Multi-Period Hub Set Covering Problems with Flexible Radius: a Modified Genetic Solution

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Highlights

- A multi-period hub set covering problem with flexible covering radius is presented.
- A dynamic model is proposed to solve the hub set covering problem with flexible covering radius.
- A modified genetic algorithm (GA) is proposed.
- A real world case study is presented for hub set covering problem with flexible covering radius.

Multi-Period Hub Set Covering Problems with Flexible Radius: a Modified Genetic Solution

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Abstract

Traditionally, in hub covering problems, it is assumed that the covering radius is an exogenous parameter which cannot be controlled by the decision maker. In many real-world cases, with a negligible increase in covering radius, considerable savings in hub establishment costs are possible. On the contrary, changes in problem parameters during the planning horizon cause the results of theoretical models to be impractical in real-world situations. This article proposes a mixed integer model for a multi-period single-allocation hub set covering problem in which the covering radius is a decision variable. The proposed model is validated through a real world case study. Also, due to the NP-Hardness of the problem a modified genetic algorithm (GA) is proposed for solving that. The proposed GA benefits from a dynamic stopping criteria and immigration operator. The performance of the proposed GA is compared with the original GA and imperialist competitive algorithm (ICA). Computational results corroborated efficiency of the proposed algorithm in achieving high-quality solutions in a reasonable time.

Keywords: Facility location, Dynamic hub set covering problem, Flexible covering radius, Mathematical modelling, Dynamic genetic algorithm, Immigration operator.

1- Introduction and literature review

The concept of hub nodes arises when there are many origin-destination (O/D) pairs in a transportation network to benefit from the economies of scale. Hubs are nodes in which the flow from various origins is gathered, and after reorganization, it is dispatched to the destinations. Hub location problems, initially introduced by O'Kelly [1], have many applications in cargo delivery systems, airline networks and telecommunication systems. O'Kelly proposed a quadratic model for hub median problem which was aimed at minimizing total flow costs [2]. Subsequent researches by Campbell [3], Ernst et al. [4], and O'Kelly et al. [5] proposed some linearized versions for the problem. Hub location problems are generally classified into three categories: hub center, hub median and hub covering problems. This article investigates on hub-covering problem. Readers can refer to the review papers by Alumur et al. [6], Farahani et al. [7], and Campbell and O'Kelly [8] for the other categories of hub location problem. Initially, Campbell proposed mathematical formulations for single and multiple-allocation versions of hub set covering problem (HSCP) and hub maximal covering problem (HMCP) [9]. He also proposed that an O/D pair i, j, might be covered by hubs k, l in three ways:

1. Total transport costs (time or distance) from origin i to destination j (via origin i to hub k, hub k to hub l, with a discount factor α; $0 \leq \alpha \leq 1$, and hub k to destination j) do not exceed a given covering radius.

2. Transport costs (time or distance) for each of the links in the path from origin i to destination j via hubs k and l do not exceed the covering radius.

3. Transport costs (time or distance) from origin i to hub k and hub l to destination j do not exceed the covering radius.

The existing models in the literature obey the above rules. They assume that the covering radius is an exogenous parameter, and the decision maker (DM) cannot control its size; however, in many real-world applications, covering radius is a decision variable. Sometimes it is possible to avoid establishing hub nodes with a slight increase in covering radii. For example, in a transportation network, equipping the existing facilities or establishing more spacious depots help increase the covering radius and consequently to serve farther customers. The capability of an airport to serve the flights has a direct proportion to the number of runways, facilities and infrastructure which can be increased when necessary. Also in a telecommunication system, the area covered by the radio waves depends on the strength of the transmitted waves, and with reinforcing the radiation, larger areas may be covered by the hub node. Accordingly, a larger covering radius, can prevent the superfluous costs of establishing new hubs. In order to capture this situation, two types of costs are considered for a hub node, fixed hub establishment and covering cost in which the latter is proportional to the selected covering radius for the hub node.

