Multi-objective multiple allocation hub location problem with multiple capacity levels considering sustainable development paradigm

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Abstract: Nowadays, the increase of flow traffic between different nodes in manufacturing and service sectors makes many researchers to work on designing hub networks for transporting the flows in order to optimise transportation and decrease transportation costs. Hence, hub location problems are considered as strategic problems and focusing on all aspects of sustainable development is necessary in such decisions. Therefore, considering environmental aspect of sustainable development with economic objective simultaneously, is one of the most important concerns of decision makers and researchers. Due to such changes observed in attitude of researchers and decision makers, a flow transportation network with economic justification and less environmental impacts could be potentially designed. This research aims to develop a multi-objective mathematical model with multiple allocation and multiple capacity levels to minimise both costs and environmental impacts simultaneously. Capacity levels are used to increase the flexibility of the model for selecting hub nodes. Since this problem is a kind of NP-hard problems, a multi-objective deferential evolution (MODE) algorithm is used to solve generated instances. The performance of the proposed model is then evaluated by analysing the sensitivity of the model in response to variations made in the model parameters.

Keywords: multiple capacity levels; environmental impacts; multi-objective differential evolution algorithm; MODE; multi-objective hub location.

Reference to this paper should be made as follows: Tavakoli, L., Manavizadeh, N. and Rabbani, M. (2014) 'Multi-objective multiple allocation hub location problem with multiple capacity levels considering sustainable development paradigm', *Int. J. Applied Decision Sciences*, Vol. 7, No. 3, pp.295–326.

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This paper is a revised and expanded version of a paper entitled 'Multiple allocation hub location problem with multiple capacity levels and limited flow in linkages' presented at 3rd International Logistics & Supply Chain Conference, Iran, Tehran, 16 December 2012.

1 Introduction

During the last two decades, a wide range of applications of hub network design problems have received increasing attention in the fields of transportation (Kara and Tansel, 2001), telecommunications (Monma and Sheng, 1986), computer networks (Gavish and Suh, 1992), postal delivery (Ernst and Krishnamoorthy, 1999), less-than-truck loading and supply chain management. Hubs are special facilities used for switching, transshipping and sorting points in distribution systems. Hub facilities are used to concentrate flows in order to take advantages of economies of scale. Three main issues are involved in designing of hub location networks, which are: locating hub facilities, allocating origin and destination nodes to hubs and routing flows through the network. In hub networks, there are n nodes and the flow between each pair of nodes is given. The hub location problem tries to identify the set of origins, destinations and potential hub locations. Since hub location problems focus on designing a network, they are a kind of strategic problems. Therefore, they should try to make decisions with considering sustainable development paradigm. In other words, considering all aspects of sustainable development in such problems is unavoidable because of their important role in performance of end networks. Usage of sustainable development word was generalised after the Rio conference in 1992. In general, it could be said that sustainable development is a process of changing in usage of resources, investment direction and technology development orientation. It is also a basic change compatible with present and future requirements. Sustainable development has some important aspects, of which that economic and environmental aspects are the most important in making strategic decisions for network design and location problems. Despite increasing attention to both economic and environmental aspects of sustainable development in different areas such as supply chain management, literature of the hub location problems is still shy.

A significant part of sustainable development literature in supply chain management focuses on environmental aspect of it. In other words, the integration of two basic aspects of sustainable development (economy and environment) leads to the definition of green supply chain. Green supply chain is a combination of economic aspect and environmental one, which contain product design, material selection and supplying, production processes, delivering to the customer and end life management (Srivastava, 2007). Therefore, green supply chain management intends to minimise resource usage, waste production and adverse effects on environment. Same as hub location problems, the most important decision in supply chain management is supply chain network design. This decision plays an important role in overall economic and environmental performance of supply chain. The design of supply chain network includes making decision about location, number and capacity of facilities and the aggregate material flow between them, (Melo et al., 2009). Recently, a comprehensive review on green supply chain management has been conducted by Srivastava (2007). Based on their research, green design for products (Kuo et al., 2001) and green operations can be identified. The current research is more related to green operations which are mainly composed of green manufacturing and remanufacturing (Sheu et al., 2005), reverse logistics and network design (Fleischmann et al., 2001; Zhu et al., 2008) and waste management (Bloemhof-Ruwaard et al., 1996; Cheng et al., 2003). Also, (Pishvaee and Razmi, 2012; Chaabane et al., 2012; Bojarski et al., 2009) studied the design of a supply chain with considering two environmental and economic objectives. In the area of hub nodes and their applications, Scholz and Cossel (2011) tried to assess the importance of hub airports for cargo carriers to manage them sustainably.

Hence, this research aims to design a hub network considering two environmental and economic aspects of sustainable development in order to minimise necessary investment for designing the network and minimise environmental impacts due to hub installation, flow shipment between linkages and their processing in hubs.

Hub location problems can be classified according to several features such as single allocation or multiple allocation, capacitated or uncapacitated hubs and one capacity level or multiple capacity levels (Correia et al., 2010) for hub nodes. In this work, we focus on a capacitated multiple allocation hub location problems with multiple capacity levels for the first time. Finally, to cover the gaps seen in the literature, a non-linear multi-objective multiple allocation hub location problem with multiple capacity levels is proposed in this research in which the capacity is related to the hubs. The objective functions of the new proposed model intends to design a hub network with considering two important aspects of sustainable development in order to minimise economic costs and environmental impacts. Since the proposed model is NP-hard, an MODE algorithm is used to solve the generated instances on the basis of the literature.

This paper is organised as follows: Section 2 reviews the problem literature, in Section 3 a definition of the proposed problem is presented; Section 4 describes the mathematical formulation; the model validation is presented in Section 5; Section 6 describes the solution method of the problem; computational results and sensitivity analysis are presented in Section 7, and in the last section final conclusion are presented.

2 Literature review

Hub facility location is a very popular problem in last decades since it has a variety of practical applications such as telecommunication, transportation and delivery systems. fuzzy p-hub centre problem (Yang et al., 2013), uncapacitated p-hub maximal covering problem (Hwang and Lee, 2012), uncapacitated hub median problem (Filipović, 2011) and hub location inventory model for bicycle sharing system design (Lin et al., 2013) are some of the recent applications of hub location problems.

It is noteworthy to say that in these problems assigning the demand nodes to hubs is undertaken in two ways: single allocation hub location, (Campbell, 1994b; Correia et al., 2010; Camargo and Miranda, 2012) and multiple allocation hub location problems, (Campbell, 1992; Sasaki et al., 1999; García et al., 2012; Kratica, 2013; Sender, 2013). In single allocation, the total flow of each demand node is routed via one hub, but in multiple allocations, the flow of each demand node is routed via one or more hub nodes. Since location problems are, in turn, affected by allocation problems and allocation problems are affected by location problems, so it is better to consider these problems together. Sometimes there is limitation on the capacity of each facility for receiving and processing the flows from non-allocated nodes (Diabat et al., 2009). In such a condition, the hub location problem is a capacitated allocation hub location problem, (Campbell, 1994b; Ernst and Krishnamoorthy, 1999; Labbe et al., 2005; Boland et al., 2004). If there is not any constraint on the capacity of the facility, the problem is named uncapacitated allocation facility location problem, (Montoya-Torres et al., 2010) and for hub location problems it is named uncapacitated allocation hub location problem (Campbell, 1994b, 1994a; Boland et al., 2004; Abdinnour-Helm, 1998; Cunha and Silva, 2007; Topcuoglu et al., 2005; Hamacher et al., 2004; Klincewicz, 1996; Martin et al., 2006; Mayer and Wagner, 2002). In the real world, it is better to consider multiple capacity levels for each hub node and try to choose the best capacity levels for creating hub nodes based on their fixed costs. Therefore, only one capacity level for each hub should be selected. This assumption is only considered in single allocation hub location problem by Correia et al. (2010) for the first time.

As mentioned before, although there are different applications of hub location problems, considering environmental aspect of sustainable development in these strategic problems are extremely weak and most of the literature of this area focuses on green supply chain. For instance, an advanced two objective model has been presented by Hugo and Pisticopoulos (2005) to design supply chain compatible with environment of chemical materials. It was an extended mixed integer linear programming model implemented in a multi-product and multi-period network. The objective functions are the maximisation of economic profit and minimisation of environmental impacts. Falcone et al. (2008) presented a two objective linear programming model for designing a paper sustainable logistic network. Since the presented model does not consider location decisions, it is not a designing problem. In presented model by Harris et al. (2009), two environment and economic objective functions were considered separately. In that model, it was assumed that environmental impacts of transportation were higher than environmental impacts of installing facilities. Also, the presented model was solved using NSGA-II. The obtained results showed that they could help decision makers for conscious selection in a good way. Wang et al. (2011) used a strategic approach of designing a supply chain with considering environmental impacts. In that research different levels of environmental impacts and necessary investment for their implementation were considered in objective functions.

