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Highlights

- A rumor spread model considering the proportion of wisemen in the crowd is established.

- We consider the speed of rumor propagation as a variable over time.

- The results of this study show that improving the people's knowledge level is beneficial to control the spread of rumors.

Rumor spreading model considering the proportion of wisemen in the crowd

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Abstract:

Similar to the spread of infectious diseases, rumor spread has a significant impact on human lives. As the saying goes, "rumors end with the wise" and the proportion of the wisemen in the crowd has a certain influence on the spread of rumors. Based on this fact, a rumor transmission model considering the proportion of wisemen in the crows is established. It provides a new angle of view on the issue of rumor spreading. Different from previous studies, we consider the speed of rumor propagation as the variable over time in our model. Then, locally asymptotic stability of each of the feasible equilibriums is presented by using Routh–Hurwitz criteria. Numerical simulations are carried out to illustrate the feasibility of our results and the influence of different parameters on the rumor spreading. The results in this paper provide theoretical support to rumor control.

Keywords: Rumor transmission model, Wisemen, Transmission rate，Equilibrium, Stability

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1. Introduction

Rumor, as a part of everyone's daily life, has often been defined as a type of social phenomenon with which some unconfirmed elaboration or annotation of the public interested events or issues spread on a large scale within a relatively short period of time through various channels, whether it's true or false [1]. Sometimes rumors or misinformation can lead to serious consequences. For instance, it can affect and shape public opinions and have impact on financial markets, which may probably cause public panic and instability afterward [2]. With the rapid development of information technology, rumors have taken great changes than before in its spreading methods, channels of transmission [3]. Rumors do not choose a particular population to spread, so it is becoming easier for people to come into contact with rumors.

Because of the public's inability to understand basic scientific knowledge, rumor spreading often causes great social panic and adverse effects on the economy.

In March 2011, Japan's Fukushima nuclear power plant exploded, and radiation began to spread outward[4]. A number of the rumors began to spread, with some saying that salt can be used to prevent people from being radiated. Others tell you that there soon will be a shortage of salt due to the pollution caused by nuclear leakage. Many Chinese in different part of the whole of the enormous country rushed towards supermarkets and stores, buying and storing salt as much as they could, causing many supermarkets running out of salt.

Later the Chinese government launched a series of propagandas, to demonstrate the public to get to know the truth about nuclear radiation[5]. In a scientific way, people began to understand the concerning knowledge and were soon confident that there was really no need for a panic purchasing of salt.

In this case, it is of vital importance for us to study the mechanism of rumor spreading, and to analyze the related factors which make rumor persist in our society, in order that we can identify a rumor as soon as possible, and reduce the harm caused by rumor spreading.

Rumor spreading was first studied by Daley and Kendall in 1965 [6], and a mathematical model of rumor spread was established, which was later named by researchers as D-K model. In D-K model, the population was divided into three groups: one group of people who have not known the rumor; those who have heard the rumor and can spread it to others; ones who have already known the rumor but will no longer spread it. These three groups were named ignorant, spreaders and stiffer respectively. After that, a lot of new progress and breakthroughs have been achieved in the study of rumor transmission. Since then this quantitative mathematical model has been substantially and widely extended [7-9]. Based on previous rumor spreading models, a number of scholars developed some new rumor spreading models [10-12]. Some scholars researched the applications of stochastic version of the D-K model on scale-free network. Their study results showed that the uniformity of the network had great influence on rumor propagation mechanism [13-15].

At the beginning of this century, it became one of the focuses of research to consider different dynamics of rumor and idea transmission. Bettencourt et al. [16] analyzed the dynamics of idea transmission using a model similar to epidemiology.

Thompson et al. [17] established a similar model considering the susceptible and the diversity of communicators in the rumor spread. Kawachi [18] and Kawachi et al. [19] established deterministic models for rumor spreading and analyzed the effects of various contact interactions among different classes in rumor spreading process. According to spreading parameters and initial conditions, Piqueira [20] proposed a rumor spreading model and carried out a equilibriums study. Considering denial and skepticism, Huang [21] built two models of the spreading process of rumors. Jaeger et al. [22] gave an interesting discovery that rumors were adopted on more frequently when the believability level was high. Recently, on the basis of considering the mechanism of forgetting and refutation, Zhao et al. [23-25]proposed a rumor propagation model on social networks.

However, it cannot be ignored that there are still some issues pertaining to their research. For instance, any inflow or outflow to the ignorant group or from the other groups is not allowed. At the same time, it also assumes that an ignorant absolutely becomes a spreader when he/she hears the rumor, which is completely out of line with the actual life.

In fact, after rumors, some people said that the reason for being deceived by rumors was mainly because of their lack of scientific knowledge. According to a website survey, the number one important measure to be taken is “to improve the citizens’ scientific literacy, so that they do not believe rumors, do not pass rumors” in the direction of the first end of the rumor[26]. The improvement of the citizen’s sphere of vision, together with the rationality of social psychology plays a vital role in ending rumors.

There is a famous saying, "rumors end with the wise. It means that a government should promote and popularize scientific knowledge with well-planned propaganda, endeavoring to educate the people on scientific knowledge. Only when the proportion of wise individuals has been increased and an institutional system of rumor prevention has been set up and consolidated with subsequent endeavors, there will be possibility for rumors to be prevented and disbelieved.

Based on this background, we would like to study in this paper the impact of large-scale enhancement of people's scientific literacy on the issue of rumor transmission.

In this paper, considering the proportion of wisemen in the crowd on rumor spreading, a dynamic model of rumor spreading is proposed based on epidemiological models. It is worth mentioning that some scholars have proposed the concept of immune factors [16, 27]. These immune agents are those who either do not believe in the rumor or not interest in spreading the rumors. The immune agents have a functional similarity with the concept of the "wisemen" that we put forward in this paper.

On the issue of infectious diseases, immune factors is human body own resistance to infectious diseases, which is congenital and cannot be obtained through external artificial means of technology.

Meanwhile, it is known that a large number of the population can understand further about how to prevent and control such diseases by introducing scientific knowledge to the public on infectious diseases. As for the prevention and control of infectious diseases, the effect of such external measures should not be overlooked.

Facing infectious diseases, however, not everyone is instinctively immune to diseases because of a lack of immune factors within the body of some people. But it is obviously easy to let more people know more or less about scientific knowledge on infectious diseases.

