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New progress of Grey System Theory in the new millennium

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Abstract

Purpose – The purpose of this paper is to summarize the progress in grey system research during 2000-2015, so as to present some important new concepts, models, methods and a new framework of grey system theory.

Design/methodology/approach – The new thinking, new models and new methods of grey system theory and their applications are presented in this paper. It includes algorithm rules of grey numbers based on the "kernel" and the degree of greyness of grey numbers, the concept of general grey numbers, the synthesis axiom of degree of greyness of grey numbers and their operations; the general form of buffer operators of grey sequence operators; the four basic models of grey model GM(1,1), such as even GM, original difference GM, even difference GM, discrete GM and the suitable sequence type of each basic model, and suitable range of most used grey forecasting models; the similarity degree of grey incidences, the closeness degree of grey incidences and the three-dimensional absolute degree of grey incidence of grey incidence analysis models; the grey cluster model based on center-point and end-point mixed triangular whitenization functions; the multi-attribute intelligent grey target decision model, the two stages decision model with grey synthetic measure of grey decision models; grey game models, grey input-output models of grey combined models; and the problems of robust stability for grey stochastic time-delay systems of neutral type, distributed-delay type and neutral distributed-delay type of grey control, etc. And the new framework of grey system theory is given as well.

Findings – The problems which remain for further studying are discussed at the end of each section. The reader could know the general picture of research and developing trend of grey system theory from this paper.

Practical implications – A lot of successful practical applications of the new models to solve various problems have been found in many different areas of natural science, social science and engineering, including spaceflight, civil aviation, information, metallurgy, machinery, petroleum, chemical industry, electrical power, electronics, light industries, energy resources, transportation, medicine, health, agriculture, forestry, geography, hydrology, seismology, meteorology, environment protection, architecture, behavioral science, management science, law, education, military science, etc. These practical applications have brought forward definite and noticeable social and economic benefits. It demonstrates a wide range of applicability of grey system theory, especially in the situation where the available information is incomplete and the collected data are inaccurate.

Originality/value – The reader is given a general picture of grey systems theory as a new model system and a new framework for studying problems where partial information is known; especially for uncertain systems with few data points and poor information. The problems remaining for further studying are identified at the end of each section.

Keywords Grey systems theory, Combined grey models, Buffer operators,
Grey forecasting models, Grey incidence analysis models, Operations of grey numbers

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1. Introduction

In 1982, the first paper "The Control Problems of Grey Systems" on grey system by Professor Julong Deng (1982) was published in the journal named *Systems & Control Letters* which was published by North Holland publishing company. In the same year, Professor Julong Deng's first grey system paper "Grey Control System" in Chinese was published by *Journal of Huazhong University of Science and Technology* (Deng, 1982). The publication of these two seminal papers indicated that a new and cross-sectional discipline named grey system theory came into the world.

The grey systems theory, established by Julong Deng in 1982, is a new methodology that focusses on the study of problems involving small samples and poor information. It deals with uncertain systems with partially known information through generating, excavating and extracting useful information from what is available. So, systems' operational behaviors and their laws of evolution can be correctly described and effectively monitored (Liu *et al.*, 2014c) In the natural world, uncertain systems with small samples and poor information exist commonly. That fact determines the wide range of applicability of grey systems theory (Andrew, 2011; Haken, 2011; Hipel, 2011; Vallee, 2008).

In 1989, an international journal, *The Journal of Grey System*, was launched by Research Information Ltd in UK. Currently, this publication is indexed by Mathematical Review of the USA, Science Citation Index, and other important indexing agencies from around the world. In 1997, a Chinese publication, named *Journal of Grey System*, is launched in Taiwan, China. It is later in 2004 that this publication becomes all English. Additionally, a new journal, named *Grey Systems: Theory and Application*, edited by the faculty of Institute for Grey Systems Studies at Nanjing University of Aeronautics and Astronautics (NUAA), has been launched by Emerald in 2011. There are currently over 1,000 different professional journals in the world that have accepted and published papers in grey systems theory, some of them are the top journals in a variety of fields. As of this writing, many journals and publishers, such as *The Journal of the Association for Computing Machinery* (USA), *Communications in Fuzzy Mathematics* (Taiwan, China), *Kybernetes: The International Journal of Systems & Cybernetics*, *Transaction of Nanjing University of Aeronautics and Astronautics*, China Ocean Press, Chinese Agricultural Science Press, Henan University Press, Huazhong University of Science and Technology Press Co. Ltd, IEEE Press, Springer-Verlag have respectively published special issues or proceedings on grey systems theory.

There are numerous universities set up the curriculums of the grey system theory all around the world. For example, in NUAA, the curriculums of the grey system theory are founded not only in PhD and master programs, but also with undergraduate programs of different disciplines of the whole university as a public elective course. In 2008, the course of grey system theory of NUAA was selected as one of the national-level model courses. In 2013, the same course was selected as the national excellent resource sharing courses, which became learning resource free opening for all of the grey system hobbyists.

There are a lot of universities recruiting and fostering doctors and postdoctoral researchers on grey systems. Such as Huazhong University of Science and Technology, NUAA, Southeast University, Wuhan University of Technology, Fuzhou University, Shantou University, America Central Florida University, Nebraska-Lincoln University, Canada Warterloo University, Toronto University, De Montfort University, Spain Pablo de Olavide University, Turkey Bogazici University, Cape Town University in

South Africa, Romania Bucharest Economics University, Janpan Kanagawa University and many universities in Taiwan. There are also tens of thousands of graduate students and doctors in worldwide colleges engaged in scientific research applying the grey system thinking and methods.

There are numerous publishing agencies, such as Science Press, Defense Industries Press, Huazhong University of Science and Technology Press Co. Ltd, Jiangsu Science and Technology Press, Shandong People's Press, Science and Technology Literature Press of China, China Science and Technology Book Press of Taiwan, Gaoli Books Limited Company of Taiwan, ASE Press of Romania, Japan Polytechnic Press, IIGSS Academic Press, CRC of Taylor & Francis Group, Springer-Verlag, Springer-Verlag London Ltd, and so on, published hundreds of academic works on grey system in a variety of different languages, including Chinese, English, Japanese, Korean, Romanian and German.

A group of emerging edge disciplines such as the grey hydrology, the grey geology, the grey breeding, the grey medical science, etc., have appeared.

Various national and local science funding agencies are actively supporting grey system research. A large number of research projects on grey system theory or applications are supported by various funds annually. There are hundreds of research projects on grey systems and applications acquiring support from National Natural Science Foundation of China, The European Commission, The Royal Society, Leverhulme Trust and Canada, Spain, Romania national funds.