Facility location decisions are frequently long-term in nature. Facilities such as airports, dams, schools, hospitals, factories, distribution centers and retail outlets are often operated for decades. Consequently, there may be a considerable uncertainty regarding the relevant parameters in the location decision [10]. For example, as time goes on, the amount of supply and demand varies in origin and destination nodes. Moreover, transportation costs among nodes may vary due to the factors such as increasing the fuel cost, depreciation or using cheaper facilities in the fleet. Also time value of money, which is affected by the inflation and the interest rate, is very important in making managerial decisions [11]. These financial factors may change during the planning horizon. Multi-period consideration of a problem enables the model to establish new facilities or close the existing ones periodically, which is proportional to the variation of the relevant parameters. From one point of view, dynamic facility location problems can be divided into two categories. The first category consists of problems in which the number of facilities is an exogenous parameter. In some cases like Zarandi et al. [12], the total number of facilities in the planning horizon is proposed by the DM and the number of facilities in each period is determined by the model. Also, in some cases like Drezner and Wesolowsky [13] and Wesolowsky and Truscott [14], the authors assumed a fixed number of facilities that can be relocated at the end of each period. The second category consists of problems in which the number of facilities is an endogenous parameter and is located such that the total costs are minimized. Contreras et al. [15] considered the un-capacitated dynamic hub location problem and proposed a mixed integer nonlinear programming model to solve that. While constructing hubs is costly, it is assumed that their closure is profitable because of the released resources. Also, Taghipourian et al. [16] studied a dynamic virtual hub location problem. The authors assumed that both closed and established hubs are costly. The hub covering problem investigated in this paper belongs to the second category. Table 1 addresses some of the major extensions to the hub covering problem. Referring to the table, existing formulations for hub covering problem are static, and the proposed dynamic model integrates previous viewpoints by considering both costs and benefits of the closed hubs. More realistic approach to the problem arises with more scrutiny on structure of the facilities. Facilities in a hub can be categorized into two types: static and movable facilities. Although, static facilities remain useless when a hub is closed, moving facilities can be transferred to newly developed hubs causing some savings. For example in a hub airport, some infrastructure facilities like building, watchtower and runways are static facilities and some like airport staff, office supplies or vehicles are moving facilities and their displacement causes savings in costs. It is assumed that the moving facilities released from the closed hubs are usable in only one of the newly established hubs in the same period. The saving associated to these movements is subtracted from the total costs of the period.

This article investigates on a multi-period HSCP with flexible covering radius. The model distinguishes between the costs and benefits of the closed hubs. Similar to the real-world situation, covering radius is assumed to be an endogenous parameter in which the amount is determined optimally. The proposed model helps the DM to design a transportation network by determining the established hubs, their covering radius and allocating ordinary nodes to the hubs. Also, multi period structure of the model enables the manager to involve changes of the problem parameters in the decision making process.

Table 1: Major contributions to the HSCP and HMCP

Article	number of periods	Major contribution	Solution approach
[9]	single period	Introduction of HSCP and HMCP	Exact
[17]	single period	New formulations for HSCP	Exact
[18]	single period	New formulations for HSCP	Exact
[19]	single period	New formulation and path relinking approach for HMCP	Heuristic
[20]	single period	Formulating an incomplete network for HSCP	Tabu search
[21]	single period	Assuming each hub as an M/M/C queue	ICA
[22]	single period	New formulations for HSCP and HMCP	Heuristics
[23]	single period	Q-coverage HSCP with mandatory dispersion	Exact
[24]	single period	Multimodal HSCP network	Heuristics
[25]	single period	New formulations for HMCP	Heuristics

[26]	single period	Time definite delivery network	Exact
[27]	single period	considering stochastic transportation time and a risk factor on the mean travel time	Multi Objective ICA (MOICA)
[28]	single period	Incomplete hub network and uncertain location of demand nodes	Variable Neighborhood Search (VNS)
[29]	single period	New formulations for HMCP and considering the problem under uncertain shipments	Non dominated sorting genetic algorithm-II (NSGA-II)
[30]	single period	partial coverage in HMCP	Exact
This article	Multi period	Flexible covering radius in HSCP	GA

The rest of this paper is organized as follows. Section 2 is dedicated to the proposed solution approaches. Initially, Section 2.1 presents a mixed integer mathematical model, and due to the computational complexity of the problem, Section 2.2 proposes a dynamic genetic algorithm to solve it in a reasonable time. Computational results are presented in Section 3 and conclusions and some guidelines for further studies are presented in Section 4.

