GPS-BASED MEASUREMENTS AND ESTIMATION OF THE TIE OF A CRYSTAL CLOCK USING AN UNBIASED FIR FILTERING ALGORITHM

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Abstract **– An unbiased finite impulse response (FIR) filtering algorithm is investigated for the GPS-based measurements of a local crystal clock. The algorithm is examined for the time interval error (TIE) measurements in presence of the uniformly distributed sawtooth noise induced by the multichannel GPS timing receiver. Based upon, we show that the unbiased FIR estimates are consistent with the reference (rubidium) measurements and fit them better than the standard Kalman filter.**
Keywords**:** GPS-based timekeeping, unbiased estimate, FIR filter

INTRODUCTION

Fast and accurate estimation and adjustment of a local clock performance, making possible for a variety of modern digital systems to operate in common time with minimum "slips", is of importance for the Global Positioning System (GPS)-based timekeeping [1,2]. To obtain filtering in an optimum way, the time interval error (TIE) model of a local clock must be known for the filter memory. In the discrete time, such a model [3] may be written as $x_1(n) = x_1(0) + x_2(0)\tau n + \frac{x_3(n)}{2}\tau^2 n^2 + w_1(n,\tau)$, where, where $n = 0,1,\ldots$; $\tau = t_n - t_{n-1}$ is the a time step multiple to 1 s; t_n is the discrete time; $x_1(0)$ is the initial time error; $x_2(0)$ is the initial fractional frequency offset of a local clock from the reference frequency; $x_3(0)$ is the initial linear fractional frequency drift rate; and $w_1(n,\tau)$ is the random component caused by the oscillator noise and environment. In GPS-based measurements, the model is observed via the mixture $\xi_1(n) = x_1(n) + v_1(n)$, in which $v_1(n)$ is the noisy component induced at the receiver (noise of a measurement set is usually small). In modern receivers [4], a random variable $v_1(n)$ is uniformly distributed owing to the sawtooth noise caused by a principle of the 1 PPS (one pulse per second) signal formation. To estimate the states of the clocks, we have studied several filtering algorithms [5-10], among which, an unbiased moving average filter for the linear clock model was proposed in [11]. An unbiased approach was then generalized in [12] in the finite impulse response (FIR) unbiased filtering algorithms for the clock model of the K-degree. In this paper, we investigate this algorithm for the GPS-based measurements of the TIE model of a local crystal clock in presence of the sawtooth noise induced by the receiver.

UNBIASED FIR FILTERING ALGORITHM

Most commonly, the TIE polynomial model projects ahead on a horizon of N points from the start point $n = 0$ with the K-degree Taylor polynomial

$$x_1(n) = \sum_{p=0}^{K} x_{p+1} \frac{\tau^p n^p}{p!} + w_1(n,\tau), \tag{1}$$

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where $x_{l+1} \equiv x_{l+1}(0)$, $l \in [0,K]$, are the initial states of the clock and $w_1(n,\tau)$ is the noise with known properties. By extending the time derivatives of the TIE model to the Taylor series, the signal and observation equations become, respectively,

$$\lambda(n) = \mathbf{A}(n)\lambda(0) + \mathbf{w}(n,\tau), \text{ (2)}, \qquad \xi(n) = \mathbf{C}(n)\lambda(0) + \mathbf{v}(n), \text{ (3)}$$

where $\lambda(n) = [x_1(n)x_2(n)...x_{K+1}(n)]^T$ is the vector, $(K+1)\times 1$, of the clock states and a time-varying transition matrix, $(K+1)\times(K+1)$, is

$$\mathbf{A}(n) = \begin{bmatrix} 1 & \tau n & \tau^2 n^2 & \dots & (\tau n)^K / K! \\ 0 & 1 & \tau n & \dots & (\tau n)^{K-1}/(K-1)! \\ 0 & 0 & 1 & \dots & (\tau n)^{K-2}/(K-2)! \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}. \tag{4}$$

For $M=K-1$, the observation vector is $\xi(n) = [\xi_1(n)\xi_2(n)...\xi_M(n)]^T$ and a measurement matrix $\mathbf{C}$ of $(K+1)\times(K+1)$ is the typically unit. Here $\mathbf{w}(n,\tau) = [w_1(n,\tau)w_2(n,\tau)...w_{K+1}(n,\tau)]^T$ is the vector of the oscillator noise and $\mathbf{v}(n) = [v_1(n)v_2(n)...v_M(n)]^T$ is the measurements noise that is not obligatory Gaussian.