Since 2000, 18 regional conferences on grey system theory and applications have been held, which were supported by Leverhulme Trust, Institute for Grey System Studies, NUAA, De Montfort University, Wuhan University of Technology, Educational Society of Pudong, Shanghai, and China Center of Advanced Science And Technology which Mr Tsung-Dao Lee, a Nobel Prize winner hold the post of the director, and two of the former president of Chinese Academy of Sciences, Mr Zhou Guangzhao and Mr Lu Yongxiang hold the post of vice director. The development of the grey system theory promoted greatly with a large number of young scholars joint the events.

A lot of special sessions or tracks on grey system theory had been organized at numerous significant international conferences, such as International Conference on Uncertain System Modeling, International Conference on System Forecast and Control, International Conference on General System Studies, International Congress of World Organisation of Systems and Cybernetics, IEEE International Conference on Systems, Man and Cybernetics and so on. Becoming a hot concerning and discussion point in many important international conferences surely play an active role on further promoting grey system theory among the world system science peer.

In 2007, 2009, 2011, 2013, and 2015 the first, second, third, fourth and fifth IEEE International Conference on Grey Systems and Intelligent Services were held in Nanjing, Macao and Leicester, respectively. Each conference has received a large number of' submissions from many countries or regions, such as China, America, England, Germany, France, Spain, Switzerland, Hungary, Poland, Japan, South Africa, Russia, Turkey, Romania, Holland, Malaysia, Iran, Ukraine, Kazakhstan, Pakistan, Iran, Taiwan, Macao, Hong Kong, etc. More than one thousand articles featured in the five conferences were indexed by EI database, among which more than 300 excellent papers were published by *Kybernetes*, *Grey Systems: Theory and Application*, *The Journal of Grey System*, *Transaction of Nanjing University of Aeronautics and Astronautics* (English version) and Springer-Verlag.

Many famous scholars have given a high evaluation to the grey system research, such as Professor Qian Xuesen, Professor Lotfi A. Zadeh (America), the founder of

fuzzy mathematics; Professor Herman Haken (Germany), the founder of synergetics; Professor James M. Tien, former vice president of IEEE and member of the National Academy of Engineering, USA; Professor Robert Valee (France), the president of World Organisation of Systems and Cybernetics, Professor Alex Andrew (England), the secretary general of World Organisation of Systems and Cybernetics, the president of the Canadian Royal Academy of Sciences (Canada); and a lot of Academicians of Chinese Academy of Sciences and Chinese Academy of Engineering, say Professor Yang Shuzi, Professor Xiong Youlun, Professor Lin Qun, Professor Chen Da, Professor Zhao Chunsheng, Professor Hu Haiyan, Professor Xu Guozhi, Professor Wang Zhongtuo, Professor Yang Shanlin, etc.

In 2005, Grey System Society of China, the CSOOPEM was approved by China Association for Science and Technology, and Ministry of Civil Affairs of China. At the beginning of 2008, the Technical Committee of IEEE SMC on Grey Systems was established. In 2012, the first Workshop of European grey system research collaboration network was held by De Montfort University. In 2013, Professor Sifeng Liu was selected as a Marie Curie International Incoming Fellowship (FP7-PEOPLE- IIF-GA-2013-629051) of the Seventh Research Framework Programme of the European Commission. In 2014, an International Network project entitled "Grey Systems and its Applications" (IN-2014-020) was granted by Leverhulme Trust. Supported by this project, a series of grey system theory cooperative research and academic exchange activities will be held in Europe, North America and China.

As an emerging discipline, the grey system theory is standing in the science forest with its strong vitality.

2. Grey numbers and their operations

A grey number is a kind of figure that we only know the range of values and do not know exact value. The range of a grey number can be an interval or a general number set. A grey number is usually expressed in symbol "⊗," called gray. A grey number can be applied to represent the degree of uncertainty of information. As the basic unit and cell of the grey system theory, the research on grey numbers and gray measure has attracted significant attention.

In 2004, an axiomatic definition of the grey degree of grey numbers was put forward based on measure of grey numbers and its background or domain by Liu and Lin (2004). This definition of the grey degree of a grey number satisfying the requirement of standardability, which provided the bedrock for us to cognize the uncertainty of grey information. In 2010, the introduction of unreduction axiom and a new definition of degree of greyness of grey numbers was put forward. Then the operations of grey numbers and grey algebraic system is built based on the grey "kernel" and the degree of greyness of grey numbers (Liu *et al.*, 2010b). On these grounds, the operation of grey numbers has been transformed to the operation of real numbers. So, the difficult problem of setting up the operation of grey numbers and the grey algebraic system has been solved to a certain degree.

In 2012, Liu *et al.* and so on came up with the concept of general grey number.

Definition 1. Let:

$$g^{\pm} \in \bigcup_{i=1}^{n} \left[\underline{a}_i, \overline{a}_i\right] \tag{1}$$

then $g^{\pm}$ is called a general grey number.

Among them, any interval grey number $\otimes_i \in [\underline{a}_i, \overline{a}_i] \subset \cup_{i=1}^{n} [\underline{a}_i, \overline{a}_i]$, satisfy $\underline{a}_i, \overline{a}_i \in \mathfrak{R}$ and $\overline{a}_{i-1} \leqslant \underline{a}_i \leqslant \overline{a}_i \leqslant \underline{a}_{i+1} g^- = \inf_{\underline{a}_i \in g^{\pm}} \underline{a}_i, g^+ = \sup_{\overline{a}_i \in g^{\pm}} \overline{a}_i$ are called the lower and upper limits of $g^{\pm}$.

They also found that summation and subtraction operation about degree of greyness of grey numbers do not satisfy the introduction unreduction axiom. Then they verified summation and subtraction operations about grey numbers and grey synthesis axiom as follows (Liu *et al.*, 2012).

Axiom 1 (the synthesis axiom of degree of greyness). When plus and minus are operated on n general grey numbers $g_1^{\pm}, g_2^{\pm}, \ldots, g_n^{\pm}$, then the degree of greyness g° of the operation results $g^{\pm}$ can be got as follows:

$$g^{\circ} = \frac{1}{\sum_{i=1}^{n} \hat{g}_i} \sum_{i=1}^{n} g_i^{\circ} \hat{g}_i = \sum_{i=1}^{n} w_i g_i^{\circ} \tag{2}$$

where:

$$w_i = \frac{\hat{g}_i}{\sum_{i=1}^{n} \hat{g}_i}, i = 1, 2, \ldots, n$$

are the weights of g_i°.

In addition, Yang *et al.* (2014) defined new measurements of uncertainties of grey numbers and grey sets, consisting of both absolute and relative uncertainties to give a comprehensive representation of uncertainties in a grey number and a grey set. The relationships between grey sets and interval-valued fuzzy sets are also analyzed from the point of view of the proposed uncertainty representation. They demonstrated that grey sets and interval-valued fuzzy sets provide different but overlapping models for uncertainty representation in sets.

