

# Seismic Performance Assessment Uses Incremental Dynamic Analysis

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## ABSTRACT

Incremental dynamic analysis (IDA) has developed as an efficient approach to illustrate the level of structural damage in different seismic intensities. IDA applies to an existing tall building with a fundamental period of one second. A set is used which contains 31 near-source ground motion records. They are considered separately to study the influence of different seismic source characteristics on the outcomes of the IDA. In order to evaluate seismic vulnerability of structures, the responses of the structure for each record of earthquake to global instability point (capacity point) are analyzed incrementally. The purpose of this study is to conduct an efficient response and exact with to apply incremental dynamic analysis. Finally, the seismic fragility curve of structure in different seismic intensities is shown.

**KEYWORDS:** Incremental Dynamic Analysis, spectral acceleration, Median of response, Standard deviation, dispersion

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## 1- INTRODUCTION

The displacement is an appropriate criterion to define damage measure and performance based design. The structure shows nonlinear behavior responses while imposed to high intensity measure, but the responses are different in earthquake records. To achieve appropriate response, it is required to identify earthquakes which are corresponding to soil type, distance from site, magnitude, fault mechanism and other parameters. In this Study Consider a structure with the fundamental period of 1.0 (s) and the critical damping ratio of 5%. The results of the nonlinear time history analysis of the structure are summarized in a ".mat" file. The SDOF.IM variable is the intensity measure in terms of PGA (g), and the SDOF.DM is the damage measure in terms of displacement (cm). The intensity of earthquake is typically expressed in terms of first mode spectral acceleration. IDA applies to an existing tall building with a fundamental period of one second. 31 sets of ground motions representing near-field records are considered separately to study the influence of different seismic source characteristics on the outcomes of the IDA. The studies demonstrate that, if  $S_a(T_1, 5\%)$  be used to represent IDA curves, the results will be more efficient than PGA [1, 2]. It is required in several parameters, such as choosing suitable ground motion Intensity Measures (IM) and representative Damage Measures (DM). At the first step of this approach, a damage measure (DM) considered for estimation of structural performance. Damage measures are included, displacement, inter story drift, ductility and other parameters that employed nonlinear behavior of structures. In this study, drift is applied to this target. In addition, proper interpolation and summarization techniques for multiple records need to be employed, to provide an approach of estimating the probability distribution of the structural demand given the seismic intensity [3]. In order to evaluate seismic vulnerability of structure, the responses of structure for each record of earthquake to global instability point (capacity point) are analyzed incrementally. Three important parameters for seismic vulnerability assessment of structure are hazard, demand and capacity. In the first step, hazard and demand are combined. In the second step, first step and second step results are combined [4].

## 2. Plotting Incremental dynamic analysis (IDA) curves

With increasing intensity measure (IM), the behavior of structure will be in several limit states. Some of these limit states are; immediate occupancy, life safety and collapse prevention. The period of the fundamental mode of structure is one second and the damping ratio is five percent. The results of nonlinear dynamic analysis are based on the drift for 31 earthquake records upon PGA [5, 6]. Studies show that if IDA curve uses the spectral acceleration value of fundamental mode ( $T_1=1\text{sec}$ ) instead of PGA, The result will be much more efficient. Also HAZUS system is used  $S_a(T_1)$  as a one of the important measure because of structural characteristic is seen.

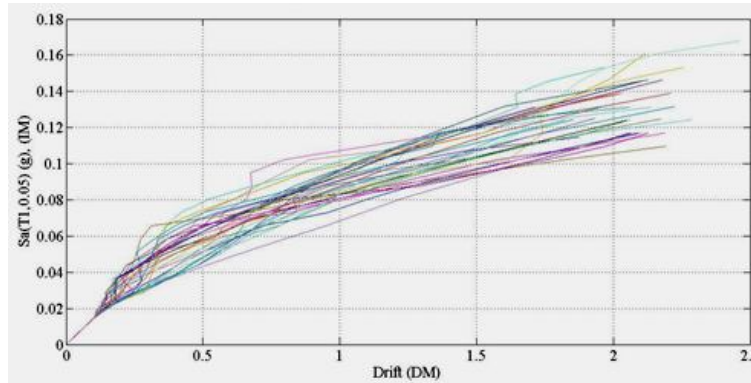
$$c(i) = \frac{SaT_1(i)}{PGA(i)}$$

$$IM_{modified} = c(i) \times IM$$

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Thus all IM amounts of each IDA curve are multiplied by  $c_i$  coefficient. IDA curves will be presented for PGA and Sa (T1). The values of PGA, Sa (T1=1) and  $c_i$  corresponding to each record are provided in the following table. IDA curves for 31 records are presented in figure 1.