Briefly, to cover the considerable gap in the literature of hub location problems in the area of sustainable development, this paper proposes a new model for a capacitated multiple allocation hub location problem considering environmental and economic aspects of sustainable development. Also, for the first time multiple capacity levels for hubs are considered in multiple allocation problems. Problem details are defined in the next section.

3 Problem definition

In the past three decades, considerable attention has been paid to environmental topics, sustainable development and development management. This has led to establish rules in national and international levels to make a trade off between development and environment concerns. Environmental Impact Assessment (EIA) is a good example in this regard so that its rules were created about 40 years ago in the USA. In general, EIA is a systematic process that studies environmental impacts of development before its implementation. Unlike many environmental protection methods, EIA focuses on prevention. EIA is usually undertaken in nine steps including:

1 screening
2 scoping
3 impact analysis
4 corrective actions
5 reporting
6 reviewing
7 decision making
8 follow up
9 public involvement.

Based on this procedure an EIA report contains project recognition and its environment, project impact forecasting in environment and corrective action presentation for decreasing the impacts. For more information about EIA see Morgan (1999).

Since hub network design is a strategic decision making process about location of hubs and assignment of non-hub nodes to the hubs environmental concerns should be considered in this process. Application of an EIA report is a good way for making the best decision compatible with environment. Therefore, in this research each potential hub and linkage are assessed based on an EIA report containing environmental impacts, corrective actions and also necessary cost for implementing these actions. It should be noted that some of EIA reports model environmental impacts in midpoint impact category, endpoint impact category or both of them. For example, climate change is a midpoint impact affecting human health which is an endpoint impact. Therefore, to model the defined problem it is assumed for each potential hub there is a set of environmental impacts and their amounts related to processing each unit of the flows and another set related to installing hubs.

A set of corrective actions is used for decreasing both environmental impact sets related to hubs and necessary cost for implementing each element of corrective action set existed. Also, for each potential linkage there is a set of environmental impacts related to shipment of the flows in the linkages. A set of corrective actions is used for decreasing declared impacts and necessary cost for implementing each element of corrective action set exist, too. In addition, it is assumed that each corrective action set at least has one element so that one of them shows implementation non-corrective action with zero cost. For example, it is possible that installation of a hub near a river leads to destroy the river ecosystem and flow processing adds river pollution. Therefore, environmental impact set of installing hub and flow processing each have one element.

A corrective action set could be defined for both of them where each set contains two elements. The first element is non-corrective action with zero cost while the second one involves purchase of a land far from the river with a specific cost. Also, there could be other corrective action elements for decreasing these two environmental impacts. Each element of corrective action set could contain a mixture of some action plans with a defined investment.

The most important assumptions of the problem are as follows:

1. all nodes could be hubs and the location of nodes are definite
2. the capacity of linkages is limitless
3. there is no internal flow in the nodes
4. implementation of different capacity levels for each potential hub does not cause different environmental impacts for the flow processing
5. there are no limitation or standard levels on allowed environmental impacts of hubs and linkages
6. there is no predefined assumption about distances between nodes (Martin et al., 2006)
7. environmental impacts could be midpoint, endpoint impact or both of them depending to EIA report.

In the next section, a mathematical model for the defined problem is proposed.

4 Mathematical model

The parameters and variables used to formulate the defined problem are described below.

Parameters	
w_{ij}	Flow to be sent from node *i* to node *j*
c_{ijkm}	Cost of sending one unit of flow from node *i* to node *j* through hubs *k* and *m*
F_k^q	Fixed cost of installing hub *k* with capacity level *q*
Γ_k^q	Capacity amount of hub *k* with capacity level *q*
α	Collection cost coefficient
δ	Transshipment cost coefficient
χ	Distribution cost coefficient

Parameters	
$Q_k = \{1, \ldots, s_k\}$	Set of different capacity levels available for a potential hub to be installed at node k
$Aop_k = \{1, \ldots, t_k\}$	Set of environmental impact levels of installing a hub at node k
$Apro_k = \{1, \ldots, t_k\}$	Set of environmental impact levels of processing each unit of flow at node k
$B_{ij} = \{1, \ldots, u_{ij}\}$	Set of environmental impact levels of shipping one unit of flow through node i to node j
$R_k = \{1, \ldots, v_k\}$	Set of corrective action levels for reducing the environmental impacts at hub k
$S_{ij} = \{1, \ldots, y_{ij}\}$	Set of corrective action levels for reducing the environmental impacts at hub k
Gh_k^r	Implementation cost of corrective action level r in hub k
Gl_{ij}^s	Implementation cost of corrective action level s at linkages i and j
$epro_k^{apro,r}$	Environmental impact of processing one unit of flow at hub k when corrective action r is implemented
$eop_{kq}^{aop,r}$	Environmental impact of installing hub k with capacity level q when corrective action r is implemented
e_{ij}^{bs}	Environmental impact of shipping one unit of flow through node i to node j when corrective action r is implemented
Variables	
x_{ijkm}	Fraction of the flow shipped from node i to node j through hubs k and m by this order
z_k^q	1 if hub k with capacity level q is installed and 0 otherwise
h_{ik}	1 if node i is assigned to hub k and 0 otherwise
μ_{km}	1 if nodes k and m are hubs and 0 otherwise
zh_k^r	1 if corrective action r is selected for hub k and 0 otherwise
l_{km}^s	1 if corrective action s is selected for a link between two hubs k and m and 0 otherwise
o_{ik}^s	1 if corrective action s is selected for a linkage between a non-hub node i and a hub node k and 0 otherwise

Transportation cost between two nodes i and j via hub k and m is as follow:

$$c_{ijkm} = \delta\, c_{ik} + \alpha\, c_{km} + \chi\, c_{mj},\ 0 \le \alpha \le 1, \alpha < \delta, \alpha < \chi \tag{1}$$

Regarding the capacity, the relation for each node k is assumed. Clearly, a necessary condition for the feasibility of the problem is as follow:

$$\sum_k \Gamma_k^{s_k} \ge \sum_i \sum_j w_{ij} \tag{2}$$

Accordingly, a mixed integer non-linear programming model for the problem is the following:

$$\begin{aligned}Min\ & \sum_i \sum_j \sum_k \sum_m w_{ij} c_{ijkm} x_{ijkm} + \sum_k \sum_{q \in Q} F_k^q z_k^q + \sum_k \sum_{r \in R} Gh_k^r zh_k^r + \\ & \sum_i \sum_k \sum_{s \in S} Gl_{ik}^s o_{ik}^s + \sum_k \sum_{m>k} \sum_{s \in S} Gl_{km}^s l_{km}^s\end{aligned} \tag{3}$$

The first objective is an economic objective that evaluates the overall cost which is divided into five parts. The first part shows the total cost of collection, transfer and distribution of total flow through installed hubs; the second part is the sum of installing costs of hubs with defined capacity levels, the next part shows the total cost of implementing corrective action r in hub k; the forth part shows the total cost of implementing corrective action s at all linkages between all non-hub nodes and hubs and the last part indicates the total cost of implementing corrective action s at all linkages between all hubs. In the last part $m > k$ is used to prevent the calculation of the related cost between hub linkages more than once.

$$\begin{aligned}Min\ & \sum_k \sum_{r \in R} zh_k^r \sum_{apro \in Apro} epro_k^{apro,r} \sum_i \sum_j w_{ij} \sum_m x_{ijkm} \\ & + \sum_k \sum_q \sum_{aop \in Aop} \sum_{r \in R} eop_{kq}^{aopmr} z_k^q + \sum_i \sum_k \sum_{s \in S} o_{ik}^s \sum_{b \in B} e_{ik}^{bs} \sum_j \sum_m w_{ij} x_{ijkm} \\ & + \sum_k \sum_m \sum_{s \in S} l_{km}^s \sum_{b \in B} e_{km}^{bs} \sum_i \sum_j w_{ij} x_{ijkm}\end{aligned} \tag{4}$$

The second objective is an environmental objective that evaluates the overall environmental impacts of hub network. This is divided into four parts. The first part is the sum of environmental impacts resulted from processing the flows at hubs when corrective action r is selected for hub k. The second part shows the sum of environmental impacts due to installing the hubs when corrective action r and capacity level q selected for hub k. The next part is the sum of environmental impacts resulted from shipping the flows in linkages between non-hub nodes and hubs when corrective action s is selected. The last part represents the sum of environmental impacts due to shipping the flows in linkages between hub nodes when corrective action s is selected.