Fang *et al.* (2005) put forward the concept of standard interval grey number and offered the algorithm of standard interval grey numbers and Li *et al.* (2012b) raised grey number rules based on numerical coverage. They carried out beneficial exploration around the grey number and operations. Yan *et al.* (2014b) analyzed the merits and demerits of the existing methods. Then, the projection rule from ordinary interval grey numbers to standard grey numbers in universe of discourse [0, 1] is designed based on grey numbers' universe of discourse and the principle of information persisting. Moreover, the concepts of relative kernel and degree of accuracy are proposed aiming at the standard grey numbers. Based on these concepts, the ranking method of grey numbers is presented.

Aydemir *et al.* (2015) developed an EPQ model which has been extended with grey demand rate, grey cost values and allowed maximum backorder level under imperfect items in copper wires manufacturing system by using degree of greyness approach.

In the grey algebra system based on the grey "kernel" and the degree of greyness of grey numbers, for the degree of greyness of the operation outcome of "multiplication" and "division," the principles of "take the bigger one" is still used according to the introduction unreduction axiom. It is a critical problem waiting to be solved that revealing the inherent law for synthesis of degree of greyness in the process of operations of "multiplication" and "division" to structure a more exquisite principle of operations of "multiplication" and "division." In this field, the developing trends is to establish more comprehensive representation of grey numbers in one hand, and simple and more practical operations on the other hand.

3. The grey sequence operator

In order to solve the prediction problem of the shock disturbed system, Liu (1991) put forward the concept of buffer operator, built up the axioms system of buffer operator and constructed several practical buffer operators.

Thereafter, research on buffer operator is pretty active and some new results have emerged. For example, Yaoguo Dang, (Dang *et al.*, 2004) Zhengpeng Wu, (Wu *et al.*, 2009) Jie Cui, (Cui and Dang, 2009) Lizhi Cui, (Cui *et al.*, 2010) Yeqing Guan, (Guan and Liu, 2008) Xiao-Li Hu, (Hu *et al.*, 2013) Yan Gao, (Dai and Su, 2012) Zhengxin Wang, (Gao *et al.*, 2013) Xuemei Li (Wang *et al.*, 2009),Wenqiang Dai (Li *et al.*, 2012a), etc. constructed a variety of different weaken and strengthen buffer operators based on the three buffer operator axioms.

In 2011, Yong *et al.* brought forth the general form of buffer operator:

$$x(k)d = x(k)\cdot\left[x(k)/\frac{1}{\sum_{i=k}^{n}\omega_i}\sum_{i=k}^{n}\omega_i x(i)\right]^{\alpha} \tag{3}$$

and proved that buffer operator in Equation (3) can express weaken buffer operator, strengthen buffer operator and the identity operator, respectively according to the different values of α. Ye *et al.* (2014) proposed the forecasting effect of grey model (GM)(1,1) and applicability evaluation criteria of weakening buffer operators based on systemic analysis of buffer operators working process to GM(1,1) prediction.

Because of the abundant shock disturbed system, the thinking methods and technology that buffer operator make the qualitative analysis results expressed quantitatively are widely applied in practice. Such as Liu *et al.*'s (2006) research on radar target tracking, Guo *et al.*'s (2014c) research on the economic effects of meteorological disasters, Liao *et al.*'s (2012) research on the analysis of transformer oil dissolved gas content and Zhu *et al.*'s (2012) research on the grey PID forecast control, etc.

Faced with actual vibration data, how to select and construct suitable buffer operator? How to determine the weight parameters and effect index of buffer operator? How the properties of the buffer operator are changed with the change of parameters and index? All of these are the problems that need further research. The answers to these questions are certainly the next step of development in this field.

4. The grey prediction model

The grey prediction model is one type of the GM with most active research and used widely. In 2005, Xie and Liu proposed the discrete grey model (DGM) first and studied its properties. Later, Wu *et al.* (2013b, 2015) came up with a kind of the fractional accumulation DGM and completed perturbation problem of GM. Chen *et al.* (2009) set up the DHGM (2,2) coupled equations combining grey differential equation and self-memory principle based on power system self-memory principle. Guo *et al.* (2014b) proposed the interval grey number self-memory prediction model based on the degree of greyness of synthesis grey number, then studied self-memory prediction model from different views.

Various forms of developments and derived model emerged in an endless stream. Such as Dang and Liu (2004) came up with GM(1,1) model based on $x(n)$ as the initial condition. Li *et al.* (2014) proposed GM(1,1, β) model, studied the content type and parameter set form of the model and analyzed several properties of the GM(1,1, β) model, then gave its optimization algorithm. Wang (2013) provided several kinds of

forms of GM(1,1) power model and studied the characteristics of their time response function. Xiao *et al.* (2013) studied generalized accumulation GM and proposed a combined optimization method. Qian *et al.* (2012) came up with the grey GM(1,1, t^{α}) model with the time power item and studied the process of modeling and parameter estimation method. Tang (2006) proposed a new prediction model based on grey supporting vector machine. Zhang (2014) put forward the multi variable DGM based on driving control. Zeng and Liu (2014) came up with the random oscillation sequence prediction model taking smooth operator compress random oscillation amplitude. Zhang (2007) used particle swarm algorithm and provided a new method of increasing the grey GM(1,1) precision through the optimization of background value interpolation coefficient and boundary value. Yao *et al.* (2010) studied the parameter characteristics of the new information discrete GM(1,1) model and fitting properties of the geometric sequence, then put forward a new information discrete GM(1,1) model with the sectional correction. Wu *et al.* (2013a) constructed the twice time-varying parameter DGM with the features of the white index law coincidence, linear law coincidence, twice law coincidence and stretching transformation consistency. Carmona Benitez *et al.* (2013) improved the GM model and forecasted long-term trend of American air transport industry passenger flow using improved model, then get satisfied results. Evans (2014) proposed a more general grey Verhulst model and forecasted changes of the steel strength in British used this model. Xie *et al.* (2014) studied the prediction problems of grey number sequence.

Liu and Deng (2000) studied the range suitable for GM(1,1) based on simulated test. The area of validity, the area to be used carefully, the area not suitable for use and the prohibited area of GM(1,1) have been divided clearly according to the threshold of the developing coefficients. Xiao and Wang (2014) studied the influences of model relative error made by the change of the background value of grey GM(1,1,α) model based on analysis of the modeling mechanism. Liu *et al.* (2003) utilized the method of "the least square estimate" to determine the constant number c in the time response sequence of whiterization equation of GM(1,1), then got the optimum time response sequence of whiterization equation for GM(1,1). Song *et al.* (2001) given a new method to handle derivative signal and background value and derived the adjusting GM. Ji *et al.* (2001) analyzed the characteristics of the deviation of the model. Then clarified the essence of the error of GM(1,1) model. Tong *et al.* (2002) shown that accumulated generating operation (AGO) of the GM can "strengthen" the law and reform randomicity of numbers, so it has nice anti-interference. Wang *et al.* (2001) put forward a GM(1,1) modeling method by taking the optimum weighted averages of ahead difference quotient and back difference quotient as the grey derivative whiting values, and proved that the new method have the linear transformation consistency. Mao *et al.* (2015) built a time-lag GM(1,N,τ) model, and provided its least squares parameter estimation formula and analytical solution. Liu *et al.* (2014a) analyzed the solution errors of a whitenization GM(1,1) model and a connotation GM(1,1) model, then present the condition that a connotation GM(1,1) model can be replaced by a whitenization GM(1,1) model. Tien (2003) proposed deterministic grey dynamic model with multiple inputs, DGDMMI(1,1,1) which with high prediction accuracy. Xia *et al.* (2015) proposed a real-time rolling grey forecasting method to provide efficient and accurate machine health prediction, while effects of influencing factors such as operating load are considered and analyzed.

