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Fuzzy failure modes and effects analysis by using fuzzy TOPSIS-based fuzzy AHP

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ABSTRACT

Failure mode and effects analysis (FMEA) is a widely used engineering technique for designing, identifying and eliminating known and/or potential failures, problems, errors and so on from system, design, process, and/or service before they reach the customer (Stamatis, 1995). In a typical FMEA, for each failure modes, three risk factors; severity (*S*), occurrence (*O*), and detectability (*D*) are evaluated and a risk priority number (RPN) is obtained by multiplying these factors. There are significant efforts which have been made in FMEA literature to overcome the shortcomings of the crisp RPN calculation. In this study a fuzzy approach, allowing experts to use linguistic variables for determining *S*, *O*, and *D*, is considered for FMEA by applying fuzzy 'technique for order preference by similarity to ideal solution' (TOPSIS) integrated with fuzzy 'analytical hierarchy process' (AHP). The hypothetical case study demonstrated the applicability of the model in FMEA under fuzzy environment.

1. Introduction

Failure mode and effects analysis (FMEA) is a widely used engineering technique for designing, identifying and eliminating known and/or potential failures, problems, errors and so on from system, design, process, and/or service before they reach the customer (Stamatis, 1995). FMEA, providing a framework for cause and effect analysis of potential product failures (Chin, Chan, & Yang, 2008), has a purpose of prioritizing the risk priority number (RPN) of the product design or planning process to assign the limited resources to the most serious risk item (Chang, Wei, & Lee, 1999).

FMEA, designed to provide information for risk management decision-making (Pillay & Wang, 2003), was first developed as a formal design methodology by NASA in 1963 for their obvious reliability requirements and then, it was adopted and promoted by Ford Motor in 1977 (Chin et al., 2008). Since then, it has become a powerful tool extensively used for safety and reliability analysis of products and processes in a wide range of industries especially, aerospace, nuclear and automotive industries (Gilchrist, 1993; Sharma, Kumar, & Kumar, 2005).

A typical FMEA is consisted of the following components; the identification and listing of failure modes and the consequent faults, assessment of the chances of the occurrence of faults, then assessment of the chances of the detection of faults, assessment of the severity of the consequences of the faults, calculation of a measure of the risk, the ranking of the faults based on the risk, taking action on the high-risk problems, and checking the effectiveness of the action with the use of a revised risk measurement (Ben-Daya & Raouf, 1996).

Each failure mode can be evaluated by three factors as severity, likelihood of occurrence, and the difficulty of detection of the failure mode. In a typical FMEA evaluation, a number between 1 and 10 (with 1 being the best and 10 being the worst case) is given for each of the three factors. By multiplying the values for severity (*S*), occurrence (*O*), and detectability (*D*), a risk priority number (RPN) is obtained, which is RPN = $S \times O \times D$ (Chin et al., 2008). Then the RPN value for each failure mode is ranked to find out the failures with higher risks.

The crisp values of RPNs have been considerably criticized for a many reasons most of which are stated below (Ben-Daya & Raouf, 1996; Bowles, 2004; Braglia & Bevilacqua, 2000; Braglia, Frosolini, & Montanari, 2003; Chang, Liu, & Wei, 2001; Gilchrist, 1993; Pillay & Wang, 2003; Sankar & Prabhu, 2001; Wang, Chin, Poon, & Yang, 2009):

- The relative importance among the three risk factors occurrence, severity, and detection is not considered as they are accepted equally important.
- Different combinations of *O*, *S* and *D* may produce exactly the same value of RPN, although their hidden risk implications may be totally different. For instance, two different failures with the *O*, *S* and *D* values of 4, 3, 3 and 9, 1, 3, respectively, have the same RPN value of 36.
- It is mostly difficult for *O*, *S* and *D* to be precisely evaluated. However linguistic terms can be adopted to convey much information in FMEA.

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- The use of multiplication method in the calculation of RPN is questionable and strongly sensitive to variations in criticality factor evaluations.

When the traditional FMEA and the fuzzy approach are compared, the fuzzy approach has an advantage of allowing the conduction of risk evaluation and prioritization based on the knowledge of the experts (Tay & Lim, 2006).

Xu, Tang, Xie, Ho, and Zhu (2002) state the reasons for considering the fuzzy logic approach as following:

- All FMEA-related information is taken in natural language which is easy and plausible for fuzzy logic to deal with as it is based on human language and can be built on top of the experience of experts.
- Fuzzy logic allows imprecise data usage so it enables the treatment of many states.

Furthermore, fuzzy FMEA allows both quantitative data and vague and qualitative information to be used and managed in a consistent manner and makes it possible for the combination of severity, occurrence and detectability in a more flexible structure (Bowles & Pelaez, 1995; Braglia et al., 2003).