2. Multi period HSCP with flexible covering radius

It is assumed that $N = \{1,2, \dots, n\}$ is the set of origin and destination nodes in the network. Each node is a potential location for establishing a hub. Let *i*, *j* be the indices for origin and destination nodes respectively, *k*, *l* be indices for the hubs and *t* be the index for periods. Furthermore, it is supposed that the costs matrix is symmetric, this means that: $c_{ij} = c_{ji}$. The hub network is assumed to be a complete graph, and each O/D pair is connected through one or two intermediate hubs. Also, there is no limitation on the capacity of the hub nodes. Ordinary nodes may be connected to one hub (single allocation) or more than one hub (multiple allocations) in which the former is investigated here. Each hub contains two kinds of costs: fixed establishment and covering costs. Covering costs of a hub are proportional to its covering radius, and the covering radius of the hub equates to the farthest node covered by the hub. Between hubs transportation is discounted considering the use of special facilities$(0 \leq \alpha \leq 1)$. Given an O/D pair, *i*, *j*, transportation cost (c_{tij}^{kl}) in period *t* equates to the sum of costs from *i* to hub *k*, hub *k* to hub *l*, considering the discount factor α, and hub *l* to *j*.

Whenever it happens, closed hubs are costly. However, the savings occur when there is a possibility to reuse the released movable facilities in newly established hubs. The following lemma is proposed to determine the number of possible movements in each period. It is assumed that after a hub is closed, movable facilities are transferred to one of the newly established hubs and are not capable of buffering for subsequent periods.

Lemma. Number of possible movements in each period equates to the minimum of total established hubs $(\sum_k p_{tk})$ and total closed hubs in that period $(\sum_k q_{tk})$.

Proof. Generally, in each period, there are three possible cases:

(a) The sum of established hubs is larger than the sum of closed hubs $(\sum_k p_{tk} > \sum_k q_{tk})$. In this case, it is possible to reuse the released movable facilities from all of the closed hubs. Hence, number of movements will be$\sum_k q_{tk}$.

(b) Number of established hubs equals to the number of closed hubs ($\sum_k p_{tk} = \sum_k q_{tk}$). In this case, movable facilities from each of closed facilities are allocated to one of the established facilities. Therefore, number of movements will be $\sum_k q_{tk}$ or$\sum_k p_{tk}$.

(c) Established hubs are less than closed hubs ($\sum_k p_{tk} < \sum_k q_{tk}$). Despite extra supply, the demand is limiting, and it is possible to use moving facilities from $\sum_k p_{tk}$ of closed hubs in newly established ones.

Accordingly, the number of possible movements in each period equates to the minimum of total established hubs and total closed hubs.

This section contains two parts, each of which presents a solution approach to the addressed problem. The first part is devoted to the suggested mixed integer model and the second part proposes a dynamic GA for solving the problem.

2.1 Proposed formulation

Despite simpler formulations for HSCP in the literature, such as the one proposed by Ernst and Krishnamoorthy [4], the developed mixed integer model is based on the formulation by Campbell [9] for the sake of clarity. The set of model parameters is as follows:

c_{tij}^{kl} Is the present value of total transportation cost from origin *i* to destination *j* via hubs *k* and *l* in period *t*.

ec_{tk} Is the present value of fixed hub establishment cost in node *k* in period *t*.

fr_{tk} Is the present value of covering cost of a hub at node *k* in period *t*.

cc_{tk} Is the present value of hub closure costs at node *k* in period *t*, including both static and movable facilities.

ms_t Is the present value of the benefits from movable facilities in a closed hub in period *t*.

d_{ik} Is the Euclidean distance from node *i* to hub *k*.

M Is a big number.

The set of decision variables in the model is as follows:

x_{tij}^{kl} A binary decision variable which is 1 if nodes *i* and *j* are connected via hubs *k* and *l* in period *t* and otherwise equals 0.

y_{tik} A binary variable which is 1 if node *i* is connected to hub *k* in period *t* and otherwise it is 0.

r_{tk} Is the covering radius of hub *k* in period *t*.

p_{tk} Is a binary variable which is 1 if a new hub is established at node *k* in period *t* and otherwise equals 0.

q_{tk} Is a binary variable which is 1 if the existing hub in node *k* is closed in period *t* and otherwise equals 0.

z_t Is the minimum of $\sum_k p_{tk}$ and $\sum_k q_{tk}$.