In 2014d, Liu *et al.* determined four kinds of GM(1,1) basic models that are even GM(1,1) model, discrete GM(1,1) model, even difference GM(1,1) model and original

difference GM(1,1) model through the experiments of simulation, then made clear of the suitable type of sequences of the different model.

The application results of the grey prediction model are numerous.

Such as Kose and Tasci (2015) predicted the vertical displacement of the Crest of Keban Dam in Turkey by grey prediction method. Their results indicate that the grey prediction method produces better results, more in-keeping with true values. Gurden *et al.* (2001) built a spectroscopic batch process data model using GMs to incorporate external information. In their paper, different approaches to building GMs are described and some of their properties discussed. Chirwa and Mao (2006) used GM (1,1) model to estimate the accident risk based on data of UK and USA. Cempel (2008) monitored mechanical vibration state using the grey prediction model. Hsu and Yeh (2000) developed a new methodology for lossy image compression based on GM. Hao *et al.* (2012) analyzed and predicted hydrological process in Karst River Basin using the grey prediction model and gained the higher precision. Then they studied human activities effect on hydrological process in Karst River Basin using sectioned GM (Hao *et al.*, 2013). Yang and Wong (2014) made the further improvement about the unbiased GM and forecasted the amount of some city's gas supplement. Tabaszewski and Cempel (2015) developed a methodology of predicting values of vibration symptoms of fan mills in a combined heat and power plant based on grey system theory and GM(1,1) prognostic models.

Wang and Nie (2008) forecasted mechanical fatigue life using the grey system model, which made prediction error greatly reduced. Bo *et al.* (2012) applied BP neural network method and grey system model to predict Tianjin Qinhuangdao passenger dedicated line Luqiao transition section roadbed settlement. Wang and Yihua (2010) adopted the grey neural network method and set up a nonlinear prediction model of China civil aviation operation risk. Li *et al.* (2011) and Yang *et al.* (2008) forecasted spacecraft fault using the grey system model and obtained the high accuracy. Zhang *et al.* (2006) applied GM(1,1) model to study variation rule of the robot emotional state and achieved emotional robot interaction system. Li *et al.* (2010) measured fatigue crack propagation rate using the grey prediction model. Lin *et al.* (2005) established the grey prediction model about the slope rock mass deformation according to the test data of Three Gorges site slope. Then he drew the fitting and prediction curves of slope deformation, which provided reliable guarantee and theoretical basis for its prediction.

In big data area, the grey system prediction method based on small data mining as a new force suddenly rises, which becomes an effective tool for valuable information extracted from a mass of data. It is a very meaningful job to build more normal model testing standards based on the grey system prediction model testing method and statistical testing theory. The investigation on the potential of grey prediction models in Big Data is certainly a future direction in this field.

5. The grey incidence analysis models

The basic thought of grey incidence analysis model is used to judge the association between different sequences whether closely or not according to geometric shapes of the sequence curve. The early grey incidence analysis models measure similarity based on proximity. Such as the Deng's (1982) grey incidence model which based on the point incidence coefficient and Liu *et al.*'s (2013a, 2014c) grey incidence model based on the whole or global perspective.

In 2010, Liu and Xie built the new grey incidence analysis model based on the perspective of similarity and proximity, respectively.

Definition 2. Assume that two sequences X_i and X_j are of the same length, then:

$$\varepsilon_{ij} = \frac{1}{1+|s_i - s_j|} \tag{4}$$

is called the similarity degree of grey incidence of X_i and X_j, or the similarity degree of incidence for short. Where $s_i - s_j = \int_1^n (X_i^0 - X_j^0)dt$.

Definition 3. Assume that two sequences X_i and X_j are of the same length, then:

$$\rho_{ij} = \frac{1}{1+|S_i - S_j|} \tag{5}$$

is called the closeness degree of grey incidence of X_i and X_j, or the closeness degree of incidence for short. Where $S_i - S_j = \int_1^n (X_i - X_j)dt$.

Zhang and Liu (2009) proposed a two-dimensional grey incidence degree model based on absolute incidence degree and double integral. With the new model, the research object was promoted to the relationship between the surface analysis from the curve analysis.

Definition 4. Assume that both matrix $X_p = (a_{i,j})_{M\times N}$ and $X_q = (b_{i,j})_{M\times N}$ are with same shape, then:

$$\varepsilon_{pq} = \frac{1+|s_p|+|s_q|}{1+|s_p|+|s_q|+|S_p - S_q|} \tag{6}$$

is called the three-dimensional absolute degree of grey incidence. Where $s_p = \iint X_p^0 dxdy$, $s_q = \iint X_q^0 dxdy$, $s_p - s_q = \iint (X_p^0 - X_q^0) - dxdy$.

Wei and Zeng (2015) simplified the relational axioms which introduced by Professor J.L. Deng and gave an axiomatic definition of special incidence degrees. In 2005, Olson *et al.* proposed the grey incidence analysis method to solve the interval multiple attribute decision problems. In 2006, they simulated analyzed different kinds of fuzzy multiple attribute decision-making model using the grey incidence analysis (Olson and Wu, 2006). In 2010, Wu *et al.* proposed the DEA model based on the grey incidence fuzzy set to solve the location problem. Amanna *et al.* (2011) used GM model and GIA model comprehensively to study cognitive inference engine and automatic adjustment algorithm in wireless communication. Liu (2013) came up with a kind of generalized grey interval number incidence model and clarified the calculating process and feasibility through the examples. Huang (2006) put forward the new grey incidence geological evaluation model and the principle of maximum entropy to evaluate Apricot River oil field in Shaanxi Gansu Ningxia basin.