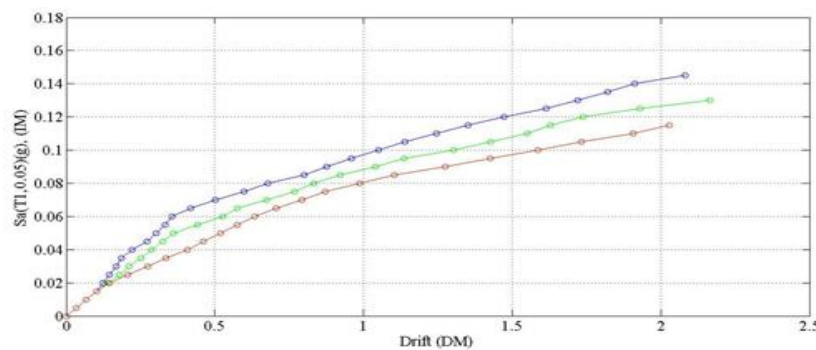
**Figure 1.** Incremental Dynamic Analysis Curves (Sa, DM)

**Table 1.** The values of PGA, Sa (T1=1) and  $c_i$  corresponding to each record

Number of EQ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
PGA	0.355	0.355	0.357	0.358	0.359	0.352	0.361	0.356	0.357	0.356	0.358	0.353	0.353	0.358	0.362	0.362
Sa(T1)	0.446	0.444	0.441	0.437	0.441	0.442	0.452	0.446	0.451	0.447	0.449	0.456	0.454	0.442	0.461	0.434
ci	1.244	1.251	1.235	1.219	1.231	1.254	1.252	1.255	1.262	1.256	1.255	1.292	1.286	1.235	1.275	1.201
Number of EQ	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
PGA	0.358	0.358	0.367	0.352	0.367	0.357	0.362	0.361	0.358	0.358	0.365	0.367	0.361	0.362	0.363	
(T1)Sa	0.442	0.439	0.45	0.449	0.457	0.457	0.452	0.428	0.459	0.451	0.443	0.442	0.442	0.446	0.469	
ci	1.235	1.224	1.226	1.275	1.246	1.28	1.249	1.187	1.282	1.259	1.214	1.205	1.225	1.23	1.291	

### 3. Calculating the median and standard deviation of a drift factor at each level of Sa

At first the values of median, standard deviation and values related to 16% and 84% probability must be determined to identify the drift hazard curve and the Probability of failure. At each level of seismic intensity, the drift values of earthquake records have different distributions. Thus the amount of each of the above parameters will vary for each level of seismic intensity, so these values will be determined for each specific level. Since the results of nonlinear analysis for all earthquake records were different based on certain levels of seismic intensity, the amount of each of the above parameters will vary for each level of seismic intensity. Therefore, the above values will be determined for each level. Since the results of nonlinear analysis for all earthquake records were not arranged based on certain levels of seismic intensity, and the applied steps in nonlinear analysis are large so one curve will be fitted to IDA curves using the interpolation method and drift values will be read for any given level of seismic intensity. To do this, the Interpl function is used for each of the 31 IDA curves. To use this function, where the values of DM, IM is the same in the IDA curve must be removed. Then the corresponding drift values must be calculated for the Sa steps (0.01) from 0 to IM. Figure 2 shows Sa (T1) versus DM [7].



**Figure 2.** Median of Incremental Dynamic Analysis Curves

In order to calculate the statistical levels of 16%, 50%, 84% of IDA curves for each Sa level, firstly the stored drift values must be stored vertically in each step for all 31 records in the column. Zero values in each column show the lack of curve cutting in that Sa. So the zero values in each column are removed. To calculate F16, the fifth sentence of each column ( $16\% \times 31$ ) is considered and to calculate F50 the 16<sup>th</sup> sentence ( $50\% \times 31$ ) is considered. To calculate F84 ( $84\% \times 31$ ) is considered. In order to fit on F50 curve, the first sentences of F84, F50 can be considered a 0.001. Figure 2 shows three levels of IDA curves.