Considering the above explanations, the proposed mathematical model is as follows.

$$Min \sum_t \sum_i \sum_k \sum_l \sum_j c_{tij}^{kl} x_{tij}^{kl} + \sum_t \sum_k ec_{tk} p_{tk} + \sum_t \sum_k fr_{tk} r_{tk} + \sum_t \sum_k cc_{tk} q_{tk} - \sum_t ms_t z_t \quad (1)$$

$$\sum_k \sum_l x_{tij}^{kl} = 1 \qquad \forall i, j, t \quad (2)$$

$$2x_{tij}^{kl} \le y_{tjl} + y_{tik} \qquad \forall t, i, k, l, j \quad (3)$$

$$y_{tik} \le y_{tkk} \qquad \forall t, i, k \quad (4)$$

$$\sum_k y_{tik} = 1 \qquad \forall t, i \quad (5)$$

$$r_{tk} \ge d_{ik} y_{ik} \qquad \forall t, i, k \quad (6)$$

$$p_{tk} - q_{tk} = y_{tkk} - y_{t-1kk} \qquad \forall k,\ t > 1 \quad (7)$$

$$z_t = \min(\sum_k p_{tk}, \sum_k q_{tk}) \qquad \forall t \quad (8)$$

$$x_{tij}^{kl}, y_{tik}, p_{tk}, q_{tk} \in \{0,1\}, r_{tk} \ge 0, z_t \ge 0 \ \& \ \text{integer} \quad (9)$$

Expression (1) is the objective function of the proposed model which is aimed at minimizing the total costs. The first part of the objective function considers transportation costs from origin *i* to destination *j* via hubs *k* and *l*. The second part is dedicated to the hub establishment costs. Covering cost of each hub in each period is the third part of the objective function. Costs associated with the closed hubs in each period are the fourth part, and the benefits from movable facilities are calculated in the fifth part. Constraints (2) guarantee that each O/D pair is connected through one or two hubs. Constraints (3) ensure that the path from *i* to *j* via hubs *k* and *l* is established if both *i* and *j* are, respectively, connected to hubs *k* and *l*. Constraints (4) ensure that in each period, ordinary node *i* may be connected to *k* if it is set as a hub. Constraints (5) ensure that each node allocates to only one hub (single-allocation constraint). Covering radius equates to the distance between the hub and farthest ordinary node allocated to it, which is calculated in (6). According to (7), for a given node *k* in period *t* when a hub is newly established, binary variable p_{tk} equals 1 and binary variable q_{tk} equals 0 and when the existing hub in a node is closed, q_{tk} equals 1 and p_{tk} equals 0. Otherwise, both variables will be zero. Constraint (8) expresses the proposed *lemma*, upon which the number of possible movements in each period equates to the minimum number of established and closed hubs. Expression (9) specifies $x_{tij}^{kl}, y_{tik}, p_{tk}$ and q_{tk} as binary variables and r_{tk}, z_t as nonnegative variables.

The following set of linear constraints may be substituted with nonlinear equation (8). Referring to (10), v_t is the subtraction of $\sum_k q_{tk}$ from$\sum_k p_{tk}$. Using constraints (11) and (12) if v_t is negative, binary variable w_t will be 1, otherwise it is 0. Constraints (13) and (14) provide an upper bound for $v_t^{'}$. Accordingly, if $w_t = 1$, then $v_t^{'} \leq v_t$ and if $w_t = 0$, then $v_t^{'} \leq 0$. Therefore, considering equation (15), the upper bound of z_t is the minimum of $\sum_k q_{tk}$ and $\sum_k p_{tk}$. However, considering the utility of larger values of z_t in the objective function, it will attain the upper bound.

$$v_t = \sum_k p_{tk} - \sum_k q_{tk} \qquad \forall t \tag{10}$$

$$v_t \geq -M w_t \qquad \forall t \tag{11}$$

$$v_t < M(1 - w_t) \qquad \forall t \tag{12}$$

$$v_t^{'} - M(1 - w_t) \leq v_t \qquad \forall t \tag{13}$$

$$v_t^{'} \leq M w_t \qquad \forall t \tag{14}$$

$$z_t = v_t^{'} + \sum_k p_{tk} - v_t \qquad \forall t \tag{15}$$

$$w_t \in \{0,1\}, v_t, v_t^{'} \geq 0 \tag{16}$$

2.2 Proposed genetic algorithm

To proof NP-hardness of a given problem, it is a common practice to show that it is at least as hard as another proven NP-hard problem [31]. Kara and Tansel proved the NP-hardness of HSCP [17]. On the other hand, the problem investigated in this article may be simplified to the classic HSCP when the number of periods in the planning horizon is limited to one and the covering radius has a predetermined level. Therefore the problem discussed here, will be NP-Hard as well. The complexity of the problem leads to a high computational time for even medium and small-sized instances. To obtain suitable solutions in a reasonable computational time, a genetic algorithm (GA) is proposed for the investigated problem. GA is a meta-heuristic algorithm, based on Darwinians theory of evolution, first introduced by Holland [32]. GA transmits a set of solutions for consecutive iterations, namely population, and in each iteration, some new individuals are added to the population and some individuals with lower utility will be eliminated. This goes on until a predetermined stopping criterion is met.