Scarlat and Delcea (2011) researched the problem of enterprise bankruptcy using the grey system theory methods and models and received series of results (Delcea *et al.*, 2012, 2013). Ejnioui *et al.* (2013) decided the priority of software requirements using the grey incidence analysis model and evaluated software projects using the grey possibility degree. Ossowski and Korzybski (2013) conducted fault diagnosis within analogous circuit using the grey system model Comparing the artificial neural network, classification and regression tree, K-nearest neighbor method, discriminant analysis method, the naive Bayes classifier, the quasi optimal algorithm and support vector

machine method with the simulation results from grey incidence classification algorithm, Twala (2014) found that grey incidence classification algorithm is the most suitable method for modeling and analysis of road traffic accident data in South Gauteng province. Zhang *et al.* (2004b) developed a differential equations of the mean area and the mean thickness of waterfilm and the adhesion force of the interface based on grey incidence analysis model. Hu *et al.* (2002) analyzed building thermal process with grey incidence analysis model.

Wei *et al.* (2013) built up the Chinese medicine chromatographic fingerprint pattern recognition model and analyzed the high-performance liquid of 56 batches of different varieties of pummelo pee medicinal material sample chromatography, which indicated that recognition rate of Mao Juhong of the different cultivars kinds closed to the herbal medicine chemical components on the type and content was over 92.85 percent.

Liu *et al.* (2007) researched the formation reaction of high-temperature sulfur retention phase calcium sulphoaluminate using the grey incidence analysis and prediction model. Xia *et al.* (2005) studied the relationship between the rolling bearings machining quality and vibration using the grey incidence analysis and found that the structure size error parameters had a larger effect on vibration of bearing. Zhang *et al.* (2012) analyzed the double tooth difference of swing movable teeth transmission failure using the grey incidence analysis model, which provided a scientific basis to improve the reliability of the double tooth difference of swing movable teeth transmission system. Xie *et al.* (2007) obtained the optimum parameters of each factor square box conservatism according to the outcome of variance analysis of the degree of grey incidence of the target sequence and every factors.

Shi *et al.* (2008) researched the main influence factors of U type steel encased concrete composite beam ductility using the grey incidence analysis model and built up the calculation formula of U-type steel encased concrete composite beam displacement ductility coefficient. Tan *et al.* (2011) combined the grey incidence analysis model with GM(1,1) model and put forward the effective method to predict the force state of cable-stayed bridge in cold area.

Xie *et al.* (2004) solved the aircraft top-level design scheme selection decision problems using the grey incidence analysis model. Cheng *et al.* (2014) studied the aircraft customized solutions using the grey incidence analysis model. Xiao and Zhang (2009) researched drone crashed fault using grey incidence analysis and fault tree method comprehensively, which provided a theoretical basis to diagnose the cause of drone crashed fault, to reduce the fault and improving the reliability of the system.

Chen *et al.* (2011) used the grey incidence analysis model and studied two kinds of the sky optical measurement method based on ASD spectroradiometer, standard grey board inversion measurement method and direct measurement method, which clearfied different methods applicable scenarios. Wang *et al.* (2011) studied cosmic ray μ sub imaging applying the grey incidence cluster analysis method and improved efficiency of material sorting.

Liang *et al.* (2014) set up multiple index grey incidence degree optimization model, then scheduled exploration and development ability of the study area complex geological parameters characteristics, which index value are interval grey number. Ronghuan *et al.* (2005) studied well logging, drilling and coring, oil testing and related geological data using the grey system theory. Then he divided lithology, physical property, oil bearing on the statistical analysis characteristic value and its accuracy, resolution through matching, fitting and extracting parameters, which provided the geological basis for oil field exploration and development. Wang *et al.* (2013) predicted

Yaojialing Zn and Au polymetallic deposit scientifically by the grey incidence analysis method. Rajesh and Ravi (2015) solved the problem of supplier selection in resilient supply chains using a grey relational analysis (GRA) approach.

Lin *et al.* (2009) solved the main influential factors of explosively formed projectile (EFP) velocity using the grey incidence analysis method, which gained results that has important reference value on the EFP cover design of explosive type and explosive charges structure design. Zhao *et al.* (2007) established the assessment model on ship antimissile incoming missile threat, which provided the decision basis on timely judgment of shipborne system target threat assessment.

The grey incidence analysis model has been used a large number and successful. There are still some problems remained to be solved. Such as the model test rule and specific quantitative criteria, etc. It is also a valuable research direction to expand the models based on the definite integral, used for sequence data analysis and based on double integral, used for matrix data analysis to the models based on the multiple integral, used for solving the problem of matrix sequence data and high-dimensional data (Liu *et al.*, 2014c).

6. The grey cluster evaluation models

In 2011, Liu and Xie improved the triangular membership function model proposed in 1993 (Liu and Zhu, 1993) and constructed the grey evaluation method based on center-point triangular whitenization weight functions. The new method reduced the base of the triangular whitenization weight function of class k, which became the straight line joining the two center points of class $k-1$ and class $k+1$, replaced the straight line joining the left end-point of class $k-1$ and the right end-point of class $k+1$. The multiple cross-phenomenon existing in the original triangular membership function clustering model is avoided effectively and the clustering vector satisfied the requirement of normalization.

In 2015a, Liu *et al.* set the weight function of grey class 1 to the whitenization weight function of lower measure and set the weight function of grey class s to the whitenization weight function of upper measure, then put forward the grey clustering evaluation model based on center-point and end-point mixed triangular whitenization weight functions (Figures 1 and 2). And the puzzle to extend the bound of value of each

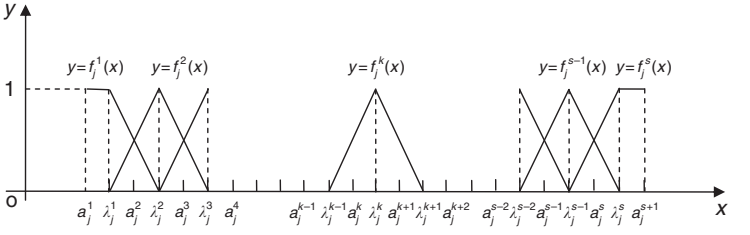

Figure 1.
End-point mixed whitenization weight function

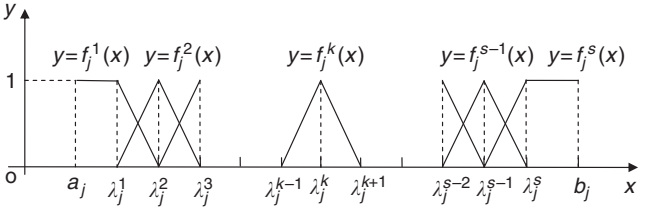

Figure 2.
Center-point mixed whitenization weight function

clustering index has been avoided. The grey cluster evaluation model based on mixed end-point triangular whitenization weight function is suitable for the situation that all the grey boundaries are clear, but the points belonging to each grey class most likely are unknown. The grey cluster evaluation model based on mixed center-point triangular whitenization weight function is suitable for the situation that the points belonging to each grey class most likely are clear, but the grey boundaries are unknown. The mixed triangular whitenization weight functions are more suitable to be used to solving the problem of clustering evaluation with poor information.