$$\sum_k \sum_m x_{ijkm} = 1 \qquad \forall i, j \tag{5}$$

Constraint (5) ensures sending the total flow from node i to node j.

$$\sum_m x_{ijkm} \le h_{ik} \qquad \forall i, j, k \tag{6}$$

$$\sum_k x_{ijkm} \le h_{jm} \qquad \forall i, j, m \tag{7}$$

Constraint (6) shows if non-hub node i is not assigned to hub k, flow of node i could not be sent from hub k and also, constraint (6) ensures only if the node j is assigned to hub m, flow could be sent to this node through hub m.

$$\sum_m \sum_{k \ne i} x_{ijkm} + h_{ii} \le 1 \quad \forall i, j \tag{8}$$

$$\sum_{k}\sum_{m\neq j} x_{ijkm} + h_{jj} \leq 1 \quad \forall i, j \tag{9}$$

Constraint (8) indicates if origin node i is hub, the flow between origin node i and destination node j could be sent directly or through another middle hub. Constraint (9) shows if destination node j is hub, the flow between origin node i and destination node j could be sent directly or through another middle hub. In other words, since there is no predefined assumption about distances between nodes, these two constraints ensure that maximum number of hubs between two nodes is 2, [34].

$$h_{ik} \leq h_{kk} \qquad \forall i, k \tag{10}$$

Constraint (10) ensures assigning non-hub node i to hub k is possible when hub k is installed.

$$\sum_{q\in Q} z_k^q = h_{kk} \qquad \forall k \tag{11}$$

Inequality (11) is used to ensure only one capacity level is selected for each installed hub.

$$\sum_{i}\sum_{j} w_{ij} \sum_{m} x_{ijkm} \leq \sum_{q\in Q} \Gamma_k^q z_k^q \qquad \forall k \tag{12}$$

Constraint (12) limits capacity of hub k with level q for processing the entering non-processed flows.

$$\sum_{r\in R} zh_k^r = h_{kk} \qquad \forall k \tag{13}$$

$$\sum_{s\in S} o_{ik}^s = h_{ik} \qquad \forall i, k \tag{14}$$

$$\sum_{s\in S} l_{km}^s = \mu_{km} \qquad \forall k, m \tag{15}$$

These three constraints ensure only one corrective action could be selected when a hub or a linkage is installed. As mentioned before, this corrective action could be a set of some corrective actions or even no necessary corrective actions.

$$2\mu_{km} \leq h_{kk} + h_{mm} \qquad \forall k, m \tag{16}$$

$$\mu_{km} \geq h_{kk} + h_{mm} - 1 \qquad \forall k, m \tag{17}$$

Two constraints (16) and (17) could show the linkages between all hubs and also they are used to prevent from making a non-linear constraint.

$$x_{ijkm} \geq 0 \qquad \forall i, j, k, m \tag{18}$$

$$z_k^q \in \{o, 1\} \qquad \forall k, q \in Q \tag{19}$$

$$h_{ik} \in \{o, 1\} \qquad \forall i, k \tag{20}$$

$$\mu_{km} \in \{0, 1\} \qquad \forall k, m \tag{21}$$

$$zh_k^r \in \{o,1\} \qquad \forall k, r \in R \tag{22}$$

$$o_{ik}^s \in \{o,1\} \qquad \forall i, k, s \in S \tag{23}$$

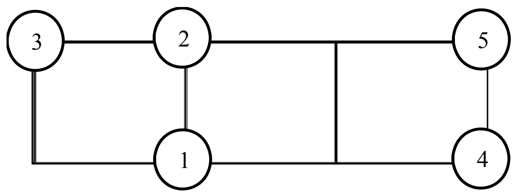
$$l_{km}^s \in \{o,1\} \qquad \forall k, m, s \in S \tag{24}$$

Constraints (18) to (24) are domain constraints. Because of existence of non-linear phrases in the objective functions, the presented model is a multi-objective mixed integer non-linear programming model.

5 Validation of the model

To validate the proposed model, it is solved using the GAMS software and BARON solver for an instance inspired of Correia et al. (2010). In this problem, there are five nodes so that each node sends two units of flows and cost of shipping the flow between each two hubs is 0.75 times of their distance and between each non-hub node and hub, it is equal their distance. As seen in Figure 1 the size of each side of square is one and distance of each two nodes is equal their Euclidean distance. Nodes 2 and 5 have two capacities 10 and 16 with installing cost 100 and 120 and the other nodes have one capacity 10 with installing cost 300.

Figure 1 Set of nodes and their distances



It is also assumed that for each potential hub and linkage there are two corrective actions and one environmental impact. Necessary investment cost of first corrective action for all nodes is zero while this cost for the second corrective action for nodes 2 and 5 is 10, 30 for other nodes and for all linkages is 20. Environmental impacts of flow processing in all nodes for first corrective action is 5, for second corrective action in nodes 2 and 5 is 4 and for other nodes is 2. In addition, installing cost of hubs for first corrective action in first capacity level in nodes 2 and 5 is 20, for second capacity level is 30, for other nodes is 20 and for second corrective action these numbers are equal 10, 20 and 10 respectively. Environmental impacts of linkages for first corrective action is equal their distances and for second corrective action is 0.8 times of them. Also, for every linkage one corrective action is selected. The results are shown in Table 1 and Figure 2. As seen, three hubs are installed. Capacity levels and corrective actions selected for each hub are shown in Table 1 and all assignments are shown in Figure 2. The result shows using multiple capacity levels and corrective action could led to less cost and environmental impacts and also multiple allocation assumption could be another result of decreasing economic cost because of routing the flow through more than one linkage. For example, in this problem the flow of node 4 to node 3 is routed via hubs 1, 2 and 5. Since the model is multi-objective – it is showed in Figure 3 by changing objective function coefficients in 0.25 scale – the model is solved by normalising the objective functions with equal coefficients based on the following equation:

$$Min \left(\frac{economic\ obj.}{economic\ obj.^*} + \frac{environmental\ obj.}{environmental\ obj.^*} \right) \quad (25)$$

So that economic obj.* and environmental obj.* are optimum amount of economic and environmental objectives. Figure 3 shows economic cost decreasing led to environmental increasing.

Table 1 Normalised results of GAMS software for proposed instances

Pro.	*(hub/Cap. Lev/Str.)*	*Economic obj.*	*Environmental obj.*	*Total obj.*
1	(1/1/2), (2/2/1), (5/2/1)	1.023 (641.4)	1.293 (353.92)	1.158 (497.66)

Figure 2 Hub network designed and assignments

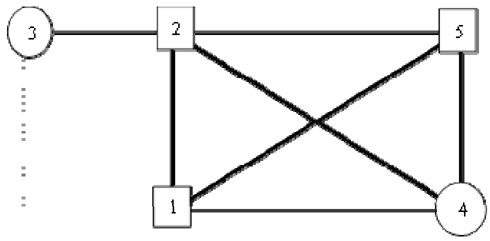
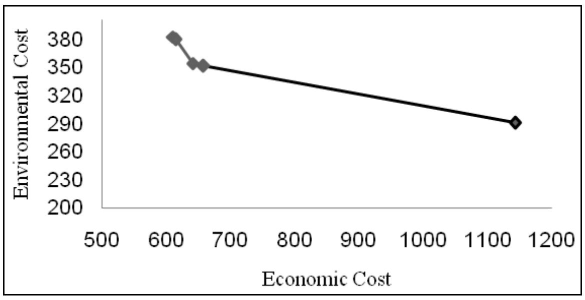


Figure 3 Contrast between economic and environmental objectives



Based on Davis and Ray (1969) and Mirchandani and Francis (1990), a capacitated location problem is a kind of NP-hard problems. Since the proposed model is a kind of capacitated location problems and is bigger than the models presented by them, and given long solution time required for solving the model using the GAMS software all ensure that the presented model is a NP-hard model. As metaheuristic algorithms are useful ways for solving NP-hard problems and given that the presented model is multi-objective, a multi-objective meta heuristic algorithm is therefore suggested to solve the model in this research.

6 Solution method

In past few years, researchers have reported that for multi-objective optimisation, multi-objective evolutionary algorithm (MOEA) can be a very useful tool for deriving reservoir operational policies. Recently, Reddy and Kumar (2007b) proposed an efficient multi-objective optimisation algorithm namely the multi-objective deferential evolution (MODE) technique, by incorporating non-dominated sorting and Pareto-optimality principles into a single-objective differential evolution algorithm. They evaluated the efficiency of the developed MODE for several test problems. It was found that the MODE technique provided superior performance to that of non-dominated sorting genetic algorithm-II (NSGAII). Besides, this algorithm not only could solve all problems with discrete, continuous and mixed integer variables but also it could be capable for solving non-linear problems. Therefore, the MODE technique was considered to solve the model developed in this study.