With similar propositions in mind, it is of vital importance to educate the common mass and enrich their knowledge based on scientific research and observations. Individuals, who have received scientific educations, tend to be more capable of distinguishing between the truth and the rumors. The more people are trained on scientific wisdom, the less likely that rumors get spread.

Generally speaking, immune factors are taken into consideration from the prospective within individual beings themselves, whereas, the wisemen factors are taken into account from the point of external environment.

These two factors are different from each other, but as different components of one complete unity, they supplement with each other in a harmonious way, fulfilling our wish to control and restrain rumors to be spread.

The remainder of this paper is organized as follows. In Section 2, we give a brief description of our rumor spreading model which takes into account the proportion of wisemen in the crowd. In section 3, we calculated the equilibrium point and discussed the existence condition of the equilibrium point, analyzed the stability of the equilibrium point of the model. In Section 4, numerical simulations are presented to demonstrate the results. Finally, we present our conclusions and discuss some implications of the results in Section 5.

2. Rumor spreading model

According to the general classical model of rumor propagation, we consider a variable population size at any time t and denote it by $N(t)$. From rumor aspect, we divide the population into three disjoint classes of individuals: Susceptible (people who never heard the rumors), Infective (people who spread rumors) and Recovered (people who know it but never spread it) individuals, denoted by $S(t)$, $I(t)$ and $R(t)$, respectively.

It is worth noting that social media also has a certain impact on rumor spread. With "science" or "theory" in their stories, rumor spreaders can usually obtain the public's trust to make rumors spread in incredibly rapid ways. However, whether it is the use of scientific theories or the rapid spread of rumors via news media, it boils down to the social psychology of the people. Facing major natural disasters ， social unrest ， or even wars, people generally lack the necessary information for the reason that most people's lives are more likely to be threatened. In these cases, rumors tend to spread with ease among the public. It can only partially solve the problem of rumor transmission to study the role of social media in the spreading of rumors. But it cannot fundamentally solve the problem. Basically, to put an end to rumors, the government as well as the media should join hand to make effort, with the mass citizens themselves involved as the major participants. The common citizens has the capable of detecting and prohibiting rumors only when the government departments make effort to strengthen publicity and educations of scientific knowledge, and experts and public intellectuals actively offer people with the right guidance. It is not difficult to find loopholes in rumors, so long as people have scientific knowledge, cautious attitude, rational judgment and calm thinking.

At the same time, in order to avoid the complexity of the problem, in this paper, the role of social media in the spread of rumors is not discussed in this paper. We assumed that rumor is spread through human contact. Rumors are being spread over a short period of time with the media. Therefore, the rumor spreading problem we considered is more similar to the "hearsay" which is inconvenient or not allowed to be spread openly, but can only be transmitted privately through human to human transmission.

Assume that the susceptible crowd has a positive constant input rate B which named immigration constant, Where B is the number of individuals entered in the whole group per unit time, and it does not represent the proportion of input individuals in the entire population. Each class has a same emigration rate which denoted by a positive constant μ .

A susceptible can know about the rumor when he/she contacted a spreader at a rate α , namely rumor transmitting rate, where $\alpha = wq$ such that w is the average number of contact per unit time and q is the probability of transmitting. In this paper, we consider the rumor propagation rate as a time dependent variable, which is an improvement on the traditional rumor propagation model. In addition, it is a logical setting that the rumor propagation rate increases with the number of infected people increases and decreases with time. Therefore, we assume that,

$$\frac{d\alpha}{dt} = \lambda I - \delta\alpha \tag{1}$$

where $\lambda > 0, \delta > 0$ are corresponding coefficients.

After knowing about the rumor from a spreader, if the susceptible class believes the rumor, he/she will convert to the infective class and start to spread rumor; if the susceptible class doesn't believe rumor or is not interested in rumor, he/she will turn into the recovered class. We set the proportion of the former to be $1-\theta$, and the proportion of the latter is θ, where $\theta \in (0,1]$. It is common sense that the higher the intelligence of a person, the less likely to believe the rumor. Therefore, in this paper, we use the parameter θ to express the proportion of the wise people in the crowd, and analyze the influence of the ratio of the wise people on the rumor spread. At any time, some infective individuals lose their interest in spreading rumors or identify rumors, and they no longer spread, thus becoming recovered individuals at a rate $\varepsilon(\varepsilon > 0)$, reducing the number of infective individuals.

Based on the above assumptions and condition of hypothesis, with the model flow diagram given in Fig.1:

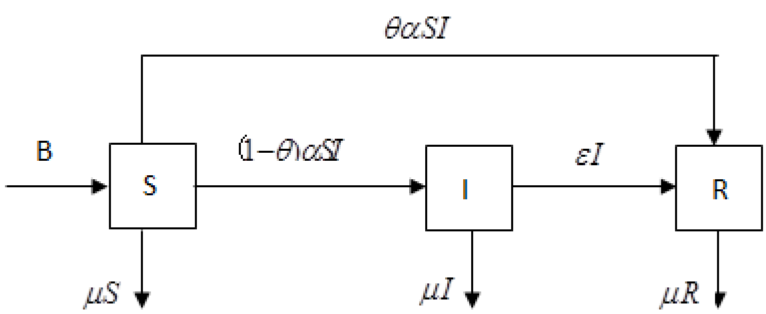

Fig.1. The flow diagram of the model.

According to the above analysis, we refer to the basic SIR epidemic model, and construct a rumor propagation model which takes into account the "wise man" factors. The system dynamics equations are described as follows:

$$\begin{cases} \frac{dS}{dt} = B - \alpha SI - \mu S \\ \frac{dI}{dt} = (1-\theta)\alpha SI - \varepsilon I - \mu I \\ \frac{dR}{dt} = \theta\alpha SI + \varepsilon I - \mu R \\ \frac{d\alpha}{dt} = \lambda I - \delta\alpha \end{cases} \qquad (2)$$

Where, $B > 0, \mu > 0, \varepsilon > 0, \lambda > 0, \delta > 0$, and $\theta \in (0,1]$

$S(t) + I(t) + R(t) = N(t)$

It is easy to know that $\frac{dN(t)}{dt} = B - \mu N$, so $N(t) = (N_0 - \frac{B}{\mu})e^{-\mu t} + \frac{B}{\mu}$, where $N_0 = N(0)$, and then $\lim_{t\to\infty} N(t) = \frac{B}{\mu}$. The positive invariant set of system (1) is:

$\Gamma = \{(\mathrm{S},\mathrm{I},\mathrm{R},\alpha) : \mathrm{S}+\mathrm{I}+\mathrm{R} \le \frac{B}{\mu}, S > 0, I > 0, R > 0, 0 < \alpha < w\}$.