The algorithm works as follows. The clock first state estimate $\hat{x}_1(n)$ is obtained with $h_K(i)$ at a horizon of N_K points. The observation $\xi_2(n)$ for the second state $x_2(n)$ is then formed by increments of $\hat{x}_1(n)$. Accordingly, $\hat{x}_2(n)$ is achieved with $h_{K-1}(i)$ at a horizon of N_{K-1} points. Inherently, the first accurate value of $\hat{x}_2(n)$ appears at $(N_K + N_{K-1} - 2)$th point starting from $n = 0$. Finally, the last state estimate $\hat{x}_{K+1}(n)$ is calculated with $h_0(i)$ at a horizon of N_0 points, using $\xi_{K+1}(n)$ that is formed in the same manner as $\xi_2(n)$. The first correct value of $\hat{x}_{K+1}(n)$ appears at $(N_K + N_{K-1} + ... + N_0 - K - 1)$th point. For $K = 2$, the 3-state unbiased FIR batch algorithm becomes

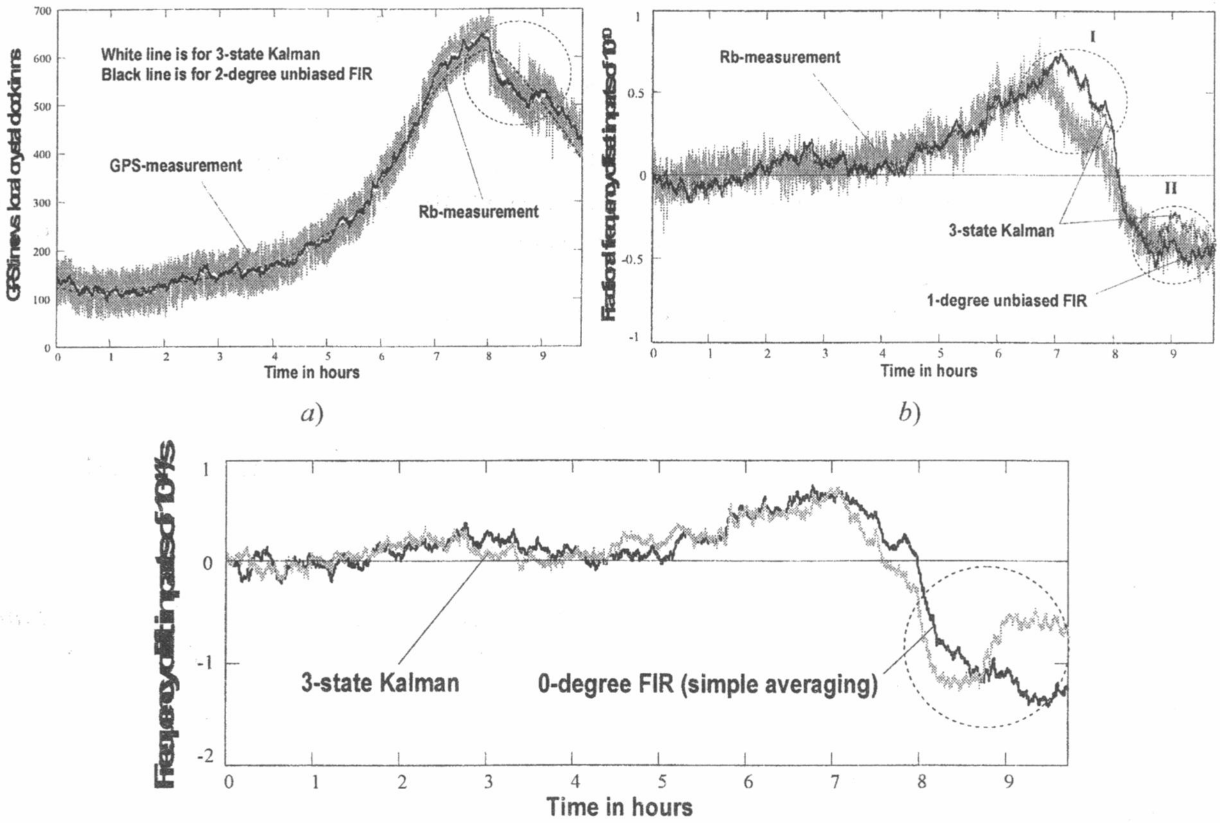
$$\hat{x}_1(n) = \sum_{i=0}^{N_2-1} h_2(i)\xi_1(n-i), \text{ (5)}, \quad \hat{x}_2(n) = \frac{1}{\tau}\sum_{j=0}^{N_1-1} h_1(j)[\hat{x}_1(n-j) - \hat{x}_1(n-j-1)], \text{ (6)}$$