In the following, at first, a brief description of differential evolution algorithm is presented and the procedure of the MODE methodology is then explained.

6.1 Deferential evolution algorithm

Differential evolution is a kind of optimisation technique proposed as a variant of evolutionary algorithms to achieve the goals of robustness in optimisation and faster convergence to a given problem, Storn and Price (1995). Deferential evolution algorithm has attracted much attention and gained a wide variety of applications in different fields, Price et al. (2005). DE is a capable algorithm in optimising all integers, discrete and continuous variables that could handle non-linear objective functions with multiple non-trivial solutions, Onwubolu and Davendra (2006).This algorithm differs from other evolutionary algorithms in the mutation and recombination phase. DE uses weighted differences between solution vectors to perturb the population, unlike some of the other metaheuristic techniques such as evolutionary strategies, where perturbation occurs in accordance with a random quantity.

Let's consider $S \subset R^n$ be the search space of the problem. The DE algorithm utilises NP (population size), n-dimensional vectors called a generation, of the algorithm as a population for each iteration.

$$X_i = \left(x_{i1}, x_{i2}, \dots, x_{in}\right)^T \in S, i = 1, \dots, NP \quad (26)$$

The initial population is usually taken to be uniformly distributed in the search space. At each generation, two mutation and crossover operators are applied on each individual to produce a new population. The next phase is selection where each individual of the new population is compared to the corresponding individual of the old population, and the better of them is selected as a member in the population of the next generation (Storn and Price, 1995).

In terms of classic DE (DE/rand/1), as used in this research, according to the mutation operator for each individual, $X_i^G \; i = 1, \dots, NP,$ the following equation is used to generate mutation vector:

$$V_i^G = X_{r1}^G + F \times \left(X_{r2}^G - X_{r3}^G\right) \quad (27)$$

where $F > 0$ is a real parameter and called mutation constant. It controls the amplification of the difference between two individuals so as to avoid search stagnation. Also, X_{r1}^{G}, X_{r2}^{G} and X_{r3}^{G} are vectors of present generation selected randomly. Applying crossover operator on the population is the next phase used to control diversity of the population after applying the mutation operator. The following equation is used to generate offspring vectors from mutation vectors:

$$U_{i,j}^{G} = \begin{cases} V_{i,j}^{G} & \text{if} \quad rand_j \leq CR;\ j = k \\ X_{i,j}^{G} & \text{otherwise} \end{cases} \tag{28}$$

where $k \in \{1, \dots, D\}$ is a random index generated for each i in order to ensure using at least one member of mutation vector in offspring vector. $CR \in [0,1]$ is a user defined crossover constant and $rand_j$ is a random number with uniform distribution compared with CR to determine offspring vector members (Storn and Price, 1997).

To decide whether the offspring vector should be a member of the population of the next generation, it is compared with the corresponding vector X. Therefore, if f denotes the objective function under consideration, then,

$$X_{i}^{G+1} = \begin{cases} U_{i}^{G} & \text{if } f\left(U_{i}^{G}\right) \leq f\left(X_{i}^{G}\right) \\ X_{i}^{G} & \text{otherwise} \end{cases} \tag{29}$$

Thus, in the selection phase, each individual of the offspring vector is compared with its parent vector and the better one is passed to the next generation, so the elitism (the best individuals in the population) is preserved. The mentioned steps are repeated until specified termination criterion is reached. Since DE is a capable technique to provide efficient solutions for complex single objective optimisation problems, it is prompted to develop MODE algorithms, Reddy and Kumar (2007b). The details are given in the following sub-section.

6.2 MODE algorithm

A multi-objective optimisation problem tends to optimise a number of objective functions simultaneously. Since an evolutionary algorithm (EA) deals with a number of population members in each generation, for a multi-objective optimisation problem an EA is an ideal candidate for finding multiple Pareto optimal solutions. In general, MOEA methods are used to perform three tasks which:

1. emphasise on non-dominated solutions for progressing towards the true Pareto-optimal front
2. emphasise on less-crowded solutions for maintaining a good diversity among the obtained solutions
3. emphasise on elites to provide a faster and reliable convergence towards the true Pareto-optimal front (Reddy and Kumar, 2007a).

The MODE procedure was evolved by combining Pareto-optimality (non-domination) criteria with the DE algorithm, MODE adapted an effective selection procedure to achieve multi-objective goals, where it uses non-dominated sorting and crowding distance assignment operators (Deb et al., 2002). Also, this methodology maintains an external archive to store the best non-dominated solutions explored over the generations.

The steps of MODE algorithm include initialisation of population, evaluation, Pareto-dominance selection, performing DE operations and reiterating the search on population to reach true Pareto optimal solutions. In this process, at first each of the members of the population is evaluated and checked for dominance relation. If the new member dominates the parent, it replaces the parent, if the parent dominates the candidate, the new member is discarded and if the parent and new member both are mutually non-dominant, then the two are added to a temporary population (tempPop). This step is repeated for all members of the population. Thereafter, the tempPop is reduced to the population size (NP) by using non-dominated sorting and crowding distance assignment procedures in order to select the population for next generation. Apart from that, it uses non-dominated elitist archive (NEA) to store the best solutions found over the generations. These operators are used to help create effective selection pressure toward true Pareto optimal solutions. The size of NEA can be set to any desirable number of non-dominated solutions. In case, the size of NEA exceeds predefined number of itself, then the crowding operator is used to select the sparse individuals to achieve good distribution of Pareto optimal solutions. The selection of best is made by choosing a solution from the elite archive randomly.

The procedure of MODE for the proposed model is explained step by step below:

Step 1 Initial MODE parameters adjustment.

Step 2 Initial solution generation involved the following steps:

a determination of hub numbers and selection of hub nodes randomly, (check the feasibility of the model for each solution based on $\sum_i \sum_j w_{ij} \leq \sum_k \Gamma_k^{q_s}$)

b assignment of non-hub nodes to hubs based on capacity constraints (for this purpose a fraction of non-assigned flow of each non-hub node is assigned to the remaining capacity of randomly selected hub)

c determination of selected capacity level for each hub, (based on assignments of each hub, if total entering flow to the hub is in $[q_i, q_i + 1]$, thus the selected capacity level of that hub is $q_i + 1$)

d selection of corrective actions for installed hubs and linkages.

- Hubs: for each corrective action of each hub with a predefined capacity level, the following probability (p_k^r – probability of selecting corrective action r for hub k) should be calculated and compared with a randomly selected number (t), then corrective action selection is done based on the following:

$$p_k^r = \exp\left(-\left(\left(Gh_k^r + \sum_{apro} epro_k^{apro,r} \sum_{aop}\sum_{q} eop_{kq}^{aop,r}\right) \Bigg/ \left(\sum_{r} Gh_k^r + \sum_{apro}\sum_{r} epro_k^{apro,r} + \sum_{aop}\sum_{q}\sum_{r} eop_{kq}^{aop,r}\right)\right)\right) \tag{30}$$

$$if\ t > 0\ and\ t < p_k^1 \quad r = 1 \tag{31}$$

$$if\ t > p_k^1\ and\ t <= p_k^1 + p_k^2 \quad r = 2 \tag{32}$$

$$if\ t > p_k^1 + p_k^2\ and\ t <= p_k^1 + p_k^2 + p_k^3 \quad r = 3 \tag{33}$$

$$if\ t > p_k^1 + p_k^2 + p_k^3 + \ldots + p_k^{r(v_k - 1)} and\ t <= 1 \quad r = r_{v(k)} \tag{34}$$

Considering this probability makes selection of corrective actions with less environmental impacts and investment cost.

- Linkages: Same as hubs, the following probability (p_{ij}^s – probability of selecting corrective action s for linkage between node i and node j) and relations should be calculated and then corrective action selection is done for each linkages:

$$p_{i,j}^s = \left(\exp\left(-\left(Gl_{i,j}^s + \sum_{b} e_{i,j}^{bs}\right)\right)\right) \Bigg/ \left(\sum_{s} Gl_{i,j}^s + \sum_{b}\sum_{s} e_{i,j}^{bs}\right) \tag{35}$$

$$if\ t > 0\ and\ r < p_{ij}^1 \quad s = 1 \tag{36}$$

$$if\ t > p_{ij}^1\ and\ t <= p_{ij}^1 + p_{ij}^2 \quad s = 2 \tag{37}$$

$$if\ t > p_{ij}^1 + p_{ij}^2\ and\ t <= p_{ij}^1 + p_{ij}^2 + p_{ij}^3 \quad s = 3 \tag{38}$$

$$if\ t > p_{ij}^1 + p_{ij}^2 + p_{ij}^3 + \ldots + p_{ij}^{s(y_{ij} - 1)} and\ t <= 1 \quad s = s_{y(ij)} \tag{39}$$

Step 3 Evaluation of present population based on domination and non-domination, selection of non-dominated solution and storing them in Pareto archive.