In this study firstly, a fuzzy approach, allowing experts to use linguistic variables for determining *S*, *O*, and *D*, is considered for FMEA by applying fuzzy TOPSIS integrated with fuzzy AHP. First Chang's (1996) fuzzy AHP is utilized to determine the weight vector of three risk factors; severity, occurrence and detectability. Then by using the linguistic scores of risk factors for each failure modes, and the weight vector of risk factors, Chen's (2000) fuzzy TOPSIS is utilized. According to the results most important failure modes are obtained. This model allowing the use of different importance weights for the risk factors (*S*, *O*, *D*) in fuzzy TOPSIS for scoring and ranking of the potential failure modes, can be taken as a contribution in the fuzzy FMEA literature.

The rest of the paper is organized as follows: In Section 2, Literature Reviews of fuzzy FMEA, fuzzy AHP and fuzzy TOPSIS are expressed. In Section 3, a fuzzy multi-criteria method, an integration of fuzzy AHP and fuzzy TOPSIS, is proposed for fuzzy FMEA. In Section 4, the proposed methodology is applied to an assembly process with 8 potential failure modes at a manufacturing facility. A sensitivity analysis is also realized. Finally, conclusions are given.

2. Literature review

2.1. Fuzzy FMEA

There are significant efforts have been made in FMEA literature to overcome the shortcomings of the traditional RPN (Wang et al., 2009). The studies about FMEA considering fuzzy approach use the experts who describe the risk factors *O*, *S*, and *D* by using the fuzzy linguistic terms. The linguistic variables were used for evaluating three risk factors *O*, *S*, and *D* as an interpretation of the traditional 10-point scale (1–10) FMEA factor scores.

In the fuzzy FMEA literature, the studies have mostly concerned with the fuzzy rule-base approach by using if-then rules (Bowles & Pelaez, 1995; Chin et al., 2008; Guimarães & Lapa, 2004, 2007; Pillay & Wang, 2003; Sharma et al., 2005; Tay & Lim, 2006; Xu et al., 2002). After the assignments of the linguistic terms to the factors, if-then rules were generated taking the linguistic variables as inputs to evaluate the risks. The outputs of the fuzzy inference system were variously named as *risk* (Chin et al., 2008; Guimarães & Lapa, 2004), the critically failure mode (Xu et al., 2002), priority for attention (Pillay & Wang, 2003), and fuzzy RPN (Sharma et al., 2005; Xu et al., 2002) in the fuzzy FMEA studies which consider the fuzzy rule-base approach.

Braglia and Bevilacqua (2000) drew attention to the doubts remained due to the difficulties in defining many rules and membership functions required by this methodology considering the applicability of the real industrial cases. They proposed the use of AHP for obtaining the rules for a particular fuzzy criticality assessment model to overcome this problem. Besides, AHP is employed in another study to cope with multiple criteria situations involving intuitive, rational, qualitative and quantitative aspects for the evaluation of the final ranking for every failure cause and this new approach is called multi-attribute failure mode analysis (MAFMA) (Braglia, 2000).

Braglia and Bevilacqua (2000) criticize that the failure modes characterized by the fuzzy if–then rules could not be prioritized or ranked and there is no way to incorporate the relative importance of risk factors into the fuzzy inference system by using fuzzy if–then rules. Therefore they develop a new fuzzy logic approach where fuzzy risk priority numbers (FRPNs) are defined as fuzzy weighted geometric means of the fuzzy ratings for *O*, *S* and *D* and can be computed using alpha-level sets and linear programming models.

The fuzzy analytic hierarchy process (FAHP) approach was considered by Hua, Hsu, Kuo, and Wua (2009) for evaluating the relative weightings of the risk factors of FMEA to analyze of the risks of green components in compliance with the European Union (EU) the Restriction of Hazardous Substance (RoHS) directive in the incoming quality control (IQC) stage. In the study, Severity factor was explained by two criteria and with considering the occurrence and the detection factors, the FAHP was utilized to determine the weights of four criteria by two experts. The traditional FMEA was modified to form green component risk priority number (GC-RPN) for the calculation of the risks with regard to each category of green components. GC-RPN was formulated by the sum of the terms of products of the factor scores and weights.

Braglia et al. (2003) proposed a fuzzy TOPSIS approach for Failure Mode, Effects and Criticality Analysis (FMECA). The fuzzy version of TOPSIS was applied allowing the traditional FMECA factors *O*, *S*, and *D* and their equally important weights to be evaluated using triangular fuzzy numbers.