#### 4. Calculating $\beta_{D|Sa}$

$\beta_{D|Sa}$  Values can be calculated at each level of Sa using the following formula:

$$\beta_{D|Sa} = \log F50 - \log F16$$

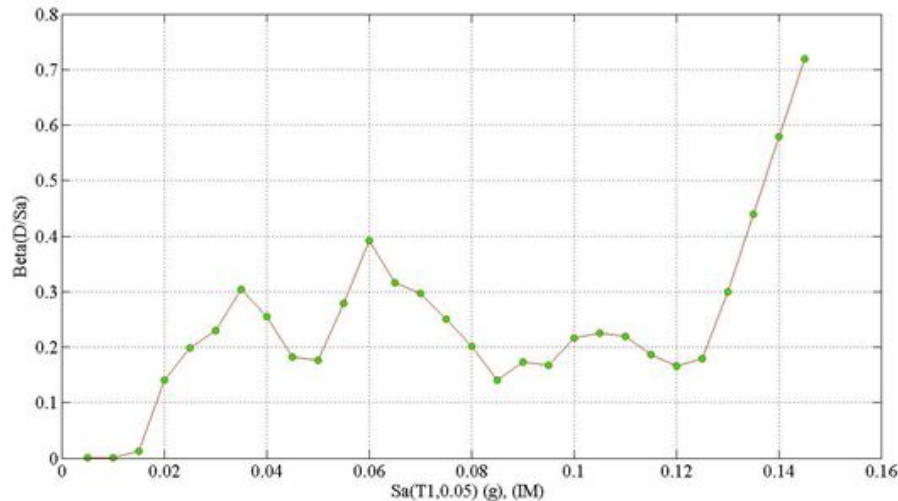


Figure 3. Illustration of Beta(D/Sa) in incremental Dynamic Analysis Curves

**4.1. Fitting the function to determine a and b in  $D = a(Sa)^b$ :** Also, as can be seen in the next step, in order to calculate the drift hazard curve, the median curve must be regressed on which a function such as  $D = a * (PGA)^b$  must be fitted. As can be seen, the regression has been done with high accuracy (95%).

General model Power 1 is  $f(x) = a * (x)^b$  And Coefficients (with 95% confidence bounds):

$a = 83.62$ ,  $b = 1.81$ . Fitting curve demonstrates in Figure 4 [8].

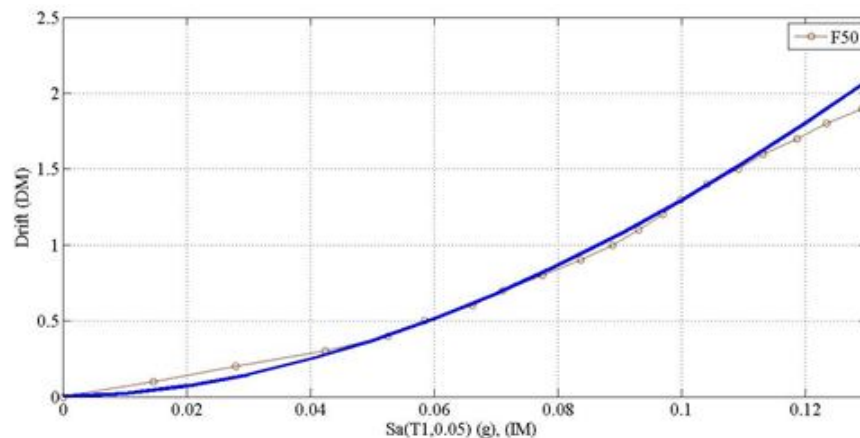


Figure 4. Fitting Curve (fx)

**5. Interpolation on IDA curves and determining Sa seismic intensity at each level from the drift on IDA curves and calculating the median and standard deviation of drift in each Sa ( $med_{16}$ ,  $med_{50}$ ,  $med_{84}$ ) and finally  $\beta_{D|Sa}$**

Like the previous step, Interpl function is used for all the 31 IDA curves. To use this function in IDA curves where the values of DM, IM are the same must be removed.

**5.1. Determining  $S_a$  seismic intensity at each level of drift on IDA curves:**  $S_a$  values are calculated for drift steps from 0 to 2.46 in all the 31 IDA curves.