3. Theoretical analysis

3.1 Existence of equilibriums

According to the system dynamics equations (2), we can calculate equilibrium $E = (S, I, R, \alpha)$. It is easy to observe that the positive equilibriums of system (2) are $E_0 = (\frac{B}{\mu}, 0, 0, 0)$ and $E^* = (S^*, I^*, R^*, \alpha^*)$. E_0 (Rumor-free equilibrium point) always exists.

With regard to the positive equilibrium $E^* = (S^*, I^*, R^*, \alpha^*)$ of the system (2), it should satisfy:

$$\begin{aligned} &B-\alpha SI-\mu S=0\\ &(1-\theta)\alpha SI-\varepsilon I-\mu I=0\\ &\theta\alpha SI+\varepsilon I-\mu R=0\\ &\lambda I-\delta\alpha=0 \end{aligned} \quad (3)$$

By calculating the equations, we can get

$$c_2 I^2 + c_1 I + c_0 = 0 \quad (4)$$

$c_2 = \lambda(\mu+\varepsilon) > 0$, $c_1 = -B\lambda(1-\theta) < 0$, $c_0 = \mu\delta(\mu+\varepsilon) > 0$,

The equilibrium points E^* exists if $B^2(1-\theta)^2\lambda \geq 4\mu\delta(\mu+\varepsilon)^2$, which details are shown in Table 1

Table 1: The positive equilibriums of system (2)

Cases	Positive equilibrium
$\Delta=0$	$E_1^*(S_1^*, I_1^*, R_1^*, \alpha_1^*), S_1^* = \frac{1}{2}\frac{B}{\mu}, I_1^* = \sqrt{\frac{\mu\delta}{\lambda}}, R_1^* = \frac{1}{2}\frac{B}{\mu}\theta + \frac{\varepsilon}{\mu}\sqrt{\frac{\mu\delta}{\lambda}}, \alpha_1^* = \sqrt{\frac{\mu\lambda}{\delta}}$
$\Delta>0$	$E_2^*(S_2^*, I_2^*, R_2^*, \alpha_2^*)$, $S_2^* = \frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)I_2^*}, I_2^* = \frac{B\lambda(1-\theta)+\sqrt{\Delta}}{2\lambda(\mu+\varepsilon)}, R_2^* = [\theta\frac{\mu+\varepsilon}{1-\theta} + \frac{\varepsilon}{\mu}]I_2^*, \alpha_2^* = \frac{\lambda}{\delta}I_2^*$
	$E_3^*(S_3^*, I_3^*, R_3^*, \alpha_3^*)$, $S_3^* = \frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)I_3^*}, I_3^* = \frac{B\lambda(1-\theta)-\sqrt{\Delta}}{2\lambda(\mu+\varepsilon)}, R_3^* = [\theta\frac{\mu+\varepsilon}{1-\theta} + \frac{\varepsilon}{\mu}]I_3^*, \alpha_3^* = \frac{\lambda}{\delta}I_3^*$

3.2 Stability of equilibriums

Theorem3.1. *The equilibrium point E_0 is locally asymptotically stable.*

Proof. We can use the Jacobin matrix of system (2) to illustrate.

$$J = \begin{bmatrix} -\alpha I-\mu & -\alpha S & 0 & -SI \\ (1-\theta)\alpha I & (1-\theta)\alpha S-(\mu+\varepsilon) & 0 & (1-\theta)SI \\ \theta\alpha I & \theta\alpha S+\varepsilon & -\mu & \theta SI \\ 0 & \lambda & 0 & -\delta \end{bmatrix} \quad (5)$$

The above Jacobin matrix at the equilibrium $E_0(\frac{B}{\mu}, 0, 0, 0)$ can be written as

$$J(E_0)=\begin{bmatrix}-\mu & 0 & 0 & 0\\ 0 & -(\mu+\varepsilon) & 0 & 0\\ 0 & \varepsilon & -\mu & 0\\ 0 & \lambda & 0 & -\delta\end{bmatrix} \tag{6}$$

It is easy to know that the characteristic roots for $J(E_0)$ are: $h_{01}=h_{02}=-\mu<0$, $h_{03}=-(\mu+\varepsilon)$, $h_{04}=-\delta$, based on Routh-Hurwitz criteria, the equilibrium point E_0 is locally asymptotically stable.

Theorem3.2. *The equilibrium point E_1^* is not stable.*

Proof. The Jacobin matrix at the equilibrium $E_1^*(\frac{1}{2}\frac{B}{\mu},\sqrt{\frac{\mu\delta}{\lambda}},\frac{1}{2}\frac{B}{\mu}\theta+\frac{\varepsilon}{\mu}\sqrt{\frac{\mu\delta}{\lambda}},\sqrt{\frac{\mu\lambda}{\delta}})$ is

$$J(E_1^*)=\begin{bmatrix}-2\mu & -\frac{1}{2}B\sqrt{\frac{\lambda}{\mu\delta}} & 0 & -\frac{1}{2}B\sqrt{\frac{\delta}{\mu\lambda}}\\ (1-\theta)\mu & (1-\theta)\frac{1}{2}B\sqrt{\frac{\lambda}{\mu\delta}}-(\mu+\varepsilon) & 0 & (1-\theta)\frac{1}{2}B\sqrt{\frac{\delta}{\mu\lambda}}\\ \theta\mu & \theta\frac{1}{2}B\sqrt{\frac{\lambda}{\mu\delta}}+\varepsilon & -\mu & \theta\frac{1}{2}B\sqrt{\frac{\delta}{\mu\lambda}}\\ 0 & \lambda & 0 & -\delta\end{bmatrix} \tag{7}$$

We describe the characteristic equation of matrix $J(E_1^*)$ as

$$\left|J(E_1^*)-hE\right|=\begin{vmatrix}-2\mu-h & -\frac{1}{2}B\sqrt{\frac{\lambda}{\mu\delta}} & 0 & -\frac{1}{2}B\sqrt{\frac{\delta}{\mu\lambda}}\\ (1-\theta)\mu & (1-\theta)\frac{1}{2}B\sqrt{\frac{\lambda}{\mu\delta}}-(\mu+\varepsilon)-h & 0 & (1-\theta)\frac{1}{2}B\sqrt{\frac{\delta}{\mu\lambda}}\\ \theta\mu & \theta\frac{1}{2}B\sqrt{\frac{\lambda}{\mu\delta}}+\varepsilon & -\mu-h & \theta\frac{1}{2}B\sqrt{\frac{\delta}{\mu\lambda}}\\ 0 & \lambda & 0 & -\delta-h\end{vmatrix} \tag{8}$$

From the model, we can obtain one of characteristic value $h_{11}=-\mu$.