2.2. Fuzzy AHP

AHP is one of the well-known multi-criteria decision making techniques that was first proposed by Saaty (1980). The classical AHP takes into consideration the definite judgments of decision makers (Wang & Chen, 2007). Although the classical AHP includes the opinions of experts and makes a multiple criteria evaluation, it is not capable of reflecting human's vague thoughts (Seçme, Bayrakdaroğlu, & Kahraman, 2009).

As the uncertainty of information and the vagueness of human feeling and recognition, it is difficult to provide exact numerical values for the criteria and to make evaluations which exactly convey the feeling and recognition of objects for decision makers. Therefore, most of the selection parameters cannot be given precisely. Thus experts may prefer intermediate judgments rather than certain judgments. So the fuzzy set theory makes the comparison process more flexible and capable to explain experts' preferences (Kahraman, Cebeci, & Ulukan, 2003).

Different methods for the fuzzification of AHP have been proposed in the literature. AHP is firstly fuzzified by Laarhoven and Pedrycz (1983) and in this study, fuzzy ratios which were defined by triangular membership functions were compared. Buckley (1985) used the comparison ratios based on trapezoidal membership functions. Chang (1996) introduces a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pair-wise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons. Kahraman, Ulukan, and Tolga (1998) proposed a fuzzy objective and subjective method based on fuzzy AHP. Kulak and Kahraman (2005) made a selection among the transportation companies by using fuzzy axiomatic design and fuzzy AHP. They developed fuzzy multi-attribute axiomatic design approach and compared it with fuzzy AHP.

2.3. Fuzzy TOPSIS

TOPSIS one of the classical multi-criteria decision making methods was developed by Hwang and Yoon (1981). It is based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). TOPSIS also provides an easily understandable and programmable calculation procedure. It has the ability of taking various criteria with different units into account simultaneously (Ekmekçioglu, Kaya, & Kahraman, 2010).

A number of fuzzy TOPSIS methods have been developed in recent years. Chen and Hwang (1992) first applied fuzzy numbers to establish fuzzy TOPSIS. Triantaphyllou and Lin (1996) developed a fuzzy TOPSIS method in which relative closeness for each alternative is evaluated based on fuzzy arithmetic operations. Chen (2000) extends the TOPSIS method to fuzzy group decision making situations by considering triangular fuzzy numbers and defining crisp Euclidean distance between two fuzzy numbers. Chu (2002) and Chu and Lin (2002) further improved the methodology proposed by Chen (2000). Jahanshahloo, Hosseinzadeh, and Izadikhah (2006) and Chu and Lin (2009) extended the fuzzy TOPSIS method based on alpha level sets with interval arithmetic.

Fuzzy TOPSIS has been introduced for various multi-attribute decision-making problems. Yong (2006) used fuzzy TOPSIS for plant location selection and Chena et al. (2006) used fuzzy TOPSIS for supplier selection. Kahraman, Çevik, Ateş, and Gülbay (2007) utilized fuzzy TOPSIS for industrial robotic system selection. Ekmekçioglu, Kaya, and Kahraman (2010) used a modified fuzzy TOPSIS to select municipal solid waste disposal method and site. Kutlu and Ekmekçioğlu (2010) used fuzzy TOPSIS integrated with fuzzy AHP to propose a new FMEA 'failure modes & effects analysis' which overcomes the shortcomings of traditional FMEA. Kaya and Kahraman (2011) proposed a modified fuzzy TOPSIS for selection of the best energy technology alternative. Kim, Lee, Cho, and Kim (2011) used fuzzy TOPSIS for modeling consumer's product adoption process.

3. Fuzzy multi-criteria analysis

3.1. Fuzzy logic

A fuzzy set is a class of objects with grades of membership. A membership function is between zero and one (Zadeh, 1965). Fuzzy logic is derived from fuzzy set theory to deal with reasoning that is approximate rather than precise. It allows the model to easily incorporate various subject experts' advice in developing critical parameter estimates (Zimmermann, 2001). In other words, fuzzy logic enables us to handle uncertainty.

There are some kinds of fuzzy numbers. Among the various shapes of fuzzy number, triangular fuzzy number (TFN) is the most popular one. It is represented with three points as follows: $A = (a_1, a_2, a_3)$. The membership function is illustrated in Eq. (1). Let A and B are defined as $A = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3)$. Then $C = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ is the addition of these two numbers. Besides, $D = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$ is the subtraction of them. Moreover, $D = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3)$ is the multiplication of them (Klir & Yuan, 1995; Lai & Hwang, 1995; Zimmermann, 2001).

$$\mu_{\tilde{A}}(x) = \left\{ \begin{array}{c} 0, \; x < a_1 \\ \left(\frac{x-a_1}{a_2-a_1}\right), \; a_1 \leqslant x \leqslant a_2 \\ \left(\frac{a_3-x}{a_3-a_2}\right), \; a_2 \leqslant x \leqslant a_3 \\ 0, \; x > a_3 \end{array} \right\}. \tag{1}$$

3.2. Fuzzy AHP

In the following, Chang's extent analysis method is explained.