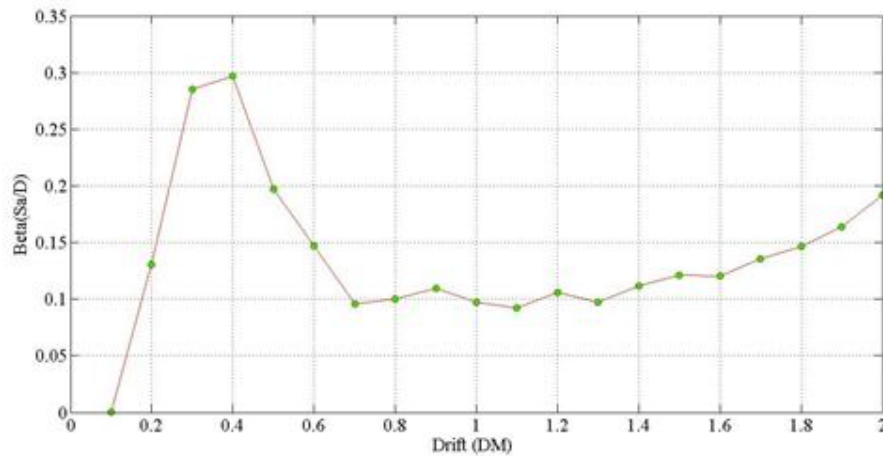
**5.2. Calculating the median and standard deviation of  $S_a$  in each drift:** In order to calculate the statistical levels of 16 %, 50 %, 84 % of IDA curves for each  $S_a$  level, firstly the drift values must be stored vertically in each step for all 31 records in the column vector and median prtile Function (MATLAB Function Reference) and the median  $\pm$  standard deviation are used [8].

### 5.3. Calculating $\beta_{Sa|D}$

$\beta_{Sa|D}$  Values can be calculated for each drift values using the following formula:

$$\beta_{Sa|D} = \log F_{50} - \log F_{16}$$

$\beta_{Sa|D}$  Values versus DM indicates in Figure 5.

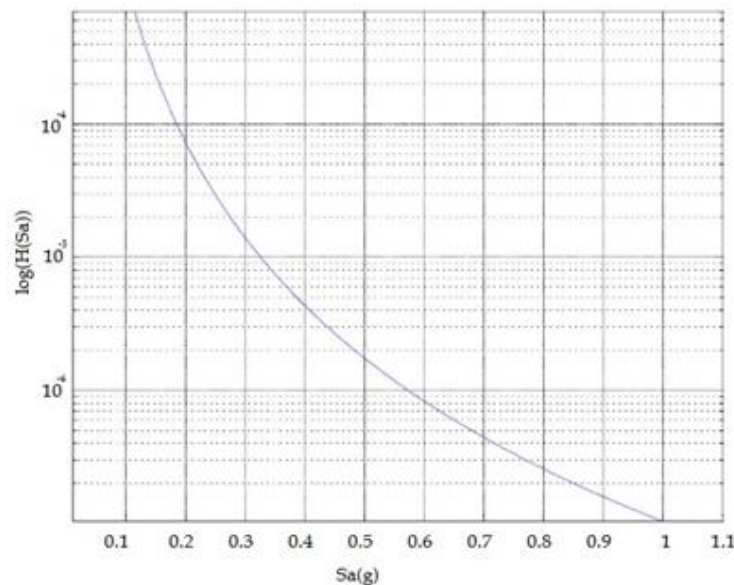


**Figure 5.** Illustration of Beta( $S_a/D$ ) in incremental Dynamic Analysis Curves

## 6. Drift hazard curve

The hazard curve extraction is evaluated in this project through the Direct and Cornell methods.

**6.1. Seismic hazard curve based on the UBC 97 for Soil Type II:** The methods require the use of seismic hazard curve in terms of seismic intensity. So the hazard curve of building design regulations for UBC 97 is used for the soil type II With a very high seismic hazard.



**Figure 6.** Seismic Hazard Curves based on UBS 97, Soil type II

**6.2. Direct method for extracting drift hazard curve:** Direct method to calculate hazard estimates based on structural drift results in the drift hazard curve in terms of seismic intensity based on conditional probability as follows.