We construct a polynomial to judge the others characteristic roots of Jacobin matrix $J(E_1^*)$:

$$h^3+a_2h^2+a_1h+a_0=0 \tag{9}$$

Where,

$$a_2 = \frac{1}{2}(1-\theta)B\sqrt{\frac{\lambda}{\mu\delta}} - 3\mu - \varepsilon - \delta \ ,$$

$$a_1 = \frac{1}{2}(1-\theta)B\sqrt{\frac{\lambda}{\mu\delta}}(\mu + 2\delta) - (3\mu + \varepsilon)\delta - 2\mu(\mu + \varepsilon)$$

$$a_0 = (1-\theta)B\sqrt{\mu\delta\lambda} - 2\mu(\mu + \varepsilon)\delta$$

According to the condition of equilibrium point $\Delta = [B(1-\theta)\lambda]^2 - 4\mu\delta\lambda(\mu+\varepsilon)^2 = 0$, we can get $B(1-\theta) = 2(\mu+\varepsilon)\sqrt{\frac{\mu\delta}{\lambda}}$, put it into a_2, a_1, a_0 , then

$$a_2 = \frac{1}{2}(1-\theta)B\sqrt{\frac{\lambda}{\mu\delta}} - 3\mu - \varepsilon - \delta = (\mu+\varepsilon) - 3\mu - \varepsilon - \delta = -2\mu - \delta < 0$$

$$a_1 = \frac{1}{2}(1-\theta)B\sqrt{\frac{\lambda}{\mu\delta}}(\mu + 2\delta) - (3\mu + \varepsilon)\delta - 2\mu(\mu + \varepsilon) = -\mu^2 - \mu(\varepsilon + \delta) + \varepsilon\delta$$

$$a_0 = (1-\theta)B\sqrt{\mu\delta\lambda} - 2\mu(\mu + \varepsilon)\delta = 2(\mu+\varepsilon)\mu\delta - 2\mu(\mu+\varepsilon)\delta = 0$$

Therefore, it does not satisfy the necessary conditions for stability, based on Routh-Hurwitz criteria, the equilibrium point E_1^* is not stable.

Theorem3.3. *The equilibrium point* E_2^* *is locally asymptotically stable if* $B^2(1-\theta)^2\lambda > 4\mu\delta(\mu+\varepsilon)^2$ and $\mu < \delta < 1$.

Proof. The Jacobin matrix at the equilibrium $E_2^*(S_2^*, I_2^*, R_2^*, \alpha_2^*)$ is

$$S_2^* = \frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)I_2^*}, I_2^* = \frac{B\lambda(1-\theta)+\sqrt{\Delta}}{2\lambda(\mu+\varepsilon)}, R_2^* = [\theta\frac{\mu+\varepsilon}{1-\theta} + \frac{\varepsilon}{\mu}]I_2^*, \alpha_2^* = \frac{\lambda}{\delta}I_2^*$$

$$J(E_2^*)=\begin{bmatrix} -\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2} & -\frac{\mu+\varepsilon}{1-\theta} & 0 & -\frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)} \\ (1-\theta)[\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}-\mu] & 0 & 0 & \frac{\delta(\mu+\varepsilon)}{\lambda} \\ \theta[\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}-\mu] & \frac{\mu\theta+\varepsilon}{1-\theta} & -\mu & \theta\frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)} \\ 0 & \lambda & 0 & -\delta \end{bmatrix} \quad (10)$$

We describe the characteristic equation of matrix $J(E_2^*)$ as

$$\left|J(E_2^*)-hE\right|=\begin{vmatrix} -\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}-h & -\frac{\mu+\varepsilon}{1-\theta} & 0 & -\frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)} \\ (1-\theta)[\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}-\mu] & 0-h & 0 & \frac{\delta(\mu+\varepsilon)}{\lambda} \\ \theta[\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}-\mu] & \frac{\mu\theta+\varepsilon}{1-\theta} & -\mu-h & \theta\frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)} \\ 0 & \lambda & 0 & -\delta-h \end{vmatrix} \quad (11)$$

From the model, we can obtain one of characteristic value $h_{21}=-\mu$.

We construct a polynomial to judge the others characteristic roots of Jacobin matrix $J(E_2^*)$:

$$h^3+b_2h^2+b_1h+b_0=0 \quad (12)$$

Where,

$$b_2=\delta+\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2},$$

$$b_1=(\delta+1)\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}+(1-\mu)(\mu+\varepsilon),$$

$$b_0=\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2(\mu+\varepsilon)^2}+\mu(1-\delta)(\mu+\varepsilon),$$

$$b_2b_1-b_0=\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}[\delta^2+(1-\mu)(\mu+\varepsilon)]$$

$$+(1+\delta)[\frac{B^2\lambda(1-\theta)^2+B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}]^2+(\delta-\mu)(\mu+\varepsilon),$$

It is obvious that $b_2>0$. If $\mu<\delta<1$, then $b_1>0, b_0>0, b_2b_1-b_0>0$.

According to the Routh–Hurwitz stability judgment, E_2^* is locally asymptotically stable

Theorem3.4. *The equilibrium point* E_3^* *is locally asymptotically stable if* $B^2(1-\theta)^2\lambda>4\mu\delta(\mu+\varepsilon)^2$ and $\mu<\delta<1$.