Let $X = \{x_1, x_2, \ldots, x_n\}$ be an object set, and $U = \{u_1, u_2, \ldots, u_n\}$ be a goal set. According to the method of extent analysis, each object is taken and extent analysis for each goal is performed, respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs: $\tilde{M}^1_{gi}, \tilde{M}^2_{gi}, \ldots, \tilde{M}^j_{gi}$, where all the $\tilde{M}^j_{gi}$ ($i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$) are TFNs.

The steps of extent analysis can be given as in the following:

Step 1: The value of fuzzy synthetic extent with respect to the ith object is defined as

$$\tilde{S}_i = \sum_{j=1}^{m} \tilde{M}^j_{gi} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{M}^j_{gi} \right]^{-1}. \tag{2}$$

To obtain $\sum_{j=1}^{m} M^j_{gi}$ perform the fuzzy addition operation of m extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} \tilde{M}^j_{gi} = \left[\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right]^{-1} \tag{3}$$

and to obtain $\sum_{i=1}^{n}\sum_{j=1}^{m} \tilde{M}^j_{gi}$ the fuzzy addition operation $\tilde{M}^j_{gi}$ ($j = 1, 2, \ldots, m$) values is performed such as

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{M}^j_{gi} = \left(\sum_{i=1}^{n} \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right) \tag{4}$$

and then the inverse of the above vector is computed in such as

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{M}^j_{gi} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{l} u_i} \right). \tag{5}$$

Step 2: As $\tilde{M}_2$ and $\tilde{M}_2$ are two triangular fuzzy numbers, the degree of possibility of $\tilde{M}_2 \leqslant \tilde{M}_1$ is defined as

$$V(\tilde{M}_2 \geqslant \tilde{M}_1) = sup_{y \geqslant x} \left| min\mu_{\tilde{M}_1}(x), min\mu_{\tilde{M}_2}(y) \right| \tag{6}$$

and can be equivalently expressed as follows:

$$V(\tilde{M}_2 \geqslant \tilde{M}_1) = \mu(d) = \begin{cases} 1, & \text{if } m_2 \geqslant m_1 \\ 0, & \text{if } l_2 \geqslant u_2, \\ \frac{l_2 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise,} \end{cases} \tag{7}$$

where d is the ordinate of the highest intersection point D between $\mu_{\tilde{M}_2}$ and $\mu_{\tilde{M}_2}$ as shown in Fig. 1. To compare $\tilde{M}_2$ and $\tilde{M}_1$, we need both values of $V(\tilde{M}_2 \geqslant \tilde{M}_2)$ and $V(\tilde{M}_1 \geqslant \tilde{M}_2)$

$$d'(A_i) = minV(\tilde{S}_i \geqslant \tilde{S}_k). \tag{8}$$

Step 3: The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers $\tilde{M}_i$ can be defined by

$$V(\tilde{M} \geqslant \tilde{M}_1, \tilde{M}_2, \ldots, \tilde{M}_k) = minV(\tilde{M} \geqslant \tilde{M}_i), \tag{9}$$

where $i = 1, \ldots, k$.

Assume that

$$d'(A_i) = minV(\tilde{S}_i \geqslant \tilde{S}_k). \tag{10}$$

For $k = 1, 2, \ldots; k \neq i$. Then the weight vector is given by

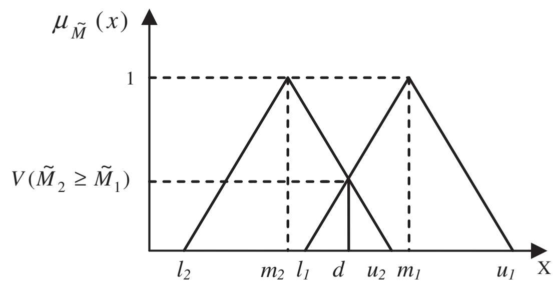

Fig. 1. The intersection between $\tilde{M}_1$ and $\tilde{M}_2$.

$$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T, \tag{11}$$

where A_i ($i = 1, 2, \ldots, n$) are n elements.
Step 4: Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), \ldots, d(A_n))^T, \tag{12}$$

where W is a non-fuzzy number.

Weight vector of risk factors can be obtained by either directly assigning or indirectly using pair-wise comparisons. Here, it is suggested that the decision makers use the linguistic variables in Table 1 to evaluate the weight vector risk factors.