Zhang (2002) studied the relation between a grey clustering analysis result and the entropy of the weight sequence, and proposed a measure method of the grey characteristics of a grey clustering analysis result. Zhou *et al.* (2013) constructed the integral mean value function of interval grey number set, and then gave the grey interval number variable and certain weight grey clustering assessment models. Liu *et al.* (2013b) combined the grey cluster model based on center-point triangular whitenization weight functions with the variable precision dominance based on rough set approach and built up a new mixed method.

Peng *et al.* (2005) set up a new method of the cover layer quantitative evaluation based on grey clustering analysis, then they applied these method to evaluate three main exploration area, four sets of mudstone in southeast basin in Hainan Province, totally 12 kinds of cap rock objects and conclusion met with the exploration results. Han *et al.* (2014) comprehensively evaluated the air defense and antimissile missile warhead development scheme using the grey cluster model. According to the characteristics and operational task of the ground wave radar which beyond visual range, Yao and Hu (2008) evaluated the combat effectiveness using the grey cluster model. Fang *et al.* (2012) predicted the coal and gas outburst risk using the multidimensional grey evaluation model, which provided a basis for safety in production in coal mine. Zhang *et al.* (2010) evaluated the safety of the carrier borne machine system applying the grey clustering evaluation model, which played a positive role to discover the hidden danger of system security in advance, prevent and reduce the accident.

Liu *et al.* (2015b) established an evaluation index system for vendor performance at development stage and mass production stage, where the index system with weights are determined based on grey system theory and the expert investigation method. Then the significant vendors who undertake the development task of C919 program of COMAC have been evaluated comprehensively by grey cluster evaluation model based on mixed end-point triangular whitenization functions. Li *et al.* (2015a) improved grey clustering measurement and applied it to evaluate the safety performance at organizational or project levels. They compared grey methodology with analytic hierarchy process (AHP) and analytic network process and conducted to comprehend extent of out-performance.

The late former academician Sun (2005) with Chongqing University and his research group applied the grey cluster model to monitor online and assess the health state of transformer, which achieved a series of important results (Yuan *et al.*, 2005).

The grey cluster models based on mixed triangular whitenization weight functions are applicable to evaluation, classification of the poor information object, which have broad application prospects. As the next step development, it is meaningful to construct the model test criteria and quantitative criterion.

7. The grey decision models

In 2010c, Liu *et al.* put forward the multi-attribute intelligent grey target decision model (Deng, 2002) based on the thought of grey target model due to Professor Julong Deng (1990).

As the basis of the new model, a grey target is defined as a satisfying region, which a decision maker wants to reach, with an inside ideal point across multiple objectives first. To facilitate the uniform distance measure of a decision strategy to the pre-defined grey target, four kinds of measure procedures are designed including the effect measures for benefit-type objectives and cost type objectives, the lower effect measure for moderate-type, and the upper effect measure for moderate type according to three types of decision objective including benefit objective, cost objective, and non-monotonic objective with a most preferred middle value. Then, a matrix of synthetic effect measures can be easily obtained based on the uniform distance measure of a decision strategy to the grey target over different objectives. Based upon the obtained matrix information, different decision strategies can be easily evaluated comprehensively. The proposed method has a clear physical meaning as missing target, hitting target as well as hitting performance:

Definition 5. Assume that k is a benefit-type objective.

that is, for objective k the larger the effect sample value is, the better the decision grey target of the objective k is $u_{ij}^{(k)} \in [u_{i_0j_0}^{(k)}, \max_i \max_j \{u_{ij}^{(k)}\}]$, namely $u_{i_0j_0}^{(k)}$ is the effect critical value of the objective k. So:

$$r_{ij}^{(k)} = \frac{u_{ij}^{(k)} - u_{i_0j_0}^{(k)}}{\max_i \max_j \left\{u_{ij}^{(k)}\right\} u_{i_0j_0}^{(k)}} \tag{7}$$

is referred to as the effect measure of a benefit-type objective.

Assume that k is a cost-type objective, that is, for objective k the smaller the objective effect sample values is, the better the decision grey target of the objective k is $u_{ij}^{(k)} \in [\max_i \max_j \{u_{ij}^{(k)}\}, u_{i_0j_0}^{(k)}]$, namely $u_{i_0j_0}^{(k)}$ is the effect critical value of the objective k. So:

$$r_{ij}^{(k)} = \frac{u_{ij}^{(k)} - u_{i_0j_0}^{(k)}}{\max_i \max_j \left\{u_{ij}^{(k)}\right\} - u_{i_0j_0}^{(k)}} \tag{8}$$

is referred to as the effect measure of cost-type objective.

Assume that k is a moderate-type objective, that is, for objective k the closer to a moderate value A the effect sample values is, the better the decision grey target of the objective k is $u_{ij}^{(k)} \in [A - u_{i_0j_0}^{(k)}, A + u_{i_0j_0}^{(k)}]$, namely, $A - u_{i_0j_0}^{(k)}$, $A + u_{i_0j_0}^{(k)}$ are the upper limit and the lower limit effect critical value of the objective k, respectively. So:

(1) when $u_{ij}^{(k)} \in [A - u_{i_0j_0}^{(k)}, A]$,

$$r_{ij}^{(k)} = \frac{u_{ij}^{(k)} - A + u_{i_0j_0}^{(k)}}{u_{i_0j_0}^{(k)}} \tag{9}$$

is referred to as the lower effect measure of moderate-value type objective.

(2) When $u_{ij}^{(k)} \in [A, A + u_{i_0j_0}^{(k)}]$,

$$r_{ij}^{(k)} = \frac{A + u_{i_0j_0}^{(k)} - u_{ij}^{(k)}}{u_{i_0j_0}^{(k)}} \tag{10}$$

is referred to as the upper effect measure of moderate-value type objective.

The measurement given in Definition 5 has some good properties, such as satisfying the requirement of normalization, dimensionless, and more ideal the effect is, the greater of measure value is. And the physical meaning is very clear. In 2009, the writer first reported the initial idea on intelligent grey target and gained confirming from professor Deng Julong, the founder of the grey system theory and other participating experts.

In 2014b, Sifeng Liu *et al.* constructed a new decision model of two-stage grey comprehensive measure. A kind of decision paradox that the comparison between the maximum components δ_i^k and δ_j^k of decision coefficient vector δ_i and δ_j may be conflicts with the comparison between δ_i and δ_j has been solved.

In 2015, Liu and Yang defined a general synthetic weight vectors for decision making and the decision coefficient vectors with grey synthetic measures. Then a novel two-stage decision model with the synthetic weight vectors for decision making and grey synthetic measures is put forward, and several functional synthetic weight vectors for decision making are given. This method can effectively solve the decision paradox and produce consistent results.

Dang Luo studied grey decision models of different types and obtained a series of achievements (Luo and Wang (2012a, b). Guo *et al.* (2014a) researched the grey double layers and multi-objective linear programming and solving problems. Yan *et al.* (2014a). proposed a new method to determine the weights of decision makers and attributes for group decision making with interval grey numbers.