Step 4 Implementation of mutation and crossover operators on present population as the following:

Mutation operator: in this problem DE/rand/1/bin mutation operator is used, therefore the mutation vector is:

$$V_i^G = X_{r1}^G + F \times \left(X_{r2}^G - X_{r3}^G\right) \tag{40}$$

Also, the following equation is used to limit mutation vector between 0 and 1.

$$V_i^G = Min\left(1, \max\ (0, V)\right) \tag{41}$$

Crossover operator: implementation of crossover operator based on the following relation:

Step 5 Check the feasibility of the individuals after implementing mutation and crossover operator for:

a selected hubs and assignments

b capacity of hubs and updating infeasible solutions in terms of capacity

c calculation of the other variables based on assignment matrix.

Step 6 Population selection for entering to the next generation by comparing each offspring with its parent, if one of them could dominate the other one, it is selected for entering to the next generation and if any of them could not dominate the other one, both of them enter to the temporary population. Now, the temporary population compared in terms of non-dominated sorting and crowding distance and the population is selected for entering to the next generation.

Step 7 Updating Pareto archive.

Step 8 Moving to the next generation and check for termination criteria. If the termination criterion is not satisfied, move to step 4 and otherwise show the Pareto solutions. Figure 4 shows the MODE pseudo code used for the proposed model.

Figure 4 Pseudo code of MODE procedure for the proposed model

Start
Input the required DE parameters;
Generate initial feasible solutions
$P^G = X_i^G = X_{i,j}^G \quad i = 1,\ldots,NP; \; j = 1,\ldots,D; \; G = 1,\ldots,G_{\max}$
Evaluate non-dominated solutions and store them in the Pareto archive;
Set G (generation counter) = 0;
While G ≠ a given number of generations
For *each member of the population*
Choose $r1, r2, r3 \in \{1,\ldots,NP\}$ *randomly so that* $r1 \neq r2 \neq r3 \neq i$;
Calculate $V_i^G = X_{r1}^G + F \times \left(X_{r2}^G - X_{r3}^G\right)$
Choose $k \in \{1, \ldots, D\}$ *randomly*
Generate $rand_j \in [0,1]$
If $rand_j < CR$ *or* $j = k$
$U_{i,j}^G = V_{i,j}^G$
Else $U_{i,j}^G = X_{i,j}^G$
End
Check for the feasibility
Check for dominance
End
Choose the population for entering to the next generation based on non-dominated sorting and crowding distance;
Update Pareto archive;
G = G + 1;
End while;
End;

7 Experimental results and sensitivity analysis

In this section computational experiments are performed to evaluate the proposed model and presented metaheuristic algorithm. To show the performance of the proposed model, sensitivity analysis is performed on the generated data. In addition to that, an evaluation of the MODE algorithm is performed based on three quality, diversity and spacing metrics.

7.1 Quality metric

Quality metric represents the ratio of Pareto solutions of each algorithm to the optimal frontier. Since achieving to the final optimal frontier is impossible, this metric is calculated based on final non-dominated solutions.

7.2 Spacing metric

This metric obtain the uniformity of distribution law of non-dominated solutions achieved by each algorithm. This metric is equal:

$$S = \frac{\sum_{i}^{N-1} \left| \bar{d} - d_i \right|}{(N-1)\bar{d}} \tag{42}$$

In the above equation, d_i is the Euclidean distance between each two consecutive solutions on the optimal frontier achieved by the algorithm and $\bar{d}$ is the average of all achieved values (Srinivas and Deb, 1994).

7.3 Diversity metric

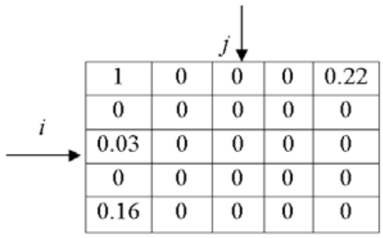
This metric is used to determine non-dominated solutions on the optimal frontier defined as the following:

$$D = \sqrt{\sum_{i=1}^{N} \max\left(\left\| x_t^i - y_t^i \right\| \right)} \tag{43}$$

On the above equation $(\| x_t^i - y_t^i \|)$ shows the Euclidean distance between each two neighbour solutions x_t^i and y_t^i on the optimal frontier (Zitzler and Thiele, 1999).

Since x_{ijkm} is the main variable of the model, in the proposed algorithm each initial solution is a four dimensional matrix so that *i* is the first dimension, *j* is the second one, *k* is the third one and *m* is the last one. An example of initial solution for $n = 5$, $k = 1$ and $m = 1$ is presented in Figure 5. In this example nodes 1, 2 and 4 are hubs and 3% of flow of node 3 to 1 is sent directly.

Figure 5 Initial solution for $n = 5$, k and $m = 1$



In Subsection 7.4 generated data and all instances are presented and in Subsection 7.5 obtained results are presented and discussed.

7.4 Test data

In this research three instances with 10, 20 and 26 nodes are presented to evaluate the proposed algorithm. For each instance three capacity levels, one environmental impact level, two corrective action levels and three values for α are presented to analysis the sensitivity of the model. Therefore, in this research 54 problems are presented. Table 2 shows the values of q, α, r and s of each problem for three nodes 10, 20 and 26.

Table 2 The feature of presented instances for nodes 10, 20 and 26

No.	*q*	α	*R, S*
1	1	0.4	1
2			2
3		0.6	1
4			2
5		0.8	1
6			2
7	2	0.4	1
8			2
9		0.6	1
10			2
11		0.8	1
12			2
13	3	0.4	1
14			2
15		0.6	1
16			2
17		0.8	1
18			2

Since the literature is shy in data provision in the area of hub location problem, the presented model is then parameterised using other sources to generate data of instance problems. Based on Mohammadi et al. (2011), it is assumed that we assume flow and transportation cost between each two nodes have uniform distribution between 0 and 10. For each hub, it is assumed $\Gamma_k^{s_k} = \Gamma_k \ \forall\, k \in N$ and $\Gamma_k^q = 0.7 \times \Gamma_k^{q+1} \ \forall\, q = 1,\dots,s_{k-1}$ and Γ_k has uniform distribution in [12.5(n^2)] and $F_k^{s_k} = F_k \ \forall\, k \in N$ and $F_k^q = \rho \times \Gamma_k^q \, F_k^{q+1} / \Gamma_k^{q+1} \ \forall\, q = 1,\dots,s_{k-1}$ so that F has uniform distribution in [200 600] and ρ chooses values between 1.1 and 1.2 (Correia et al., 2010). Environmental data are generated using research proposed by Wang et al. (2011) so that they should be capable to generate feasible solutions. The features of generated environmental data are shown in Table 3.

Table 3 Generation of environmental data

$r = 1$	$Gh_k^r = 0$		
$r = 2$	$Gh_k^r = r_{cost} F_k^{s_k}$	$r_{cost} \sim U(0.4, 0.6)$	r_{cost} cost fixed coefficient
$s = 1$	$Gl_{ij}^s = 0$		
$s = 2$	$Gl_{ij}^s \sim U(1,100)$		
$b = 1$, $s = 1$	$e_{ij}^{bs} = r_e c_{ij}$	$r_e \sim U(0.9, 1.2)$	r_e environmental impact fixed coefficient
$b = 1$, $s = 2$	$e_{ij}^{bs} = y_e \, r_e \, c_{ij}$	$y_e \sim U(0.7, 0.9)$ $r_e \sim U(0.9, 1.2)$	y_e environmental impact coefficient
$r = 1$ and $r = 2$	$eop_{kq}^{aop,r} \sim U\left(48/2^{r-1}, 72/2^{r-1}\right)$, $q = 1$	$eop_{kq}^{aop,r} = eop_{k,q-1}^{aop,r} \times \Gamma_k^q / \Gamma_k^{q-1}$, $q = 2,\dots,s_k$	It is assumed relationship between environmental impact and capacity increasing is linear
$r = 1$	$epro_k^{apro,r} \sim U(0, 0.01)$		
$r = 2$	$epro_k^{apro,r} \sim r_{epro} U(0, 0.01)$	$r_{epro} \sim U(0.7, 1)$	

Based on the above, it is assumed implementation cost of corrective action level 2 at hub k is a ratio of operating fixed cost, environmental impact of linkages is a ration of their transportation cost and environmental impact of flow processing under corrective action 2 in hub k is a ratio of it under corrective action level 1.