After comparison is made, it is necessary to check the consistency ratio of the comparison. To do so, the graded mean integration approach is utilized for defuzzifying the matrix. According to the graded mean integration approach, a fuzzy number $\tilde{M} = (m_1, m_2, m_3)$ can be transformed into a crisp number by employing the below Eq. (13):

$$P(\tilde{M}) = M = \frac{m_1 + 4m_2 + m_3}{6}. \tag{13}$$

After the deffuzification of each value in the matrix, 'consistency ratio' (CR) of the matrix can easily be calculated and checked whether CR is smaller than .10 or not.

3.3. Fuzzy TOPSIS

In the following, Chen's fuzzy TOPSIS method is explained.

Chen (2000) extends the TOPSIS method to fuzzy group decision making situations by considering triangular fuzzy numbers and defining crisp Euclidean distance between two fuzzy numbers. In Chen's fuzzy TOPSIS, linguistic preferences can easily be converted to fuzzy numbers which are allowed to be used in calculations (Ekmekçioğlu et al., 2010; Önüt & Soner, 2008; Kutlu & Ekmekçioğlu, 2010).

It is suggested that the decision makers use linguistic variables to evaluate the ratings of alternatives with respect to criteria. Table 2 gives the linguistic scale for evaluation of the alternatives. Assuming that a decision group has K people, the ratings of alternatives with respect to each criterion can be calculated as

Table 1
Fuzzy evaluation scores for the weight vector.

Linguistic terms	Fuzzy score
Absolutely strong (AS)	(2, 5/2, 3)
Very strong (VS)	(3/2, 2, 5/2)
Fairly strong (FS)	(1, 3/2, 2)
Slightly strong (SS)	(1, 1, 3/2)
Equal (E)	(1, 1, 1)
Slightly weak (SW)	(2/3, 1, 1)
Fairly weak (FW)	(1/2, 2/3, 1)
Very weak (VW)	(2/5, 1/2, 2/3)
Absolutely weak (AW)	(1/3, 2/5, 1/2)

Table 2
Fuzzy evaluation scores for alternatives.

Linguistic terms	Fuzzy score
Very poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very good (VG)	(9, 10, 10)

$$\tilde{x}_{ij} = \frac{1}{K}[\tilde{x}_{ij}^1(+)\tilde{x}_{ij}^2(+)\cdots(+)\tilde{x}_{ij}^K], \tag{14}$$

where $\tilde{x}_{ij}^K$ is the rating of the Kth decision maker for ith alternative with respect to jth criterion (Chen, 2000).

Obtaining weights of the criteria and fuzzy ratings of alternatives with respect to each criterion, the fuzzy multi-criteria decision-making problem can be expressed in matrix format as

$$D = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \ldots & \tilde{x}_{1n} \\ \vdots & \vdots & \ldots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \ldots & \tilde{x}_{mn} \end{bmatrix}, \tag{15}$$

$$W = [w_1, w_2, \ldots, W_n], j = 1, 2, \ldots, n, \tag{16}$$

where $\tilde{x}_{ij}$ is the rating of the alternative A_i with respect to criterion j (i.e. C_j) and w_j denotes the importance weight of C_j. These linguistic variables can be described by triangular fuzzy numbers: $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$. To avoid the complicated normalization formula used in classical TOPSIS, the linear scale transformation is used here to transform the various criteria scales into a comparable scale. Therefore, we can obtain the normalized fuzzy decision matrix denoted by $\tilde{R}$

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n}, \tag{17}$$

where B and C are the set of benefit criteria and cost criteria, respectively, and

$$\tilde{r} = \left(\frac{\tilde{a}_{ij}}{c_j^*}, \frac{\tilde{b}_{ij}}{c_j^*}, \frac{\tilde{c}_{ij}}{c_j^*}\right), \quad j \in B; \tag{18}$$

$$\tilde{r} = \left(\frac{a_j^-}{c_{ij}}, \frac{b_j^-}{b_{ij}}, \frac{c_j^-}{a_{ij}}\right), \quad j \in C; \tag{19}$$

$$c_j^* = \max_i c_{ij} \quad \text{if } j \in B; \tag{20}$$

$$a_j^- = \min_i a_{ij} \quad \text{if } j \in C. \tag{21}$$

The normalization method mentioned above is to preserve the property that the ranges of normalized triangular fuzzy numbers belong to [0; 1].