Proof. The Jacobin matrix at the equilibrium $E_3^*(S_3^*, I_3^*, R_3^*, \alpha_3^*)$ is

$$S_3^*=\frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)I_3^*}, I_3^*=\frac{B\lambda(1-\theta)-\sqrt{\Delta}}{2\lambda(\mu+\varepsilon)}, R_3^*=[\theta\frac{\mu+\varepsilon}{1-\theta}+\frac{\varepsilon}{\mu}]I_3^*, \alpha_3^*=\frac{\lambda}{\delta}I_3^*$$

$$J(E_3^*)=\begin{bmatrix} -\frac{B^2\lambda(1-\theta)^2-B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2} & -\frac{\mu+\varepsilon}{1-\theta} & 0 & -\frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)} \\ (1-\theta)[\frac{B^2\lambda(1-\theta)^2-B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}-\mu] & 0 & 0 & \frac{\delta(\mu+\varepsilon)}{\lambda} \\ \theta[\frac{B^2\lambda(1-\theta)^2-B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}-\mu] & \frac{\mu\theta+\varepsilon}{1-\theta} & -\mu & \theta\frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)} \\ 0 & \lambda & 0 & -\delta \end{bmatrix} \quad (13)$$

We describe the characteristic equation of matrix $J(E_3^*)$ as

$$\left|J(E_3^*)-hE\right|=\begin{vmatrix} -\frac{B^2\lambda(1-\theta)^2-B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}-h & -\frac{\mu+\varepsilon}{1-\theta} & 0 & -\frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)} \\ (1-\theta)[\frac{B^2\lambda(1-\theta)^2-B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}-\mu] & 0-h & 0 & \frac{\delta(\mu+\varepsilon)}{\lambda} \\ \theta[\frac{B^2\lambda(1-\theta)^2-B(1-\theta)\sqrt{\Delta}}{2\delta(\mu+\varepsilon)^2}-\mu] & \frac{\mu\theta+\varepsilon}{1-\theta} & -\mu-h & \theta\frac{\delta(\mu+\varepsilon)}{\lambda(1-\theta)} \\ 0 & \lambda & 0 & -\delta-h \end{vmatrix} \quad (14)$$

From the model, we can obtain one of characteristic value $h_{31}=-\mu$.

We construct a polynomial to judge the others characteristic roots of Jacobin matrix $J(E_3^*)$:

$$h^3 + k_2 h^2 + k_1 h + k_0 = 0 \quad (15)$$

Where,

$k_2 = \delta + \frac{B(1-\theta)[B\lambda(1-\theta) - \sqrt{\Delta}]}{2\delta(\mu+\varepsilon)^2}$,

$k_1 = (\delta+1)\frac{B(1-\theta)[B\lambda(1-\theta) - \sqrt{\Delta}]}{2\delta(\mu+\varepsilon)^2} + (1-\mu)(\mu+\varepsilon)$,

$k_0 = \frac{B(1-\theta)[B\lambda(1-\theta) - \sqrt{\Delta}]}{2\delta(\mu+\varepsilon)^2} + \mu(1-\delta)(\mu+\varepsilon)$,

$$k_2 k_1 - k_0 = \frac{B(1-\theta)[B\lambda(1-\theta) - \sqrt{\Delta}]}{2\delta(\mu+\varepsilon)^2}[\delta^2 + (1-\mu)(\mu+\varepsilon)]$$

$$+(1+\delta)[\frac{B(1-\theta)[B\lambda(1-\theta) - \sqrt{\Delta}]}{2\delta(\mu+\varepsilon)^2}]^2 + (\delta-\mu)(\mu+\varepsilon),$$

Based on the assumptions of $\sqrt{\Delta} = \sqrt{[B(1-\theta)\lambda]^2 - 4\mu\delta\lambda(\mu+\varepsilon)^2} < B(1-\theta)\lambda$, then we know $B\lambda(1-\theta) - \sqrt{\Delta} > 0$, so it is obvious that $k_2 > 0$. If $\mu < \delta < 1$, then $k_1 > 0$, $k_0 > 0$, $k_2 k_1 - k_0 > 0$. According to the Routh–Hurwitz stability judgment, E_3^* is locally asymptotically stable.

4. Numerical simulation

In this section, we need to illustrate some numerical simulations that we performed to validate the theoretical model and results of the previous sections. In the similar literature on rumor propagation, the range of these parameters has not been explicitly given. Most of them are limited to positive numbers. In the numerical simulation, we refer to the values in other similar literatures and combine the requirements of stability conditions, and give the numerical values of the parameters in the model [24, 25, 28, 29].

Let $B = 1$, $\mu = 0.35$, $\varepsilon = 0.1$, $\theta = 0.3$, $\lambda = 0.1$, δ=0.2, then, figure 2 depicts the locally asymptotically stability of system (2) about E_0 with different initial value. The results of figure 2 clearly show that different initial values do not affect the stability of the E_0.

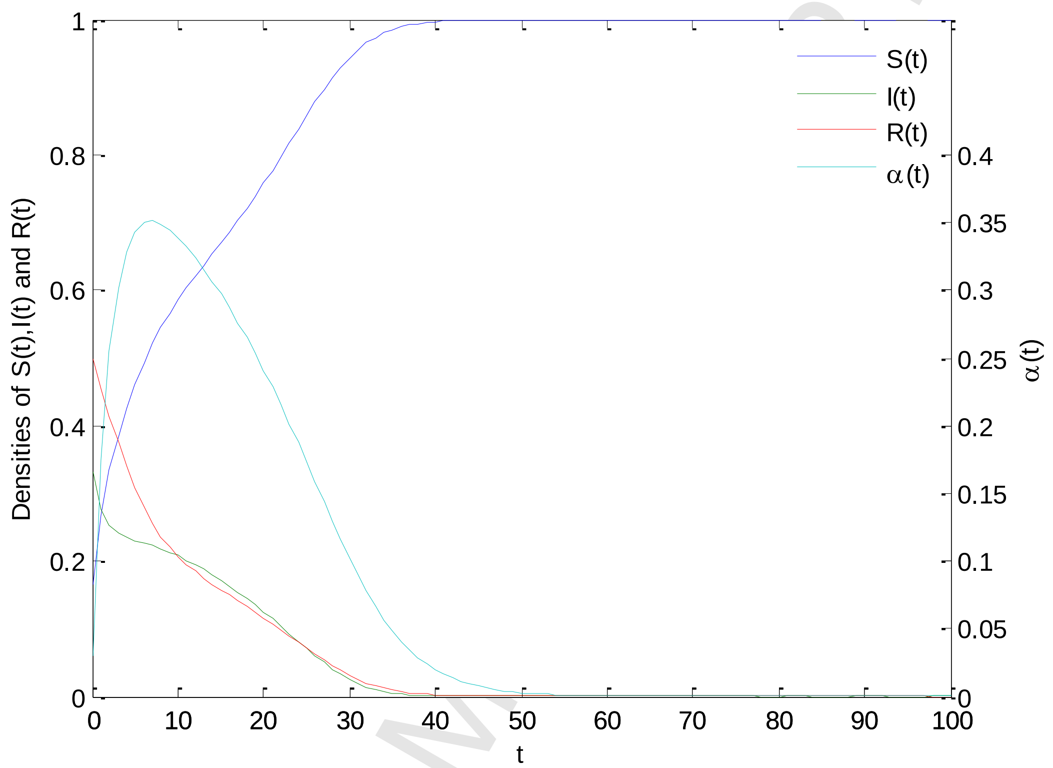

Fig.2. The stability of system (2) about E_0 with different initial value.