The algorithm parameters are tuned after multiple running and the best values are selected as detailed below:

CR	0.6
F	0.7
Population size	25
Pareto archive capacity	100
Number of iteration to stop the algorithm	25.

7.5 Computational results

As mentioned above, the proposed model is NP-hard and the GAMS software could not solve the problems with even small sizes, implying that proposed instances with 10, 20 and 26 are medium and large size problems for the proposed model. To measure the efficiency of the proposed algorithm, MODE results are presented in Tables 4, 5 and 6 for three spacing, diversity and quality metrics. It is remarkable that minimum quality metric and maximum spacing and diversity metrics are associated with high efficiency of the algorithm. As seen, elapsed time for solving each problem is shown in the last column of Tables 4, 5 and 6.

Table 4 Comparison of metrics' results for $n = 10$

No.	*Spacing metric*	*Diversity metric*	*Quality metric*	*Time (second)*
1	0.3662	76.4852	9	91.6676
2	0.7254	33.8866	5	59.6332
3	0.7468	73.8894	7	57.8
4	0.9958	43.643	8	51.226
5	0.8429	59.0318	5	79.746
6	0.136	26.191	4	95.1383
7	0.5881	104.8732	8	56.3361
8	0	11.083	2	53.0555
9	0.7882	74.8731	9	47.0331
10	1.2946	38.1327	4	63.2092
11	0.8162	89.4637	7	57.4646
12	1.0219	51.578	7	46.4367
13	0.9464	45.0435	4	55.5784
14	1.4952	61.0847	5	72.3532
15	0.6408	96.7931	8	41.425
16	1.0402	64.261	3	57.434
17	0.2715	56.977	4	43.4572
18	0.3609	75.6925	5	63.709

Table 5 Comparison of metrics' results for $n = 20$

No.	*Spacing metric*	*Diversity metric*	*Quality metric*	*Time (second)*
1	0.8797	88.4628	7	424.4627
2	0.8460	34.6590	3	513.5228
3	0.6485	89.8463	8	543.6891
4	0.8849	74.2994	5	602.1418
5	0.5079	73.56	5	398.2891
6	0.7261	94.7255	6	429.442
7	1.0108	107.0585	7	343.8606
8	0.6458	56.1908	6	323.1186

Table 5 Comparison of metrics' results for n = 20 (continued)

No.	*Spacing metric*	*Diversity metric*	*Quality metric*	*Time (second)*
9	0.3788	59.8242	5	342.604
10	0.1513	44.8538	3	272.5344
11	0.2477	58.5886	5	423.3281
12	10.109	39.729	3	166.8675
13	0.2214	149.776	8	849.9607
14	0.6057	114.0519	7	318.5315
15	0.8541	134.5829	11	309.66
16	0.8334	288.8451	12	293.0198
17	0.7088	143.5659	8	241.0281
18	0.6185	341.0255	14	317.1788

Table 6 Comparison of metrics' results for n = 26

No.	*Spacing metric*	*Diversity metric*	*Quality metric*	*Time (second)*
1	0.6461	114.1437	7	1,112.0643
2	0.5914	62.6884	6	683.4011
3	0.2126	91.6136	6	1,048.4451
4	0.4741	113.7891	5	1,665.0654
5	0.3805	93.5134	5	980.814
6	0.9906	227.5886	6	918.6476
7	0.9642	97.5723	5	796.1265
8	1.0717	143.8894	7	628.505
9	0.5372	71.6265	4	805.2575
10	0.7869	72.9427	4	831.5323
11	0.2928	97.8155	8	1,136.2348
12	0.8105	119.6129	6	791.3001
13	0.3637	85.7391	4	866.3526
14	0.7323	213.939	8	673.5154
15	0.5686	123.2913	6	643.4648
16	0.8711	180.3349	8	622.148
17	0.8931	101.3117	6	656.5153
18	0.9681	380.1938	11	614.5047

This section tries to use a non-parametric test to show significant statistical differences among the three different sizes of proposed problems for each metric. Non-parametric tests, besides their original definition for dealing with nominal or ordinal data, can be also applied to continuous data by conducting ranking-based transformations and adjusting the input data to the test requirements (Hollander and Wolfe, 1999). A pairwise statistical procedure is used to perform individual comparisons between the proposed problems, obtaining a p-value independent from another one in each application. In order to use a statistical test, two hypotheses, the null hypothesis H0 and the alternative hypothesis H1, are defined. H0 is a statement of no difference and the alternative hypothesis represents the presence of a difference (in our case, significant differences between each two sizes of problems). In order to reject a hypothesis, a level of significance α is used to determine at which level the hypothesis may be rejected (Manavizadeh et al., 2013).

In this study, a sign test that is one of the pairwise comparison procedures is used. It is a popular way to compare the overall performances of metrics that count the number of cases on which a problem is the overall winner. Also, it uses these counts in inferential statistics, with a form of two-tailed binomial test. If both problems compared are equivalent, as assumed under the null hypothesis, each should win on approximately $n/2$ out of n problems. The number of wins is distributed according to a binomial distribution. For a greater number of cases, the number of wins is under the null hypothesis distributed according to $n\left(n/2, \sqrt{n/2}\right)$, which allows for the use of the z-test: if the number of wins is at least $\left(n/2 + 1.96\sqrt{n/2}\right)$ (or, for a quick rule of a thumb, $n/2 + \sqrt{n}$), then the problem is significantly better with $p < 0.05$ (Derrac et al., 2011). Table 7 shows the critical number of wins needed to achieve both $\alpha = 0.05$ and $\alpha = 0.1$ levels of significance. A problem is significantly better than another if it performs better on at least the cases presented in each row of Table 7. Since tied matches support the H0, they should not be discounted when applying this test, but if there is an odd number of them, one should be ignored. Applied problems are compared based on sign test in Table 8. H0 shows the equivalence of each two sizes of problems and third column is number of wins of the first metric of each comparison. Fourth and fifth columns show acceptance or rejection of H0 in terms 18 considering tie matches. The obtained results show in the three metrics in terms of two proposed significant levels there are significant differences between all small, medium and large size problems. Also, the results show that the algorithm performs better for all three metrics in problems with larger sizes. Figures 6, 7 and 8 illustrate efficient solutions of Pareto frontier for some of the instance problems.

Table 7 Critical values for the two-tailed sign test at $\alpha = 0.05$ and $\alpha = 0.1$

Cases	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*
$\alpha = 0.05$	5	6	7	7	8	9	9	10	10	11	12	12	13	13	14	15	15	16	17	18	18
$\alpha = 0.1$	5	6	6	7	7	8	9	9	10	10	11	12	12	13	13	14	14	15	16	16	17

Table 8 Results of sign test

Metric	H_0	*Number of wins*	$\alpha = 0.05$	$\alpha = 0.1$
Spacing	$n(10) = n(20)$	9	Reject	Reject
	$n(10) = n(26)$	8	Reject	Reject
	$n(20) = n(26)$	6	Reject	Reject
Diversity	$n(10) = n(20)$	3	Reject	Reject
	$n(10) = n(26)$	2	Reject	Reject
	$n(20) = n(26)$	5	Reject	Reject
Quality	$n(10) = n(20)$	9	Reject	Reject
	$n(10) = n(26)$	12	Reject	Reject
	$n(20) = n(26)$	8	Reject	Reject

Figure 6 Efficient solutions of Pareto frontier for n = 10, q = 2, α =0.4 and r,s = 1

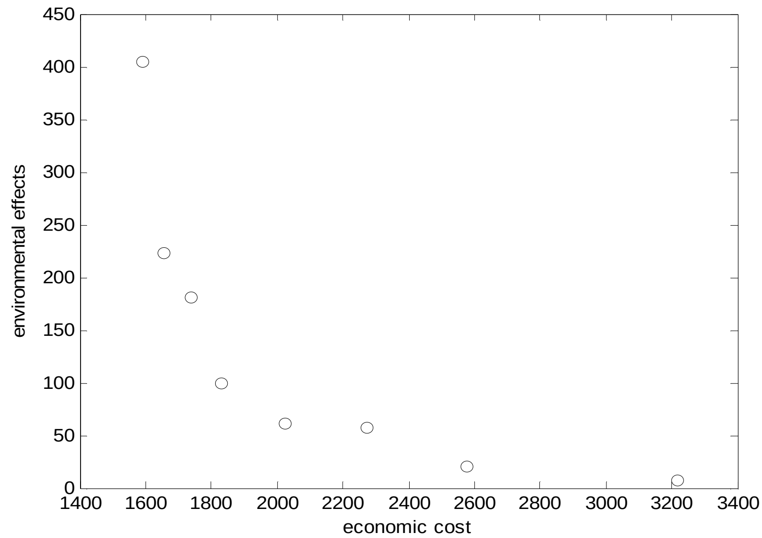


Figure 7 Efficient solutions of Pareto frontier for n = 20, q = 1, α = 0.4 and r,s = 1

