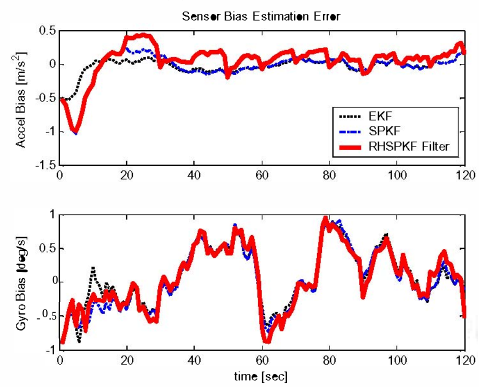
$$\nabla_k = \nabla_{k-1} + w_{\nabla,k}, \; w_\nabla \sim N(0, \sigma_\nabla) \quad (22a)$$



(a) Position error



(b) Azimuth error



(c) Sensor bias estimation error

Figure 4. Results of the situation I

$$\varepsilon_k = \varepsilon_{k-1} + w_{\varepsilon,k}, \; w_\varepsilon \sim N(0, \sigma_\varepsilon) \quad (22b)$$

where the process noise must be set by σ_∇ and σ_ε.

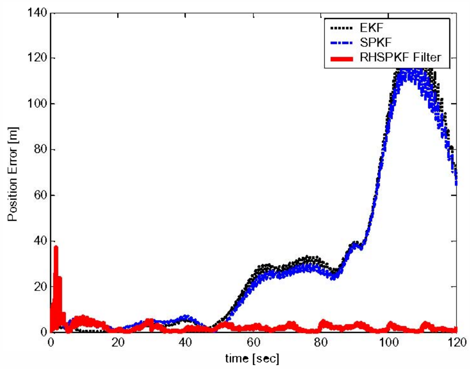
As can be seen in Figure 4, the performance of the filters is similar to one another.

B. Situation II

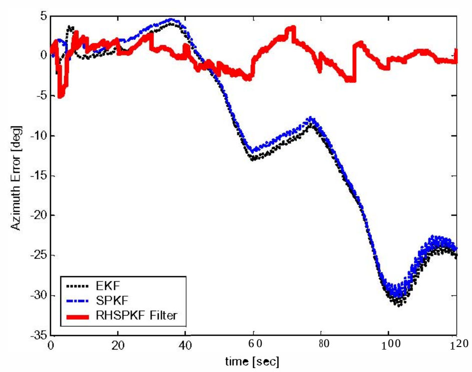
- Sensor bias is random walk.
- The filters consider the sensor bias as random constant.

In general, the biases of inertial sensors can be modeled as random constant. However, the biases of low-cost inertial sensors may have random walk process. In this situation, the biases are modeled as random walk. But the filters are considered the sensor biases as random constant.

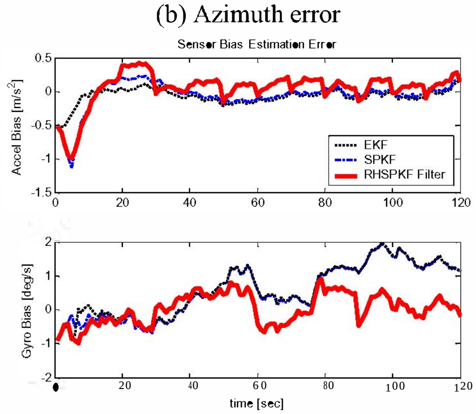
As can be seen in Figure 5, the estimation errors of the EKF and the SPKF diverge gradually. First, the gyro bias estimation error increases with time. Second, the azimuth error is expanded under the influence of the gyro bias estimation error. Finally, the position data diverges. On the other hand, the RHSPKF filter has bounded errors. Therefore, the RHSPKF filter is robust against the model uncertainty.



(a) Position error



(b) Azimuth error



(c) Sensor bias estimation error

Figure 5. Results of the situation II

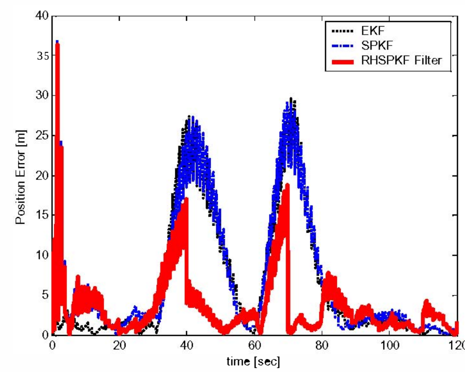
C. Situation III

- Sensor bias is random walk.
- The filters consider the sensor bias as random walk.
- Accelerometer error has a temporary unknown bias.