$$\hat{x}_3(n) = \frac{1}{\tau N_0}\sum_{r=0}^{N_0-1}[\hat{x}_2(n-r) - \hat{x}_2(n-r-1)], \tag{7}$$

where the unique FIRs $h_1(i)$ and $h_2(i)$ are given in [12], respectively,

$$h_1(i) = \frac{2(2N-1)-6i}{N(N+1)}, \text{ (8)}, \qquad h_2(i) = \frac{3(3N^2-3N+2)-18(2N-1)i+30i^2}{N(N+1)(N+2)}. \text{ (9)}$$

Below, we use this algorithm to estimate the TIE model of an oven crystal clock embedded to the Stanford Frequency Counter SR620. The measurement is done with the GPS timing sensor SynPaQ III and SR620 for $\tau = 1$ s (GPS-measurement). Simultaneously, to get a reference trend, the TIE of the same crystal clock is measured, by SR625, for the rubidium clock (Rb-measurement). The initial time and frequency shifts between two measurements are then eliminated statistically and a transition to $\tau = 10\,\mathrm{s}$ is provided by the data thinning in time. At the early stage, the TIE model was identified to be quadratic, $K = 2$, and the horizon N for each estimate is determined in the minimum MSE sense. We also compare the unbiased FIR estimates to those obtained with the 3-state standard Kalman filter.



c)

Fig. 1 – Short-term measurements and estimation of the crystal clock TIE model with the 3-state unbiased FIR algorithm and the 3-state Kalman filter: (*a*) TIE, (*b*) fractional frequency offset, and (*c*) linear fractional frequency drift rate.

Several hours measurements. In this experiment, a short-term measurement of the TIE has been done during several hours (Fig. 1*a*). The algorithm then was run. The horizons were identified for $\tau = 10\,\mathrm{s}$ to be $N_1 = 155$ or 0.43 hours, $N_2 = 950$ or 2.64 hours, and $N_3 = 860$ or 2.39 hours for the Rb-measurements. Thereafter, we set the values of q's in the Kalman filter to obtain the minimum MSEs for the FIR estimates. Figure 1 illustrate the studies, showing that the unbiased FIR estimates, $\hat{x}_1(n)$, $\hat{x}_2(n)$, and $\hat{x}_3(n)$, and the relevant Kalman estimates, $\hat{x}(n)$, $\hat{y}(n)$, and $\hat{z}(n)$, respectively, are consistent with, however, some differences. It follows that the FIR filter works accurately. Figure 1*a* shows that $\hat{x}_1(n)$ and $\hat{x}(n)$ track the mean value of the GPS-measurement and that their offsets from the Rb-measurement are coursed mostly by the GPS time uncertainty. In this experiment, a maximum estimate error of about 60 ns was indicated between 8^{th} and 9^{th} hours when a time shift in the 1 PPS signal has occurred. It follows (Fig. 1*b*) that $\hat{x}_2(n)$ and $\hat{y}(n)$ fit well weighted by $1/\tau$ the increments of the Rb-measurement. Even so, there are two special ranges (dashed). In the range I, the frequency shift of about 3×10^{-11} has occurred in the span between 7^{th} and 8^{th} hours and no appreciable error is indicated in a range of large time shifts (between 8^{th} and 9^{th} hours in Fig. 1*a*). We associate it with the frequency shift in SR625. In the range II, the Kalman filter demonstrates a brightly pronounced instability caused likely by the temporary model uncertainty, whereas the FIR estimate is still consistent. We watch for a bit shifted trends of $\hat{x}_3(n)$ and $\hat{z}(n)$ in Fig. 1*c* that may be explained by some inconsistency between the q's and N_1. It is also seen that $\hat{z}(n)$ traces much upper $\hat{x}_3(n)$ after about 8.7 hours. We associate it with the Kalman filter instability, like the case of a range II in Fig. 1*a*.

Conclusions

In this paper, we presented the results of investigations of an unbiased FIR filter for the GPS-based measurements of the TIE K-degree polynomial model of a local crystal clock. The trade-off between the 3-state unbiased FIR algorithm and the 3-state standard Kalman algorithm has shown their consistency. However, as it was demonstrated experimentally, the FIR filter produces a smaller error and a lower Allan variance for the sawtooth noise.

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