Fig.3 shows the locally asymptotic stability of system (2) about equilibrium E_2^* by taking $B=1$, $\mu=0.355$, $\varepsilon=0.12$, $\theta=0.2$, $\lambda=0.2$, δ=0.2, which satisfies the locally asymptotic stability condition in Theorem 3.3.

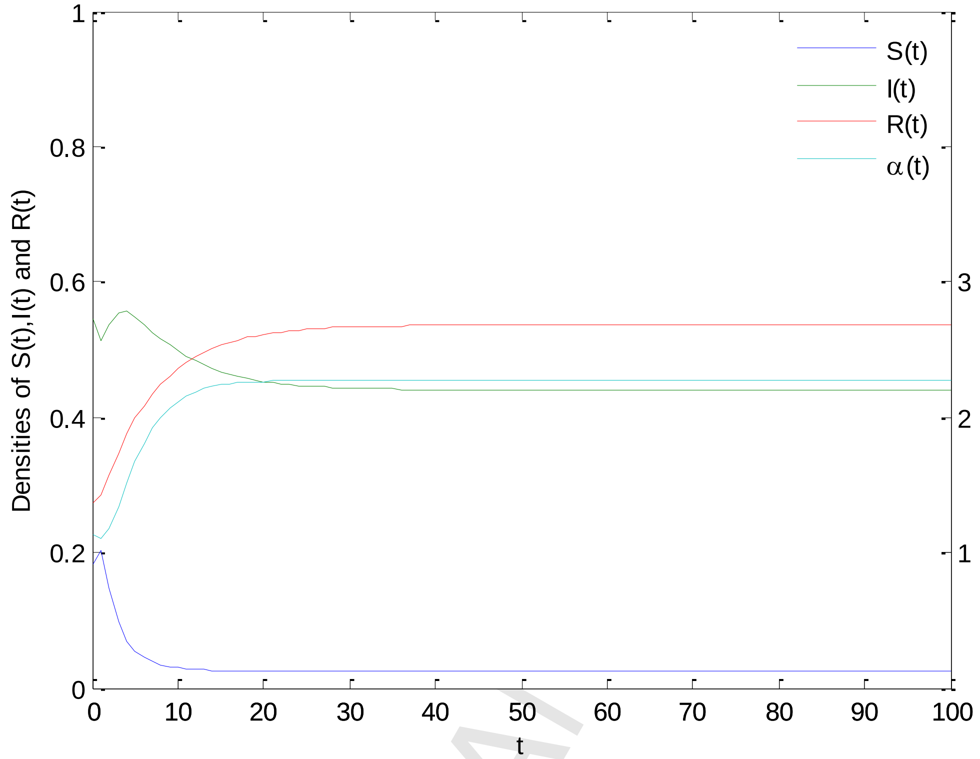

Fig.3. The densities of three groups over time at E_2^* with different initial value.

Fig.4 describes the local asymptotic stability of system (2) about equilibrium E_3^* by taking Let $B=0.5$, $\mu=0.16$, $\varepsilon=0.12$, $\theta=0.2$, $\lambda=0.09$, δ=0.2, which satisfies the local asymptotic stability condition in Theorem 3.4. From the figure, we can see that the density of $S(t)$ increased slowly, close to 1, while the density of $I(t)$ and $R(t)$ decreased slowly, approaching 0. The situation about E_3^* is similar to the case of E_0.

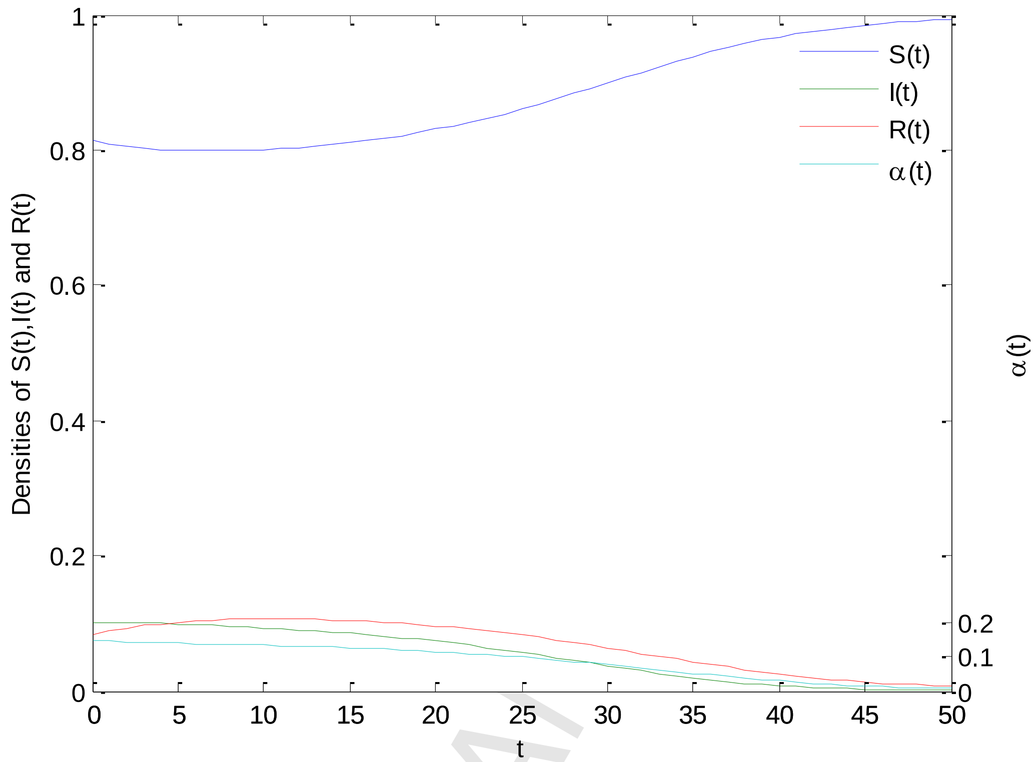

Fig.4. The densities of three groups over time at E_3^* with different initial value.

Fig.5 illustrates how the densities of $S(t), I(t)$ and $R(t)$ change with different θ by taking $B=5$, $\mu=0.155$, $\varepsilon=0.12$, $\lambda=4$, δ=0.2. Given the initial value (1,1,1,1), when $\theta=0.96$, the density of R is 0.8994, this maximum can be obtained by software Matlab2014.