Considering the different importance of each criterion, we can construct the weighted normalized fuzzy decision matrix as

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n}, \quad i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n, \tag{22}$$

where

$$\tilde{v}_{ij} = \tilde{r}_{ij}(\cdot)d(C_j). \tag{23}$$

According to the weighted normalized fuzzy decision matrix, we know that the elements $\tilde{v}_{ij}$ $\forall i, j$ are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval [0, 1]. Then, we can define the fuzzy positive-ideal solution ($FPIS, A^*$) and fuzzy negative-ideal solution ($FPIS, A^-$) as

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \ldots, \tilde{v}_n^*), \tag{24}$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-), \tag{25}$$

where

$$\tilde{v}_j^* = (1,1,1) \text{ and } \tilde{v}_j^- = (0,0,0), \quad j = 1,2,\ldots,n. \tag{26}$$

The distance of each alternative from A^* and A^- can be currently calculated as

$$d_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*), \quad i = 1,2,\ldots,m, \tag{27}$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i = 1,2,\ldots,m, \tag{28}$$

where $d(.,.)$ is the distance measurement between two fuzzy numbers calculating with the following formula:

$$d(\tilde{\rho}, \tilde{\tau}) = \sqrt{\frac{1}{3}[(\rho_1 - \tau_1)^2 + (\rho_2 - \tau_2)^2 + (\rho_3 - \tau_3)^2]} \tag{29}$$

where $\tilde{\rho} = (\rho_1, \rho_2, \rho_3)$ and $\tilde{\tau} = (\tau_1, \tau_2, \tau_3)$ are two triangular fuzzy numbers.A closeness coefficient is defined to determine the ranking order of all alternatives once the $\tilde{d}_j^*$ and $\tilde{d}_j^-$ of each alternative A_i ($i = 1, 2, \ldots, m$) are calculated. The closeness coefficient of each alternative is calculated as

$$CC_i = \frac{\tilde{d}_j^-}{\tilde{d}_j^* + \tilde{d}_j^-}, \quad i = 1,2,\ldots,m. \tag{30}$$

Obviously, an alternative A_i is closer to the $(FPIS, A^*)$ and farther from $(FPIS, A^-)$ as CC_i approaches to 1. Therefore, according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

3.4. Proposed methodology

Fuzzy logic is the tool for transforming the vagueness of human feeling and recognition and its decision-making ability into a mathematical formula. It also provides meaningful representation of measurement for uncertainties and vague concepts expressed in natural language. So a fuzzy multi-criteria decision making methods is preferred instead of crisp decision making methods for overcoming the FMEA procedure.

For determining the importance of failure modes a modified fuzzy approach proposed by Ekmekçioğlu et al. (2010) is used in this section. Firstly, a group of decision-makers identifies the failure modes. Second, a pair-wise comparison matrix for risk factors is constructed, and Chang's fuzzy AHP is utilized to determine the weight vector of these risk factors. Later, experts' linguistic evaluations of each failure mode with respect to risk factors are aggregated to get a mean value. Then by using the linguistic scores of risk factors for each failure modes, fuzzy decision matrix is constructed for the implementation of TOPSIS. After that, by using the weight vector of risk factors, and the fuzzy decision matrix weighted normalized fuzzy decision matrix is constructed. Subsequently, FPIS and FNIS and the distance of each failure mode from FPIS and FNIS are calculated, respectively. At last step of Chen's fuzzy TOPSIS closeness coefficients of processes are obtained. According to the closeness coefficients, the ranking order of all failure modes is determined.

Fig. 2 represents proposed fuzzy FMEA model.

To sum up the most important failure modes are determined through succeeding the following steps:

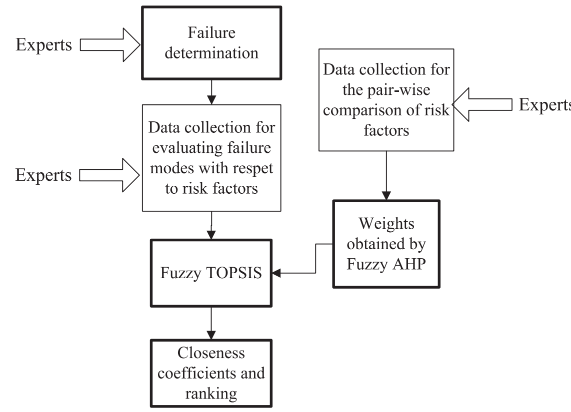

Fig. 2. Flowchart of the fuzzy FMEA model.

Step 1. A group of decision-makers identifies the failure modes.

Step 2. Chang's fuzzy AHP approach is used to obtain the weights of the risk factors.
- Appropriate linguistic variables for risk factors of each failure mode are determined.
- A pair-wise comparison matrix for severity, occurrence, and detectability is constructed, and experts' linguistic evaluations are aggregated to get a mean value for each pair-wise comparison.
- Consistency of pair-wise comparison matrix for S, O, and D is checked after the defuzzification of each value in the matrix according to graded mean integration approach.