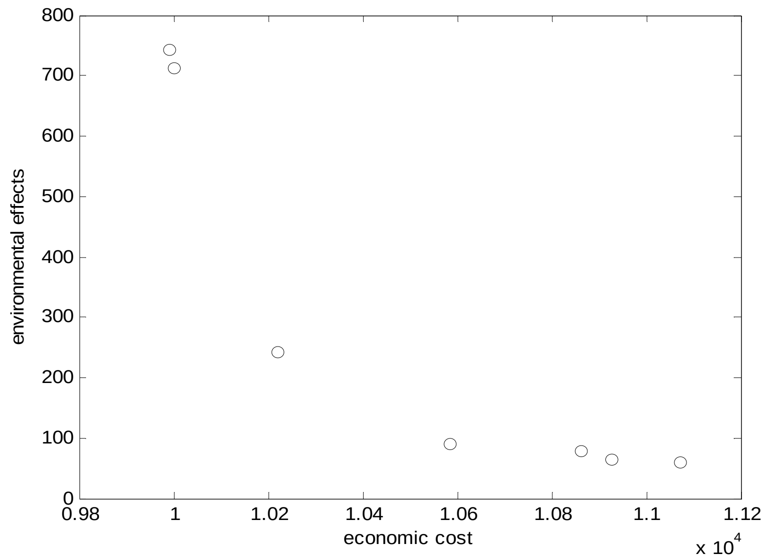
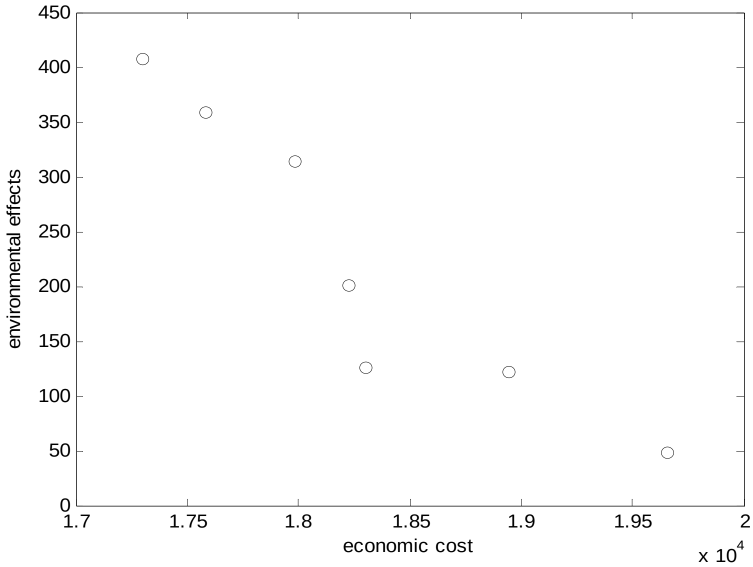


Figure 8 Efficient solutions of Pareto frontier for $n = 26$, $q = 1$, $\alpha = 0.4$ and $r,s = 1$



To analyse the sensitivity of the model toward parameters variation, the average of economic and environmental objectives and their combined average are used as shown in Tables 9, 10 and 11. F_1 is the average of economic objective of all Pareto optimal solutions of each problem, F_2 is the average of environmental objective of all Pareto optimal solutions of each problem and F is the average of them.

Table 9 Average of objective functions for $n = 10$

No.	F_1	F_2	F
1	3,737.1909	125.7643	1,931.4776
2	4,010.4975	113.508778	2,062.00314
3	4,361.946	154.1672	2,258.0566
4	6,923.8804	241.2765	3,582.5784
5	4,381.0973	125.7905	2,253.444
6	6,974.495	125.8081	3,550.1515
7	2,113.366	131.7101	1,122.538
8	4,267.461	114.4828	2,190.9719
9	2,640.3258	166.4133	1,403.3696
10	5,513.1995	140.3513	2,826.7754
11	4,109.444	107.846	2,108.645
12	6,206.8811	150.99	3,178.9356
13	2,876.26	261.5823	1,568.921
14	3,376.663	165.5123	1,771.088
15	2,193.1978	249.9418	1,221.57
16	3,476.4727	195.3663	1,835.92
17	1,852.4885	264.5017	1,058.495
18	2,085.5704	215.8736	1,150.722

Table 10 Average of objective functions for n = 20

No.	F_1	F_2	F
1	10,523.0689	284.1591	5,403.614
2	17,123.5412	285.9041	8,704.7227
3	12,913.5951	142.5299	6,528.063
4	23,729.7883	576.4906	12,153.1394
5	15,850.9146	222.642	8,036.7783
6	24,177.40265	699.7191	12,438.5609
7	7,168.4657	538.8972	3,853.6815
8	17,722.7343	423.6727	9,073.2035
9	9,475.534	171.4021	4,823.468
10	20,909.026	184.1012	10,546.5636
11	11,785.1628	256.1521	6,020.6575
12	20,460.718	294.1599	10,377.4389
13	4,675.091	715.6044	2,695.348
14	15,652.5163	678.6992	8,165.608
15	5,373.31	606.4529	2,989.882
16	18,020.139	543.211	9,281.675
17	10,424.6372	784.1614	5,604.399
18	19,022.878	663.5124	9,843.195

Table 11 Average of objective functions for n = 26

No.	F_1	F_2	F
1	18,286.0419	225.1855	9,255.6137
2	26,344.1815	239.09615	13,291.6388
3	19,568.7311	392.7166	9,980.7238
4	36,674.4567	188.40227	18,431.4295
5	24,730.1559	532.5386	12,631.3472
6	40,987.06972	322.31369	20,654.6917
7	11,818.1576	306.098	6,062.1278
8	19,245.9062	116.88163	9,681.3939
9	14,475.3585	557.8289	7,516.5937
10	32,257.838	553.0212	16,405.4296
11	18,408.6902	619.973	9,514.3316
12	35,232.3897	426.401	17,829.3953
13	10,893.898	1,039.6887	5,966.7933
14	17,678.7222	993.6985	9,336.2104
15	12,722.984	966.7122	6,844.8481
16	30,471.1182	866.8478	15,668.983
17	15,289.6103	928.1073	8,108.8588
18	30,712.9551	872.6901	15,792.8226

Figures 9, 10 and 11 presents the sensitivity of the model toward *r* and *s* parameters' variation for $n = 10$, $n = 20$ and $n = 26$. Figures' trend show decreasing of environmental objective when there are two corrective actions for environmental impacts toward when just one corrective action exists.

Figure 9 Comparison of environmental objective function average based on different corrective action levels for $q = 3$ and $n = 10$ (see online version for colours)

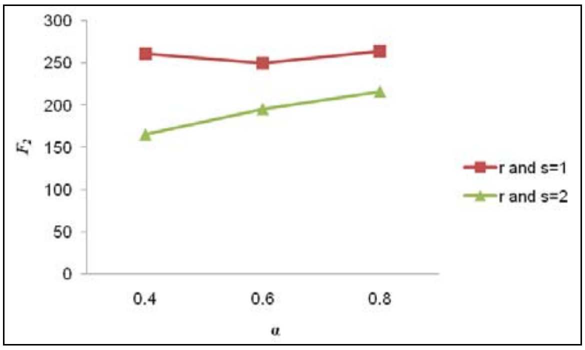


Figure 10 Comparison of environmental objective function average based on different corrective actions levels for $q = 3$ and $n = 20$ (see online version for colours)

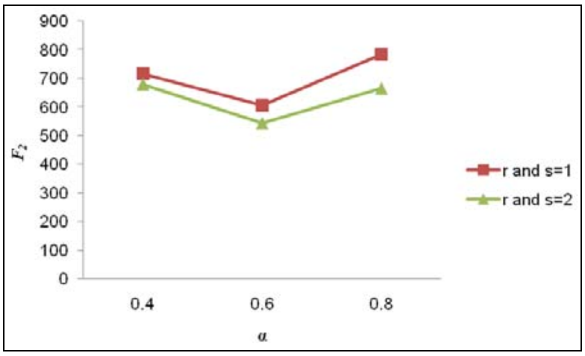
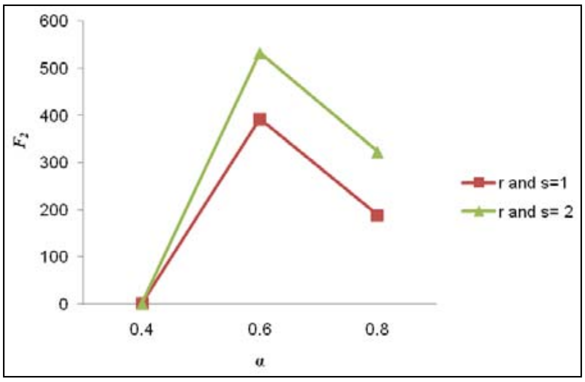


Figure 11 Comparison of environmental objective function average based on different corrective actions levels for $q = 1$ and $n = 26$ (see online version for colours)



Figures 12, 13 and 14 show the sensitivity of the model toward variation of α for $n = 10$, $n = 20$ and $n = 26$. The results show an increase in economic objective with increasing of α.