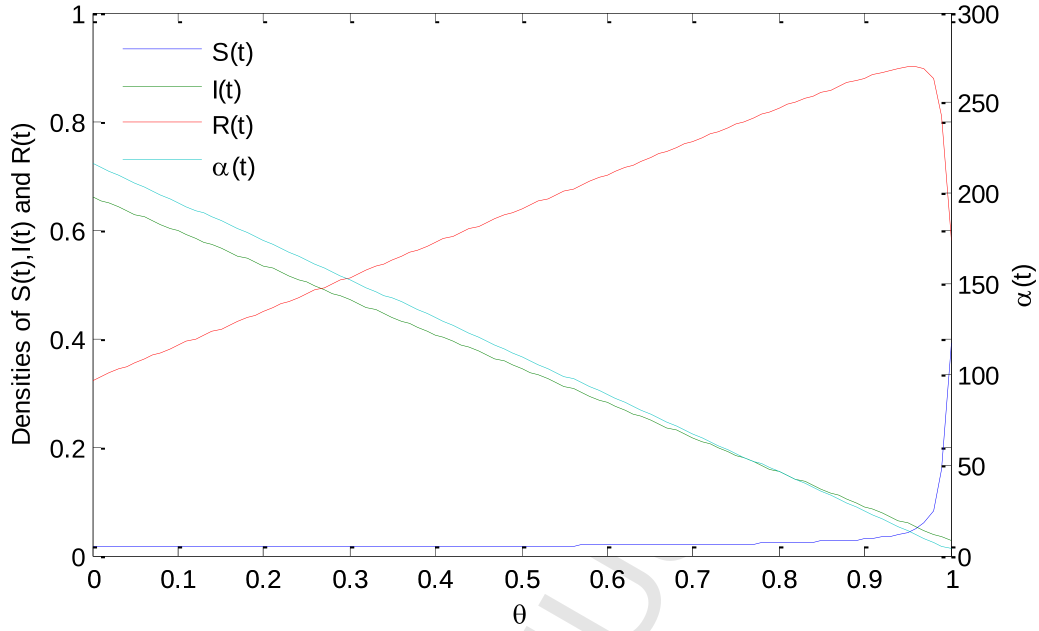

Fig.5. The density of $S(t), I(t), R(t)$ and the value of $\alpha(t)$ over time under different θ.

At the same time, it can be seen from Fig.5, with the bigger of the parameter θ, $\alpha(t)$ which means the intensity of rumors has continued to decline. From the Fig.5, we can see clearly that increasing the proportion of the wisemen in the population is one of the effective means to reduce the spread of rumors.

In addition, for practical problems, we propose to determine the specific numerical parameters, referring to the relevant professional background knowledge by investigating the actual background, according to the relevant existing literature, etc.

Fig.6 discusses how the densities of $S(t), I(t)$ and $R(t)$ change with different λ by taking $B=1$, $\mu=0.355$, $\varepsilon=0.12$, $\theta=0.2$, δ=0.2. As can be seen from Fig.6, when $\lambda<5$, $S(t)$ decreases rapidly, while $I(t)$ and $R(t)$ rise rapidly; when $\lambda>5$, $S(t), I(t)$ and $R(t)$ are in stable state and no long change. At the same time, with the bigger of the parameter λ, $\alpha(t)$ which means the intensity of rumors has continued to increase. It is not difficult to find from the figure 6 that the increase of the parameter λ is beneficial to the increase of rumor propagation intensity, but it is unfavorable to control and reduce rumor spread.

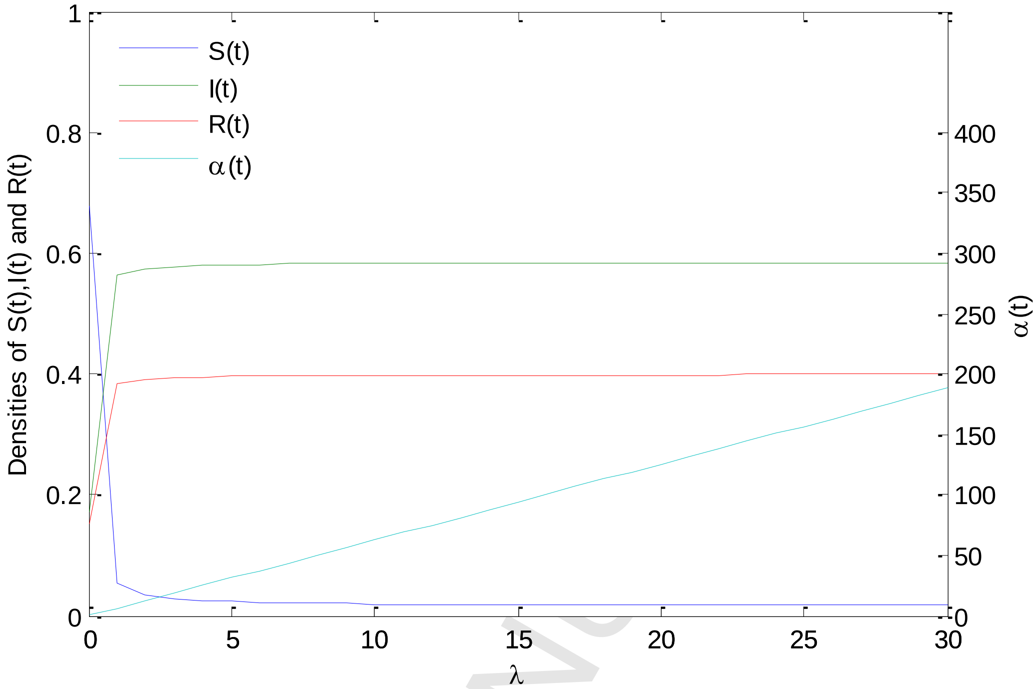

Fig.6. The density of $S(t), I(t), R(t)$ and the value of $\alpha(t)$ over time under different λ .

Fig.7 describes how the densities of $S(t), I(t)$ and $R(t)$ change with different δ by taking $B=5$, $\mu=0.355$, $\varepsilon=0.12$, $\theta=0.2$, λ=5. As can be seen from Fig.6, when $\delta<54$, $S(t)$ decreases slowly, while $I(t)$ and $R(t)$ rise less quickly; when $\delta>54$, $S(t), I(t)$ and $R(t)$ are in stable state and no long change. At the same time, with the larger of the parameter δ , $\alpha(t)$, which means the intensity of rumors, has dropped rapidly and then became stable. It is not difficult to find from the Fig.6 that the increase of the parameter δ is beneficial to the increase of rumor propagation intensity. Therefore, we know that increasing the value of parameter δ is also one of the effective measures to control and reduce rumor propagation.