Step 3. Chen's fuzzy TOPSIS is utilized to obtain the closeness coefficients of processes.
- Experts' linguistic evaluations of each failure mode with respect to risk factors are aggregated to get a mean value.
- Fuzzy decision matrix and the normalized fuzzy decision matrix are constructed for the implementation of TOPSIS.
- Weighted normalized fuzzy decision matrix is constructed.
- FPIS and FNIS are determined.
- The distance of each failure mode from FPIS and FNIS are calculated, respectively.

Step 4. According to the closeness coefficient, the ranking order of all failure modes is determined.

4. An illustrative example

The proposed methodology is applied to manufacturing facility of a SME performing in an automotive industry. Major potential failure modes (PFMs) are identified by a group of experts in an assembly process at the manufacturing facility as non-conforming material (A), wrong die (B), wrong program (C), excessive cycle time (D), wrong process (E), damaged goods (F), wrong part (G), and incorrect forms (H).

After the determination of the PFMs, by utilizing FAHP method, evaluations of three experts in linguistic variables are used to determine the importance of risk factors (S, O, and D) by pair-wise comparison as shown in Table 3. For instance, when comparing the risk factor severity and occurrence, the responses of three experts are fairly strong (FS), fairly strong (FS), and very strong (VS), respectively. As a result the weight vector for the risk factors is obtained as (.468 .201 .331). Subsequently evaluations of the experts in linguistic variables for the risk factors with respect to each failure modes are obtained as expressed in Table 4. The experts

Table 3
Evaluations of experts in linguistic variables and weights of the risk factors.

	Severity (*S*)	Occurrence (*O*)	Detection (*D*)	Weight vector
Severity	E, E, E	FS, FS, VS	SS, SS, SS	.468
Occurrence	–	E, E, E	SS, FW, E	.201
Detection	–	–	E, E, E	.331

CR for the defuzzified version of this matrix 0.0552 < 0.10.

Table 4
Evaluations of experts in linguistic variables for risk factors with respect to each PFMs.

Potential failure modes	*S*	*O*	*D*
(A) Non-conforming material	F, F, MP	F, MG, MG	G, MG, G
(B) Wrong die	P, MP, MP	VG, G, VG	MP, MP, P
(C) Wrong program	MP, P, MP	VG, G, G	VP, MP, P
(D) Excessive cycle time	MP, F, MP	F, MG, MG	G, MG, G
(E) Wrong process	F, F, MP	MG, MG, G	G, VG, G
(F) Damaged goods	MG, MG, F	MG, G, MG	MP, MP, F
(G) Wrong part	P, MP, VP	VG, VG, VG	VP, MP, P
(H) Incorrect forms	VP, VP, P	VP, VP, VP	VP, VP, VP

Table 5
Fuzzy FMEA analysis using Fuzzy TOPSIS integrated with FAHP.

PFM	Fuzzy TOPSIS			Scores
	Severity w_S .468	Occurrence w_O .201	Detection w_D .331	
(A)	(2.33, 4.33, 6.33)	(4.33, 6.33, 8.33)	(6.33, 8.33, 9.66)	.219
(B)	(.67, 2.33, 4.33)	(8.33, 9.67, 10)	(.67, 2.33, 4.33)	.146
(C)	(.67, 2.33, 4.33)	(7.67, 9.33, 10)	(.33, 1, 2.33)	.129
(D)	(1.67, 3.67, 5.67)	(4.33, 6.33, 8.33)	(6.33, 8.33, 9.66)	.207
(E)	(2.33, 4.33, 6.33)	(5.67, 7.67, 9.33)	(7.67, 9.33, 10)	.236
(F)	(4.33, 6.33, 8.33)	(5.67, 7.67, 9.33)	(1.67, 3.67, 5.66)	.216
(G)	(.33, 1.67, 3.67)	(9, 10, 10)	(.33, 1.67, 3.66)	.132
(H)	(0, 0.33, 1.67)	(0, 0, 1)	(0, 0, 1)	.028

evaluated the potential failure mode non-conforming material as fair (F), fair (F), and medium poor (MP) respectively for severity (S), fair (F), medium good (MG), and medium good (MG) respectively for occurrence (O), and good (G), medium good (MG) and good (G) respectively for detection (D). In the next step, by using weight vector of the risk factors obtained through FAHP, and the fuzzy evaluations of each risk factor with respect to PFMs, fuzzy TOPSIS is utilized as illustrated in Table 5. The closeness coefficient values found in the method are used as scores. Finally, as shown in Table 6, the scores are ranked and results show that the most important failure mode is "wrong process" (E).