Figure 12 Comparison of economic objective function average toward α for $n = 10$ and $q = 3$ (see online version for colours)

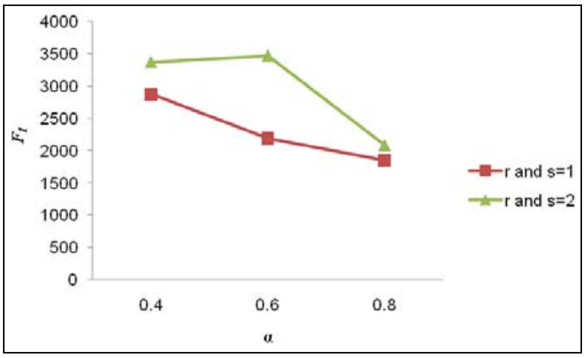


Figure 13 Comparison of economic objective function average toward α for $n = 20$ and $q = 3$ (see online version for colours)

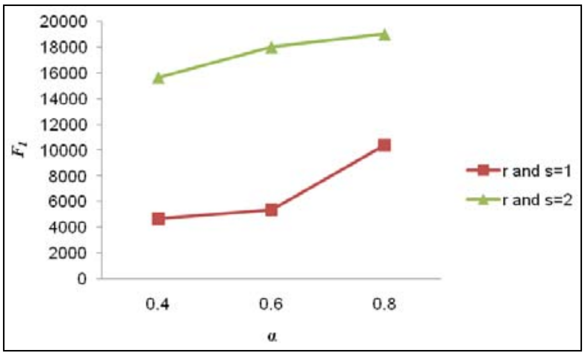
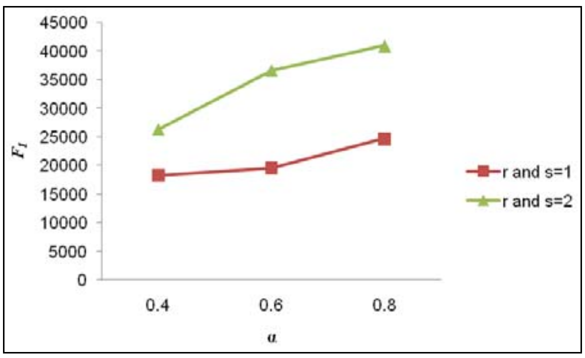


Figure 14 Comparison of economic objective function average toward α for $n = 26$ and $q = 1$ (see online version for colours)



And finally the sensitivity of the model is evaluated toward different capacity levels as shown in Figures 15, 16 and 17. Their trend shows using more capacity levels could decrease economic objective and it is because of high flexibility of the model in the present of multiple capacity levels leads to selection of less hub numbers, less installing cost and less investments cost for decreasing of environmental impacts.

Figure 15 Comparison of economic objective function average toward different capacity levels for $n = 10$ and $r, s = 2$ (see online version for colours)

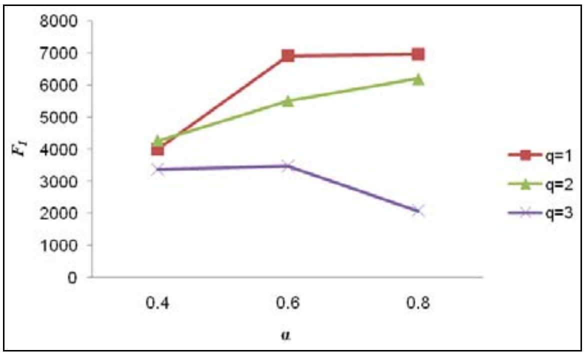


Figure 16 Comparison of economic objective function average toward different capacity levels for $n = 20$ and $r, s = 1$ (see online version for colours)

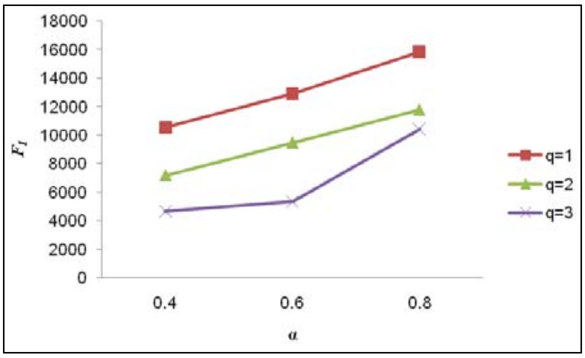
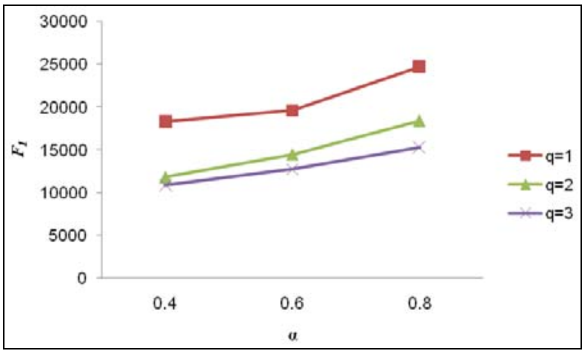
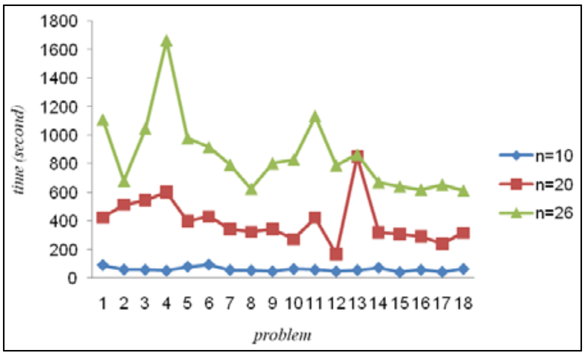


Figure 17 Comparison of economic objective function average toward different capacity levels for $n = 26$ and $r, s = 1$ (see online version for colours)



Since the proposed model is NP-hard problem, necessary time for solving the model increases with the problem size. Comparison of elapsed time for solving all problems with $n = 10$, 20 and 26 by using the MODE algorithm is shown in Figure 18.

Figure 18 Necessary time comparison for solving the model for all instances (see online version for colours)



8 Conclusions

In this research a multi-objective mathematical model with multiple allocation and multiple capacity levels is proposed. The proposed model focuses on environmental and economic aspects of sustainable development in designing hub networks as a management strategic decision. The proposed model is applicable for all the location decisions dealing with environmental impacts. Multiple capacity levels are used to increase the flexibility of the model for selecting hub nodes. Since this problem is a kind of NP-hard problems, a MODE algorithm is used to solve generated instances. Three spacing, diversity and quality metrics are used to evaluate the MODE algorithm performance. Obtained results show that the proposed algorithm outperforms for spacing, quality and diversity metric in large sized problems. Also, the performance of the proposed model is evaluated by analysing the sensitivity of the model toward parameters' variations. The results of sensitivity analysis reveals that economic objective function increases by increasing transportation cost coefficient, environmental objective function decreases by increasing corrective action levels and decreasing of economic objective function by increasing capacity levels. It is proposed to solve the model by using other metaheuristic algorithms and compare results with those obtained using MODE in order to evaluate the efficiency of this algorithm. Also, in this research it is proposed there are not any predefined limitations on environmental impacts and the goal of model is designing a hub network with minimum environmental impacts. While existence of a limitation on each kind of environmental impacts could be considered for future studies.

Acknowledgements

The authors would like to acknowledge the financial support of University of Tehran for this research under Grant No. 8109002/1/07.

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