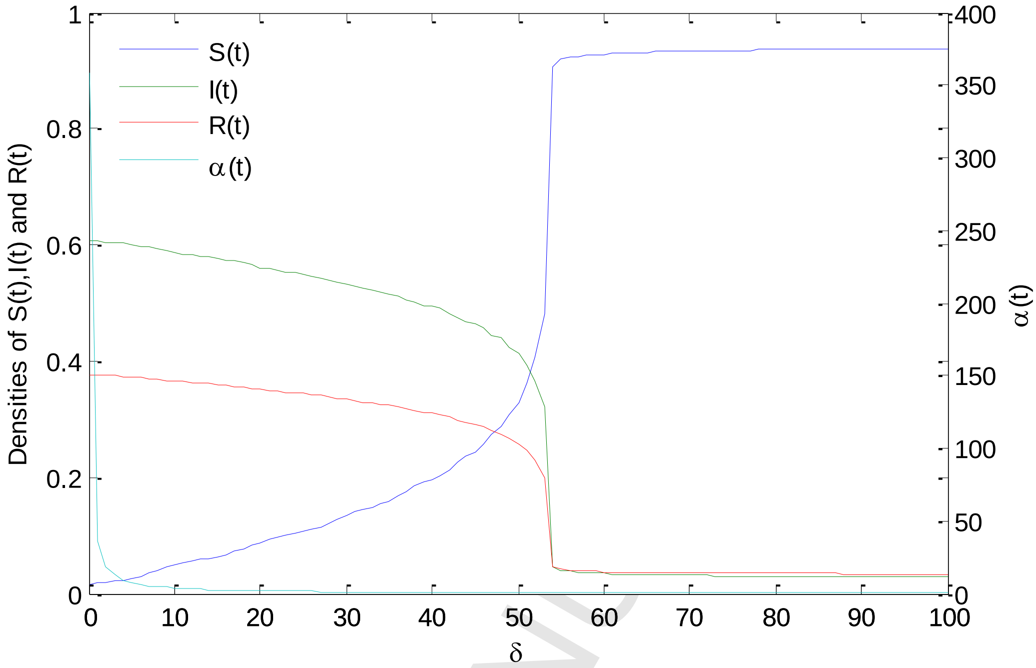

Fig.7. The density of $S(t), I(t), R(t)$ and the value of $\alpha(t)$ over time under different δ .

Fig.8 shows that the densities of $S(t), I(t)$ and $R(t)$ change with different ε by taking $B=18$, $\mu=0.255$, $\delta=0.1$, $\theta=0.2$, $\lambda=6$. In this process, the density of $S(t)$ increases slowly till becoming stable, and the density of $I(t)$ also decreases quickly till becoming stable. In addition, it is interesting in this process that the density of $R(t)$ firstly rises rapidly, reaches its peak, and then declines slowly until it reaches a steady state. In the whole process, the number of $\alpha(t)$ descends quickly till reaching the stable. From Fig.8, it is not difficult to get the result that the increase in parameter ε does help control and reduce rumor propagation, but this effect is getting weaker as the parameter ε grows.

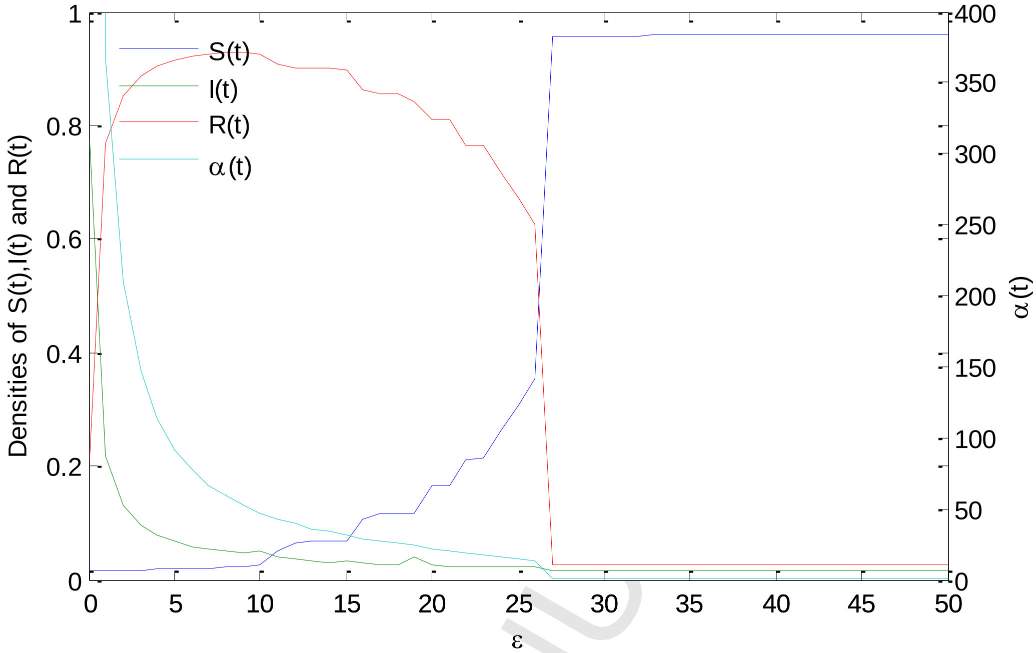

Fig.8. The density of $S(t), I(t), R(t)$ and the value of $\alpha(t)$ over time under different ε .

5. Conclusion

To solve the problem of rumor spreading systematically, we should try to educate more wise individuals among the public. On the other hand, the effects of social media on rumor spreading cannot be ignored, in that rumors can constantly cheat the public in the disguise of scientific and theoretical observations.

In this paper, a rumor spreading model was studied which considered the proportion of wisemen in the crowd, without the media factor. In contrast with previous studies, the speed of rumor propagation was considered as a variable over time rather than a constant. The equilibrium points of the model can be calculated, of which the stability condition was given.

Our proposal is to help increase the proportion of wise individuals among the public by scientific knowledge education and publication. To observe and study the specific proportion changes of wise people among the public will be one of our future work. Another aspect of our future research is to explore possibilities of taking comprehensive measures, taking both social media and the publicity of people into account.

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