GA has properties such as the chromosomes structure, initial population, selection strategies, genetic operators and stopping criteria, which determine the performance of the proposed algorithm. The following subsections describe each of these features for the proposed GA.

2.2.1 Chromosome structure

One of the most important specifications of the GA, with a noticeable effect on the efficiency of algorithm, is the chromosome structure. The proposed structure, presented in Figure 1, must capture all the features of the problem. In the first stratum of the proposed structure, each allele represents a node and its content determines the hub to which it is assigned, accordingly, self-assigned nodes are interpreted as hubs. Length of a chromosome equates to the number of nodes multiplied by the number of periods. Considering the difficulty of implementing the GA operators on the first stratum, the second stratum is introduced in which the hub nodes are appointed.

4	3	**3**	**4**	3	2	**2**	5	2	**5**	**1**	1	**3**	1	1
0	0	1	1	0	0	1	0	0	**1**	1	0	1	0	0
		period1					period2					period3		

Figure1. The chromosome structure

2.2.2 Initial population

GA is a population-based algorithm and permanently transmits a set of individuals to consecutive iterations. First, it is necessary to generate a set of solutions as initial population. To emphasize the importance of the population size, it is noticeable that extra-large size of the initial population results in trashy increase of computational time and small size of the population will cause the algorithm not achieving to optimal solution. Based on the parameters tuning results, initial population consists of 200 individuals. To create the population, some of the nodes are selected as hubs and the others are allocated to the nearest hubs.

2.2.3 Selection strategies

There are different methods to select parents for implementing crossover and mutation. Roulette wheel method, first introduced by Goldberg [33], is applied here. In this approach, after sorting all population members based on their fitness, each one will be allocated a selection probability proportional to its rank. In this situation, all the members of the population have the chance to be selected, although the chromosome with a better fitness is more likely to be chosen.

2.2.4 Genetic operators

Premature convergence to a non-optimal solution is probably the most serious problem encountered in GA [34]. The operators in GA are tools to avoid the premature convergence. Reproduction causes to add new individuals to the population which the characteristics are their parent’s patrimony. This phenomenon is presented in crossover operator. Scarcely, and due to disorder in structure, some of the individuals have salient differences with the others. Similar to the illustrious role of mutation in the human evolution, this operator also is very important for the GA in salvation from local optima. A phenomenon that many human societies are faced with, is the immigration. Inspired from the real world populations, the proposed algorithm applies the immigration as a GA operator.

Crossover is the basic operator of GA for combining two chromosomes [35]. There are multiple methods for implementing crossover. Here a modified version of single-point crossover is proposed. It is noteworthy that the investigated problem is a multi-period one; therefore, single-point crossover might not be suitable. For more efficiency a k-point crossover is proposed, where k is the number of periods. Each cut point is located randomly within one of the periods. Crossover on the first stratum might result in an infeasible solution, therefore the second stratum of the chromosome is used to do a crossover. Crossover on the second stratum of a hypothetical chromosome is presented in Figure 2.