$$H_D(d) = \sum_{all} P[D \geq d | S_a = x_i] P[S_a = x_i]$$

$P[S_a = x_i]$  Which is the annual probability of exceedance in earthquakes that is obtained from the seismic hazard curve?  $P[D > d | S_a = x_i]$  Which is obtained based on IDA curves and the intersection of intensity  $S_a = x_i$ . It operates on a simple counting of the points that satisfy the probable condition. The above conditional probability relationship shows the most likely amount of demand,  $D$ , of a certain value, such as  $d$  for different levels of seismic intensity. In the relationship  $P[S_a = x_i]$ ,  $S_a = x_i$  is the Probability of an earthquake exceedance which is determined based on earthquake hazard curve. To determine the probability of  $P[D > d_i | S_a = x_i]$  the number of drift bigger than  $d_i$  is counted for each level of spectral acceleration and is divided by the total drift. So for each  $x_i$  and  $d_i$  a number is obtained for the above probability and all the probabilities of  $P[D > d_i | S_a = x_i]$  are added for any given level of spectral acceleration and multiplied by the probability of Spectral acceleration hazard  $H_{Sa}[S_a = x_i]$ . Thus the probability of the drift hazard for any given level of spectral acceleration is obtained and finally the drift hazard curve is identified.

**6.3. Cornell method for extracting the drift hazard curve:** Based on the formulation presented by Professor Cornell, seismic hazard curves can be stated as a closed form. Here is the formulation. This method is based on the direct method [9].

$$H_D[d] = v. \sum_{all x} P[D > d | S_a = x_i] P[S_a = x_i]$$

Based on the above relationship, the continuous variable is expressed as follows.

$$H_D(d) = v. p[D > d] = \int_0^{\infty} P[D > d | S_a = x_i]. v. f_{S_a}(x_i). dx = \int_0^{\infty} P[D > d | S_a = x_i]. |v. dG_{S_a}(x)|$$

In which  $f_{S_a}(x)$  is the Probability density function in  $S_a = x_i$  and  $G_{S_a}(x)$  is its Distribution function. The following structure is arranged Based on the formulation presented by Professor Cornell in the annual probability of seismic hazard curves can be stated as a closed form.

$$H_D(d) = k_0 \left(\frac{d}{a}\right)^{-\left(\frac{k}{b}\right)} \exp\left(\frac{1}{2} \frac{k^2}{b^2} \beta_{D|S_a}^2\right) = H(S_a) \exp\left(\frac{1}{2} \frac{k^2}{b^2} \beta_{D|S_a}^2\right)$$

Comparing the results of the drift hazard curves obtained from the two methods of direct method and method Cornell indicate in Figure 7.

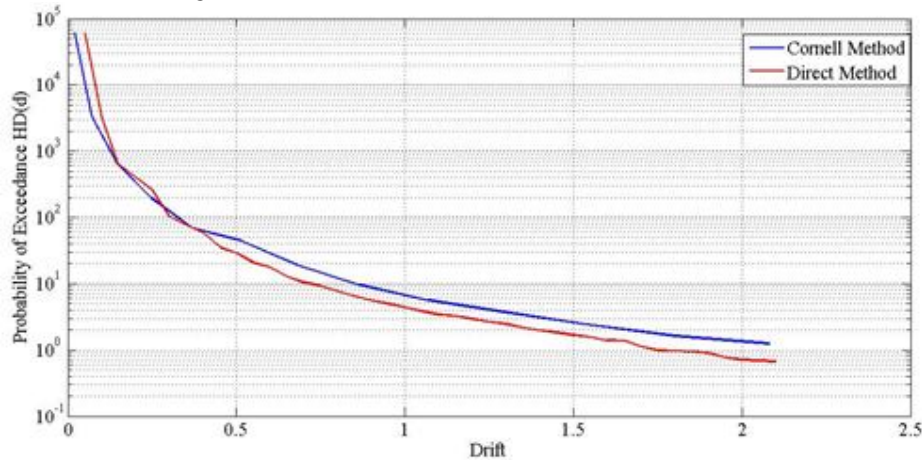


Figure 7. Drift Hazard Curve

## 7. Calculating the probability of expected non-occurrence of hazard level

Now, the final step can be taken for Structural Safety. To evaluate the probability, the Structural performance level should always be limited to a reference level. The general equation is as follows.

$$P_{PL} \leq P_0$$

In which  $P_{PL}$  is the probability of expected non-occurrence of hazard level and  $P_0$  is the performance level (reference).

$$P_{PL} = P[C \leq d] = H(S_a^e) \exp[1/2 k^2 / b^2 (\beta_{D|S_a}^2 + \beta_C^2)]$$



In which  $S_a^c = (c/a)^{1/b}$ . after replacing equation above equation, the basic relationship of structural control which is similar to the structural design by the LRFD method is obtained as follows.