Cui *et al.* (2012a) gave the weighting formula evaluation value on each stage detection based on the new information priority principle, which provided a new thinking to solve multiple stages grey decision problems. Golmohammadi and Mellat-Parast (2012) gave the grey decision model about supplier selection. Liang *et al.* (2012) came up with a case reasoning method based on the grey system theory and logistic regression model and applied it to safety assessment for thermal power plant. Yi *et al.* (2002) conducted risk evaluation of tubing in natural gas well with grey target model.

Wu and Chang (2003, 2004) studied optimization problems of the company production plan under variable environmental costs using the grey compromise programming model. They also proposed a method and procedure for optimizing textile dyeing manufacturing process via GA-based grey nonlinear integer programming and a grey input-output analysis model and applied to environmental cost allocation analysis. Cui *et al.* (2012b) researched the selection problem of ground and air missile weapon system applying the multi-objective grey decision model. Li *et al.* (2007) constructed the grey planning model of missile nuclear optimal allocation, which provided a theoretical basis for the order, storage, position allocation and operational application of the missile nuclear weapons. Yu *et al.* (2009) applied the grey cluster decision method to aircraft large parts automatic docking assembly system, which enhanced the stability of the system, reduced the risk of equipment failure and reduced repair costs as well.

The grey programming model has made little progress in recent years. Interested readers can focus on this field.

8. The combined GM

Fang *et al.* (2006) opened a new direction of combined gray model on grey game model, they carried out effective research in economic decision application around the grey game model (Fang *et al.*, 2010), they proposed the concept of functional interval grey numbers and designed its representation. Then, comparison and arithmetic rules of functional interval grey numbers are given. The functional game model based on grey

information variable is built (Fang *et al.*, 2014). Zhu and Shi (2013) researched the game behaviors in the supervising and the regulatory process of private equity investment, and put forward the corresponding supervision strategies. Chen *et al.* (2012) constructed the grey game matrix about the public and the insurance company, then they analyzed selection strategy of the public and the insurance company and influence of government insurance subsidy ratio and compensation subsidy ratio to the public and the insurance company decision. Wang (2009) established the grey game model on the farmers and a variety of companies, obtained both sides maximize Nash equilibrium strategy of performance of the perishable agricultural products supply chain.

In 2008, Li and Liu studied the grey matrix and grey input-output model in depth. Based on these, they put forward an analytical model for enterprise grey input-output in 2012 (Li *et al.*, 2012b) the grey physical input-output analysis model. Jian *et al.* (2011) developed a series of the grey rough set hybrid models. Wang and Liu (2009) conducted research on the grey DEA model, which has made significance research results. Based on the whitenization function of grey numbers and the capital assets pricing model, Liu *et al.* (2004a) proposed a new venturous capital pricing method and the synthetic utility index method.

Lin *et al.* (2001) proposed a Markov-Fourier grey prediction model. They compared the performance of the new model with different prediction schemes, such as back-propagation neural networks and fuzzy models. The simulation results show that the new approach can predict the future more accurately and also use less computation time than other methods. Kose and Forrest (2015) combined the grey system theory with the classic N-person game theory and sets up the N-person grey game with grey payoff functions. Chen and Chang (2000) proposed a new approach of grey fuzzy dynamic modeling for the prediction of solid waste generation in the urban area based on a set of limited samples. Luo *et al.* (2001) proposed a hierarchical grey fuzzy motion decision-making algorithm, which is capable of integrating multiple sequential data for decision making and for the design of the control kernel of the target tracking system. Bahrami *et al.* (2014) proposed a new model based on the combination of the wavelet transform and GM for short-term electric load forecasting. Samet and Mojallal proposed a rolling GM and a Grey-Markov method to predict the actual reactive power of Mobarakeh Steel Company in Iran. Verma *et al.* (2014) used GRA coupled with fuzzy logic to model the stator winding fault and to predict the optimal setting for running the induction motor within its parameters range. The results indicate that the proposed novel approach is very effective in predicting the stator winding fault (Aydemir *et al.*, 2015). Oztaysi (2014) proposed a AHP integrated grey-TOPSIS method, and applied in a Turkish foreign trade company. Zhang and Chen (2002) proposed a new genetic algorithm method based on random simulation to solve the general grey nonlinear programming problem.

Liu *et al.* (2010a) optimized the railway digital mobile communication system scheme under the condition of limit frequency planning based on grey cluster and rough set models. Guo *et al.* (2013) combined grey prediction and Markov chain to improve the prediction accuracy of pollutants. Yuan *et al.* (2014) forecasted fire accidents based on portfolio optimization model of grey neural network. Meng *et al.* (2012) predicted gun tube life using the grey linear regression combination model and enhanced the prediction accuracy. Mi *et al.* (2014) constructed an optimal portfolio model based on the diagnosis results of three kinds of diagnosis methods such as fuzzy fault diagnosis method, genetic algorithm and grey system theory and made the fault diagnosis on 25 Hz phase sensitive track circuit. Xu *et al.* (2010) used grey

econometrics model to predict the traffic volume of highway. Jiang (2012) diagnose the fault of wind turbine drive chain based on the grey rough set theory. Yin *et al.* (2012) evaluated groundwater quality in Taonan City based on the grey cluster method and matter element extension method. Tang *et al.* (2012) studied the main influential factors of gas well productivity of Permian Shan 2 gas reservoir in Zizhou Gas Field using grey system method, the method of principal component analysis and R cluster analysis method comprehensively.

The Chinese academician Wu *et al.* (2012) with The National Key Laboratory on Hydrology Water Resources and Hydraulic Engineering at Hehai University applied grey system theory and a variety of scientific methods to research the slope stability and dam safety service status, which made a series of vital achievements (Zheng *et al.*, 2005).

Many combined models are based on the actual problems and the relevant theoretical foundation is not solid. Such as the basic concepts about grey fuzzy, fuzzy grey, grey rough, rough grey and grey random have not formed an unified definition. It still needs time to establish consensus on several typical combined models.

9. Grey control

Su and Liu (2008, 2009) used several methods such as the Lyapuonv function, Lyapunov-Krasovskii function and model transformation and combined formula, matrix inequality, Holder inequality, Schur complement and other mathematical tools and decomposition technique of continuous matrix cover of grey matrix, and studied the robust stability problem of grey stochastic time-delay systems in depth, especially the distribution type, neutral type and neutral-distribution type exponential robust stability problem of grey stochastic time-delay systems. They investigated in details, gave the effective criterion, and obtained several useful achievements.