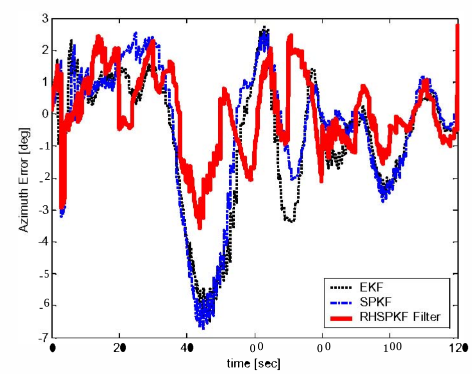
In this situation, a temporary unknown bias is occurred in the accelerometer as follows:

$$\nabla_k = \nabla_{k-1} + B_{a,k} + w_{\nabla,k},\ w_\nabla \sim N(0, \sigma_\nabla)$$
$$B_{a,k} = \begin{cases} 2[m/s^2] & ,30 \le k \le 60 \\ 0 & ,otherwise \end{cases} \tag{23}$$

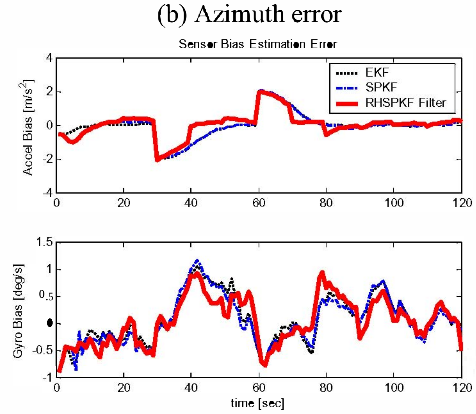
The result is shown in Figure 6. It can be seen that the errors of the RHSPKF filter are less than that of the EKF and the SPKF. The reason is that the RHSPKF filter is influenced restrictively by the unknown bias due to the FIR construction.



(a) Position error



(b) Azimuth error



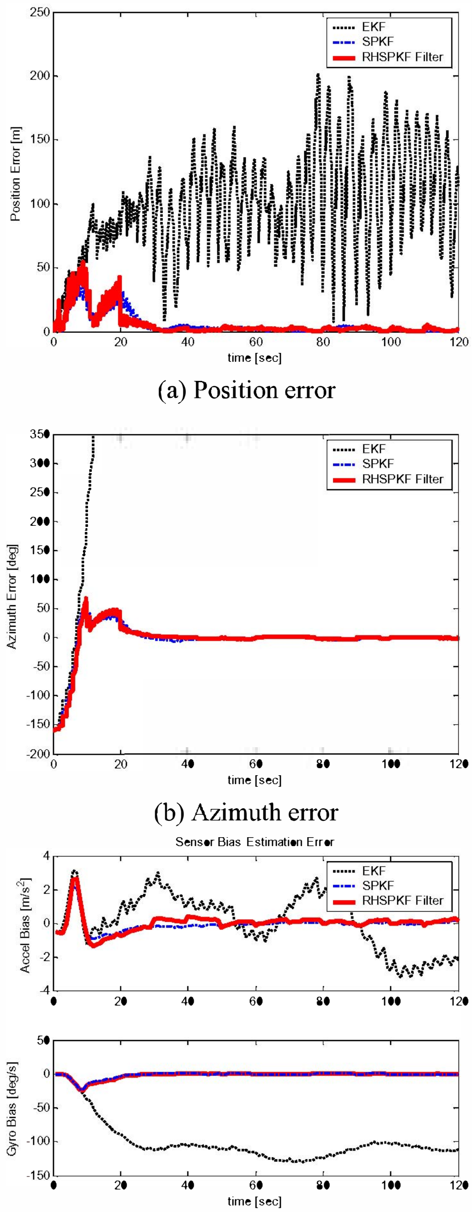
(c) Sensor bias estimation error

Figure 6. Results of the situation III

D. Situation IV

- Initial azimuth has a large error.

The initial azimuth information cannot be obtained unless a magnetic compass or a high-grade gyro module is utilized. Therefore, the initial azimuth error exists unavoidably. In this situation, the initial azimuth error is set by 160degrees. Figure 7 shows the simulation results.



(a) Position error

(b) Azimuth error

(c) Sensor bias estimation error

Figure 7. Results of the situation IV

Table 1. Results of the simulation
(Mean value of the estimation error)

		Position [m]	Azimuth [deg]	Acc Bias [m/s^2]	Gyro Bias [deg/sec]
I	(1)	1.4858	-0.3429	0.0293	0.0742
	(2)	2.3736	-0.2792	0.0444	0.0417
	(3)	2.1889	-0.0290	0.0521	0.0328
II	(1)	32.1380	-9.6005	0.0655	0.5828
	(2)	30.5878	-8.8680	0.0554	0.5376
	(3)	2.3861	-0.0038	0.0452	0.0444
III	(1)	6.3339	-0.7554	0.0410	0.0623
	(2)	7.0553	-0.4358	0.0550	0.0351
	(3)	3.5784	-0.0046	0.0413	0.0406
IV	(1)	94.4472	∞	0.1204	-99.9573
	(2)	5.7376	-3.4950	0.0702	-1.3536
	(3)	5.7813	-3.0563	0.0189	-1.6293

(1) EKF (2) SPKF (3) RHSPKF filter

As can be seen in Figure 7, the EKF errors diverge with time. However, the SPKF and the RHSPKF filter have good performance. This phenomenon is owing to the Jacobian matrix error. The SPKF and the RHSPKF filter need not to calculate the Jacobian matrix. Therefore, the proposed filter is robust to the initial large estimation error, also.

V. CONCLUSION

The RHSPKF filter for tightly coupled DR/GPS hybrid navigation system is developed and simulated in the various situations. The proposed filter has a robust estimation property by the FIR strategy. This filter also has robustness to the initial large estimation error due to the merit of the SPKF. And the flaw of the RHKF filter, heavy computational burden, is overcome in this filter. It can be expected that the RHSPKF filter can be utilized in DR/GPS hybrid navigation system with robust properties.

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