$$\varphi \hat{C} \geq \gamma \hat{D}_{P_0}$$

In order to calculate the Safety Factor at each level of  $S_a$  we have:

$$k=4.268; k_0=1.77e-4; a=83.62; b=1.81; \beta_c=0.1886; \hat{C}=1.468485.$$

The following structure is arranged based on the formulation of calculating the probability of expected non-occurrence of Hazard level and the Safety Factor.

$$\hat{C} \exp \left[ -\frac{1}{2b} \beta_c^2 \right] > \hat{D}_{P_0} \exp \left[ \frac{k}{b} \beta_{(D|S_a)}^2 \right]$$

$\hat{C}$  is median of capacity structure;  $\hat{D}$  is median of demand structure. The Safety factor is shown in figure 9[10].

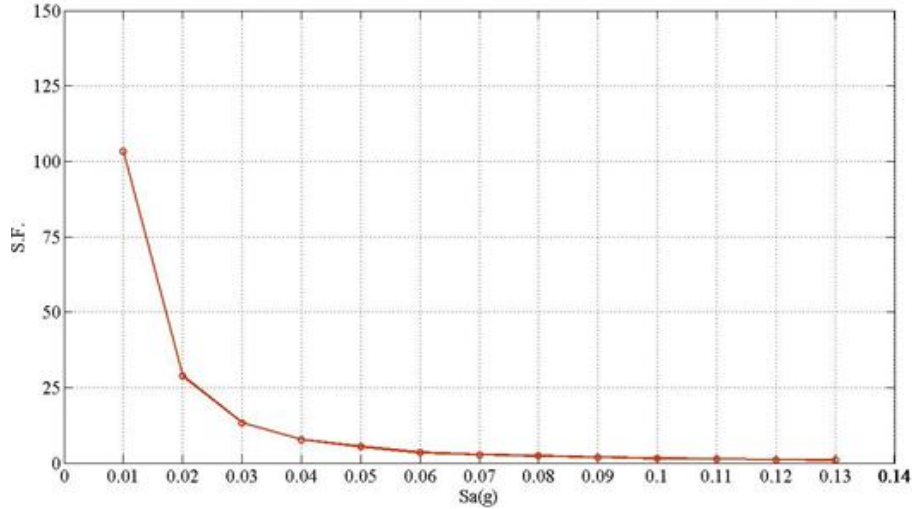


Figure 9. Safety Factor

## 8. Structural safety evaluation

The following equations are derived by Cornell et al to calculate the probability of failure. They tried to consider the both types of uncertainty namely Randomness and Uncertainty in equations [11]. The presented equation by evaluates the structural safety similar to equations of LRFD for structures. If  $P_0$  is the allowable probability of failure, the Safety condition will be as follows.

$$P_{PL}^X \leq P_0$$

By replacing these values to the first side of the unequal, we have:

$$k_0 \cdot \left( \frac{\hat{C}}{a} \right)^{-\frac{k}{b}} \cdot \exp \left[ \frac{k^2}{2b^2} \beta_{RT}^2 + k_x \beta_{PPL} \right] < P_0$$

By defining the coefficients, a much simpler equation can be stated:

$$\begin{aligned} \hat{C} \exp \left[ -\frac{k}{2b} \beta_{RT}^2 - k_x \frac{b}{k} \beta_{PPL} \right] &> a \left( \frac{P_0}{k_0} \right)^{-\frac{b}{k}} \\ \hat{C} \exp \left[ -\frac{k}{2b} \beta_{RT}^2 - \frac{k}{2b} \beta_{UT}^2 \right] - k_x \beta_{UT} &> a \left( \frac{P_0}{k_0} \right)^{-\frac{b}{k}} \exp \left[ -\frac{k}{2b} \beta_{UT}^2 \right] \\ \hat{C} \exp \left[ -\frac{k}{2b} \beta_{CR}^2 + \beta_{CU}^2 \right] &> \hat{D}_{P_0} \exp \left[ -\frac{k}{2b} \beta_{DR}^2 + \beta_{DU}^2 \right] \exp \left[ k_x \beta_{UT} - \frac{k}{2b} \beta_{UT}^2 \right] \\ \lambda_x &= \exp \left[ -\beta_{UT} \left( k_x - \frac{k}{2b} \beta_{UT} \right) \right] \end{aligned}$$