Li *et al.* (2015b) proposed an improved GM to acquire high-control system performance. Liem *et al.* (2015) set up a new method for estimating the load torque of a DC motor shaft by using a novel modeling method based on an adaptive control technique, named as online tuning grey fuzzy PID (OTGFPID). Huang and Huang (2000) proposed a grey prediction model combined with a proportional plus derivative controller to balance an inverted pendulum. Luo and Chen (2000) developed an autonomous mobile target tracking system based on grey fuzzy control algorithm. Chou *et al.* (2000) designed an optimal grey fuzzy controller of a constant turning force system by Taguchi-genetic method. Li *et al.* (2001) used the "grey system" analysis methodology for automated boiler water chemistry control in electric power plants. Lee and Liao (2003) proposed a self-tuning fuzzy control system which adopted the grey predictor to compensate the time-delayed *R*-ab caused by the low pass filter data processing.

Gao *et al.* (2012) built up high-speed train speed controller model based on the model of grey genetic algorithm according to the fitness grey number of train operation target design. Lu and Wang (2013) studied the problem on modeling and simulation of the automatic train operation speed controller. Tian and Yi (2007) set up the grey forecast model of billet heating furnace temperature and put forward billet temperature control method. In the view of the flue temperature control problems with strong nonlinearity, large time delay, multi disturbance characteristics, Wang *et al.* (2010) brought forth an improved fuzzy expert control method based on the combination of grey prediction model. Zhang *et al.* (2004a) designed self-adjustable grey prediction controller combining the traditional feedback control methods and grey prediction controlling. The simulation results showed that the new controller with more excellent dynamic performance and robustness.

In the light of the characteristics such as stochastic, nonlinear, time-varying and difficult to establish precise mathematical model of deep sea walking mechanism at the bottom of the complex operation environment of deep sea, Qiao *et al.* (2009) put forward the grey prediction and fuzzy PID control method, and realized the effective control of the deep sea walking mechanism.

Chinese academician Yexiang Liu *et al.* (2004b) and his research group with the National Key Laboratory of Powder Metallurgy at Central South University made a number of achievements using grey system methods and models on control problems of aluminum electrolysis process (Lai *et al.*, 2004).

It has been 30 years since the success of the first grey controller in 1985. Industrial process control is still led by the traditional PID control. Integrating a variety of control methods and models, the condition to establish a more effective control system seems to be ready. However, it takes time to promote the new control method. In addition, it is also a valuable research field that grey control idea and models are applied to the control and regulation of the important social and economic parameters.

10. The new framework of grey system theory

The new framework of grey system theory is shown in Table I.

11. Concluding remarks

During 2000-2015, it gradually moves toward a mature period for grey system theory to grow from adolescence to young excessive. In the meantime, the grey system theory has formed a system structure that is generally accepted, and become a course at many universities all around the world. The course of grey system theory of NUAA was selected as one of the best national curriculums and was shored as a national excellent resource, and the book *Grey System Theory and Its Application* (fourth and sixth edition) have been selected into the "Eleventh Five Years" and "Twelfth Five Years" national planning, respectively.

With the eight search terms including grey system, grey theory, GM, GM(1, 1), the grey incidence analysis, grey clustering, grey prediction and grey decision making, a search in the China Knowledge Net (CNKI), the full text of PhD theses and master degree theses, returned 10,180 and 48,185 results, respectively; and a search against the topics of those PhD theses and master degree theses returned 2,873 and 13,463 results, respectively. The full text of journals containing the search term altogether accounted for 69,276, and a search for journal papers based on the search terms obtained 39,544 papers. International famous publishers Springer-Verlag and Taylor & Francis group launched a number of grey system English works. *A Series on Grey System* publishing plan was approved officially by Science Press, and the first one of the 22 volumes has begun to come out.

In accordance with the laws of scientific development, a new subject needs to go through several generations of continuous work, and several decades or even a hundred years before it becomes mature. The theory of grey system has just been developed over thirty years, it has successful applications in many countries of the world, and has achieved a large number of achievements. However, it still needs a long way to go for further developments and improvements. Our colleagues who are interested in grey system research should welcome and treat various criticisms and suggestions, and continuously explore, and constantly excavate new growth points, so as to continuously improve grey system theory. The related problems mentioned at the end of the each section of this paper need active involvement of talented scholars.

	Main contents	Detailed thinking, methods and models of Grey system theory
Basic thinking and models	Operations of grey number and grey algebra system	Grey number, the algorithms of interval grey number, the concept of the kernel of grey number and general grey number, and the operation axiom of grey number and algebra system based on grey "kernel" and degree of greyness
	Sequence operator	Average generation operator, accumulating generation operator, inverse accumulating generation operator, shock disturbed system, axiom system of buffer operator, series of accumulating and inverse accumulating generators
	Grey incidence model	Series grey incidence analysis model including Deng's grey incidence model, absolute degree of incidence, relative degree of incidence, synthetic degree of incidence, nearness degree of incidence, similitude degree of incidence, three-dimensional grey incidence degree, etc.
	Grey cluster evaluation model	Variable weight grey cluster model, fixed weight grey cluster model, grey cluster evaluation model based on mixed end-point and center-point triangular whitenization weight function
	Family model of GM and grey system forecasting	Even GM(1,1) model, even difference GM(1,1) model, original difference GM(1,1) model, discrete GM(1,1) model, fractional-order grey model, memoryless grey model, Grey Verhulst model, multivariable discrete grey model, discrete grey model with approximate non-homogenous exponential law, sequence grey forecasting, interval forecasting, catastrophe forecasting, grey wave forecasting, and system forecasting
	Grey decision model	Grey target decision; four kinds of uniform effect measure function which is able to characterize factors for positive point and negative, multi-attribute intelligent grey target decision model, two stages grey synthetic measure decision model
Advanced thinking and models	Grey equation and grey matrix	Grey algebraic equation, grey differential equation, grey matrix, and matrix equation
	Combined grey models	Grey Econometrics Combined Mode, grey Cobb-Douglas Model, grey DEA model, grey-Markov Mode, grey rough model, etc.
	Grey game models	Grey matrix game model based on pure strategy, Grey matrix game model based on mixed strategy, duopoly strategy output-making model based on bounded knowledge and bounded rationality, solving the paradox of centipede game: a new model of grey structured algorithm of forwards induction
	Grey input and output	P-F theorems of grey non-negative matrices, regional input-output model, enterprise grey input-output, grey input-output optimization model
	Grey programming	Linear programming models with grey parameters, grey linear programming of prediction type, drift grey linear programming, grey 0-1 programming, grey multiple objective programming, grey nonlinear programming
	Grey control model	Grey control model, controllability and observability of grey systems, robust stability of grey systems , grey linear time-delay systems, grey stochastic linear time-delay systems

Table I.
The new framework of grey system theory

The grey system modeling software version 7.0 written by Professor Bo Zeng contains commonly used grey system models. Readers in need can freely download it from the website of the Institute for Grey System Studies at Nanjing University of Aeronautics and Zeng *et al.* (2011), http://igss.nuaa.edu.cn/

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