A sensitivity analysis by changing the weight of risk factors is calculated according to information given in Table 7. For example in Case 0 shows the original weight values of the risk factors while the other cases show different weight values for possible situations. The results for ranking the PFMs for different cases are represented in Table 8 and Fig. 3.

Fig. 3 and Table 8 indicates that in four of five cases the most important failure mode is *Wrong Process*. In Case 1, as the weight of severity is the highest, *Wrong Process* failure mode is the second

Table 6
Ranking of failure modes.

PFM	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
Scores	.219	.146	.129	.207	.236	.216	.132	.028
Ranking	2	5	7	4	1	3	6	8

Table 7
Weights of the risk factors with respect to the considered cases.

Risk factors	Case 0	Case 1	Case 2	Case 3	Case 4
O	.468	.6	.5	.4	.4
S	.201	.2	.25	.3	.2
D	.331	.2	.25	.3	.4

Table 8
Ranking results of Failure Modes with respect to the considered cases.

PF	Case 0	Case 1	Case 2	Case 3	Case 4
A	2	3	3	2	2
B	5	5	5	5	5
C	7	6	7	7	7
D	4	4	4	4	3
E	1	2	1	1	1
F	3	1	2	3	4
G	6	7	6	6	6
H	8	8	8	8	8

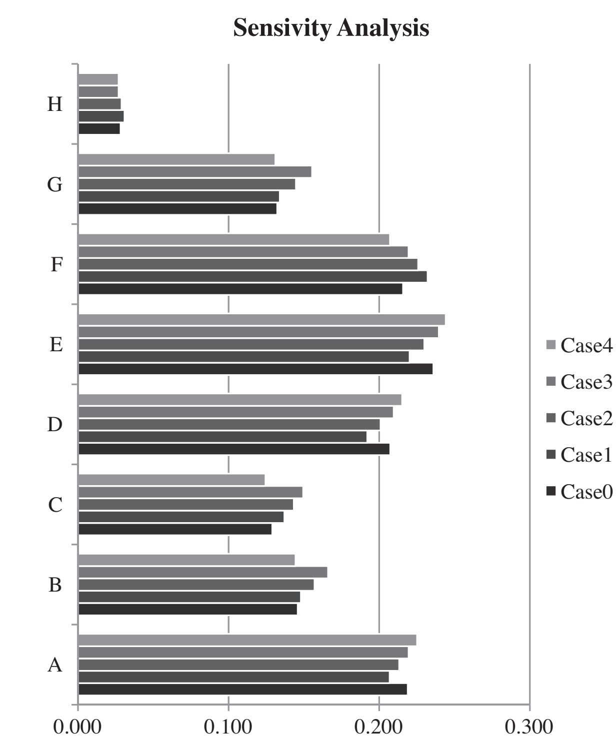

Fig. 3. Sensitivity analysis for fuzzy FMEA.

most important failure mode. In Case 0, Case 4 and Case 5 *Non-Conforming Material* is the second most important failure mode. It is also ranked the third in other cases. In all five cases, *Wrong Die* is in ranked the fifth and *Incorrect Forms* is ranked the eighth.

5. Conclusion

FMEA, designed to provide information for risk management decision-making, is a widely used engineering technique in industries. In FMEA potential failure modes are determined and can be evaluated by risk factors named severity, occurrence, and detection. In a typical FMEA, the risk priority number of each failure mode is obtained by the multiplication of crisp values of the risk factors. However, in the literature the crisp values of RPNs have been considerably criticized for a many reasons such as ignoring relative importance among the risk factors, imprecisely evaluation, questionable multiplication procedure and obtaining RPN values not big enough with two factors with very low risk value but a factor highly risky.

Due to the criticisms for RPN calculation in literature, a fuzzy approach is considered for FMEA analysis by its superiority over the traditional approach. In this study, fuzzy TOPSIS based FAHP is utilized to get the scores of PFMs, which are ranked to prioritize the failure modes. The results are used to find out the most important and risky PFM that would be handled at first glance. In the literature most of the studies consider fuzzy rule based systems for fuzzy FMEA whereas this study applies a model of fuzzy TOPSIS integrated with FAHP. In addition to allowing experts to evaluate the risk factors of each potential failure mode in linguistic variables, the advantage of using this model considers the importance of the risk factors. As a managerial implication the proposed model can be applied to any case for providing information for risk management decision-making in industrial and service organizations.

For further research, the results of our study can be compared with that of other fuzzy multi-criteria techniques like fuzzy ELECTRE, fuzzy PROMETHEE, or fuzzy VIKOR.

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