The last term is called Confidence Level. The control equation in the mentioned reference is summarized as follows.

$$\hat{C} \cdot \phi \cdot \lambda_x > \hat{D}_{P_0} \cdot \gamma$$

And  $\beta_{UT}$  is equal to:

$$\beta_{UT} = \sqrt{\beta_{UT}^2 + \beta_{UC}^2}$$

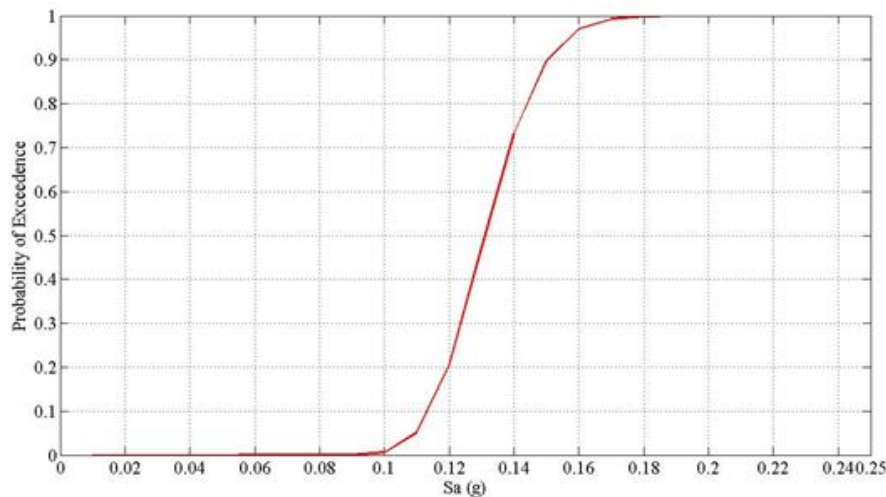
The values related to the error of calculating are Uncertainty and can be determined depending on the structural importance in FEMA350 regulation and based on the type of nonlinear static or dynamic analysis and

structural classes and type and number of floors. In this project,  $\beta_{UT} = 0.3$  is considered. In this equation, a value is determined d for  $K_x$  for each D and by determining  $\beta_{D/Sa}$  and the amount of  $\beta_{UT}$  and based on structural properties. In fact the equation  $e^{-K_x \beta_{UT}}$  shows the average probability of failure by assuming lognormal distribution. So according to  $X = \varphi(K_x)$ , the value of x shows the corresponding confidence level x for the desired return periods. The following structure is used to control the safety for the desired return periods.

### 9. Seismic Fragility

As mentioned conditional probability function which expresses the probability that a structure reaches or passes a seismic fragility is calculated from the next Equation. This Equation shows the probability that the structural demand (D), exceeds structural capacity (C). In analytical methods and especially in time history analysis the seismic demand probability models which are given for a certain seismic intensity. The recommended lognormal theory (Cornell et al-2002) offers median values of demand which conform to the following power value model. After estimation of the dispersion which is a conditional value of the intensity measure ( $\beta_{D|IM}$ ), the fragility value can be calculated as next Equation. Where d is the limit state value. Limit states are given in qualitative, descriptive or prescribed form. Qualitatively given limit states by HAZUS (FEMA 2003) describes different damage levels from slight damage to total collapse of some typical structure. Some of limit state is based on laboratory testing as well as analysis of structural components. In this study we have chosen the collapse prevention (CP) limit state. Using the mean values of the proportion of demand to seismic intensity, capacity and the damage index, fragility curve is computed. The probabilistic seismic demand model is generated using the 31 ground motions in previous steps [12].

$$P[EDP \geq d|IM] = 1 - \Phi\left(\frac{\ln(d) - \ln(aIM^b)}{\beta_{EDP|IM}}\right)$$



**Figure 10.** Seismic Fragility Curves for Collapse prevention limit state

### 10. CONCLUSION

To evaluate seismic fragility of structures, it is needed for exact identification of demand, capacity and hazard. In this study, the trend of to obtain seismic vulnerability of a structure with the mathematical and statistical formulation is represented. The purpose of this study is to conduct an efficient and exact estimation of probability of exceedance response with to apply incremental dynamic analysis method. Fragility curves in terms of spectral acceleration exceed 0.18 shows the general structural failure. The spectral acceleration values of 0.1 to 0.18 are high structural vulnerabilities quickly. It is shown; the design of a structure is not suitable.

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