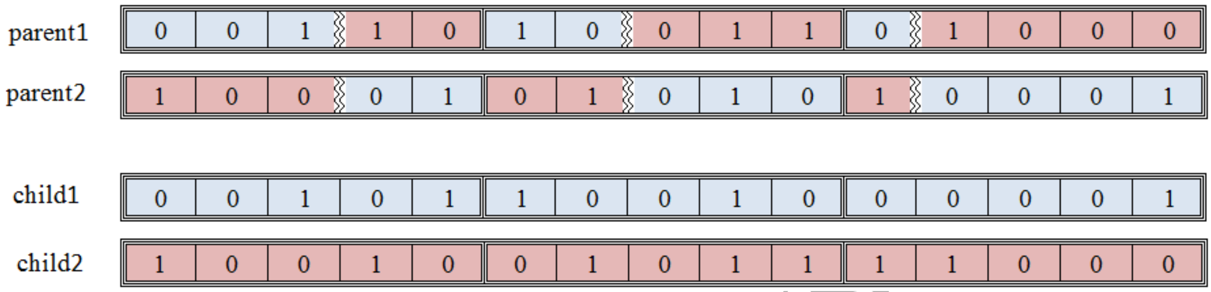

Figure2. Crossover on the second stratum of a chromosome with four periods

Mutation is the second operator of GA. When the solution remains unchanged for consecutive iterations, the algorithm might be trapped in local optima. This operator prevents the algorithm from trapping in local optima by the exploring solution space [36]. The proposed mutation operator is conducted on the second stratum of the chromosome. To mutate a chromosome, some of the genes are selected randomly. If the selected node is a hub, it is changed to an ordinary node. Otherwise it is altered to a hub. Figure 3 shows mutation on a hypothetical chromosome with three periods in which 4 nodes are mutated. One of the possible occurrences is to make a period without any hubs, like the third period in Figure 3. In this case, the same number of the genes are selected randomly to be hubs.

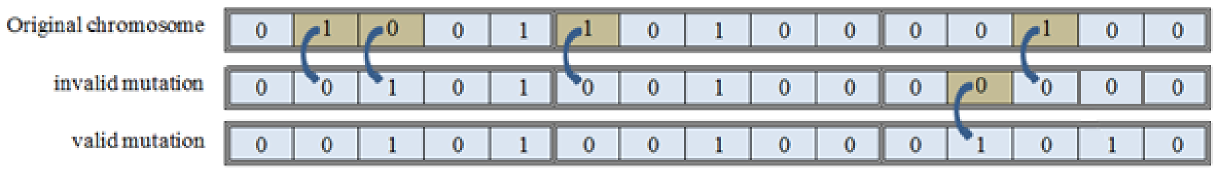

Figure3. The mutation operator

Although increasing the number of changes in a selected chromosome for mutation causes the increment of computational time, more changes provide better search in solution space. Proposed dynamic mutation operator increases the number of changes in a selected chromosome for mutation along with increasing the number of iterative solutions. The number of genes that are remodeled in a mutant (nm) is presented in (17).

$$nm = \begin{cases} \left\lceil \frac{nnode}{5} \right\rceil & \text{if IS} < \frac{\max it}{3} \\ \left\lceil \frac{nnode}{3} \right\rceil & \text{if IS} < \frac{\max it}{3} \\ \left\lceil \frac{nnode}{2} \right\rceil & \text{if IS} > \frac{\max it}{2} \end{cases} \quad (17)$$

Where nm is the number of remodeled genes in a chromosome, $nnode$ is the number of origins and destinations, IS is the number of the iterative solutions in which the best solution found by the algorithm remains unchanged, and $maxit$ is the maximum number of iterations.

The third operator introduced here is called immigration. It is assumed that there are some immigrants to the society in each period. In real-world situation, alongside with the economic and scientific growth of a society, general tendency of the people from other populations increases to immigrate to the society, similarly the designed dynamic operator increases immigration rate with increasing the probability of achieving the global optima, which the sign is a fixed solution for consecutive iterations. Increasing the immigration rate, as well as the mutation rate, increases the algorithm's capability to avoid local optima. Similar to the initial population, immigrants are created randomly and the rate of immigration is presented in (18).

$$IM_p = \frac{IS}{n_{pop}} \quad (18)$$

Where IS is the number of consecutive iterations in which the best solution remains unchanged, n_{POP} is the number of individuals in the population and IM_p is the immigration rate.

2.2.5 Stopping criteria

Various criteria have been introduced to stop the GA computational processes. The maximum number of iterations is the one most widely used as the stopping criteria. In some cases, the algorithm attains the optimum solution in primal iterations and remains unchanged until the last iteration. There are two possibilities: first, the algorithm is trapped in a locally optimal solution. In this case, as described in Section 2.2.4, the designed algorithm tries to escape the trap by increasing the severity of search in solution space with the aid of intensifying the number of permutations in a mutant and increasing the immigration rate. Second, the possibility is that the algorithm has reached optimum solution; in this case, it is ideal to stop the algorithm immediately. To reduce the computational time in the latter case, another stopping criterion is utilized alongside with the maximum number of iterations. Provided that the best solution remains unchanged for $\frac{maximum\ iterations}{2}$, the algorithm will be terminated.

To aggregate the above explanations, the flowchart of the proposed GA is presented in Figure 4.

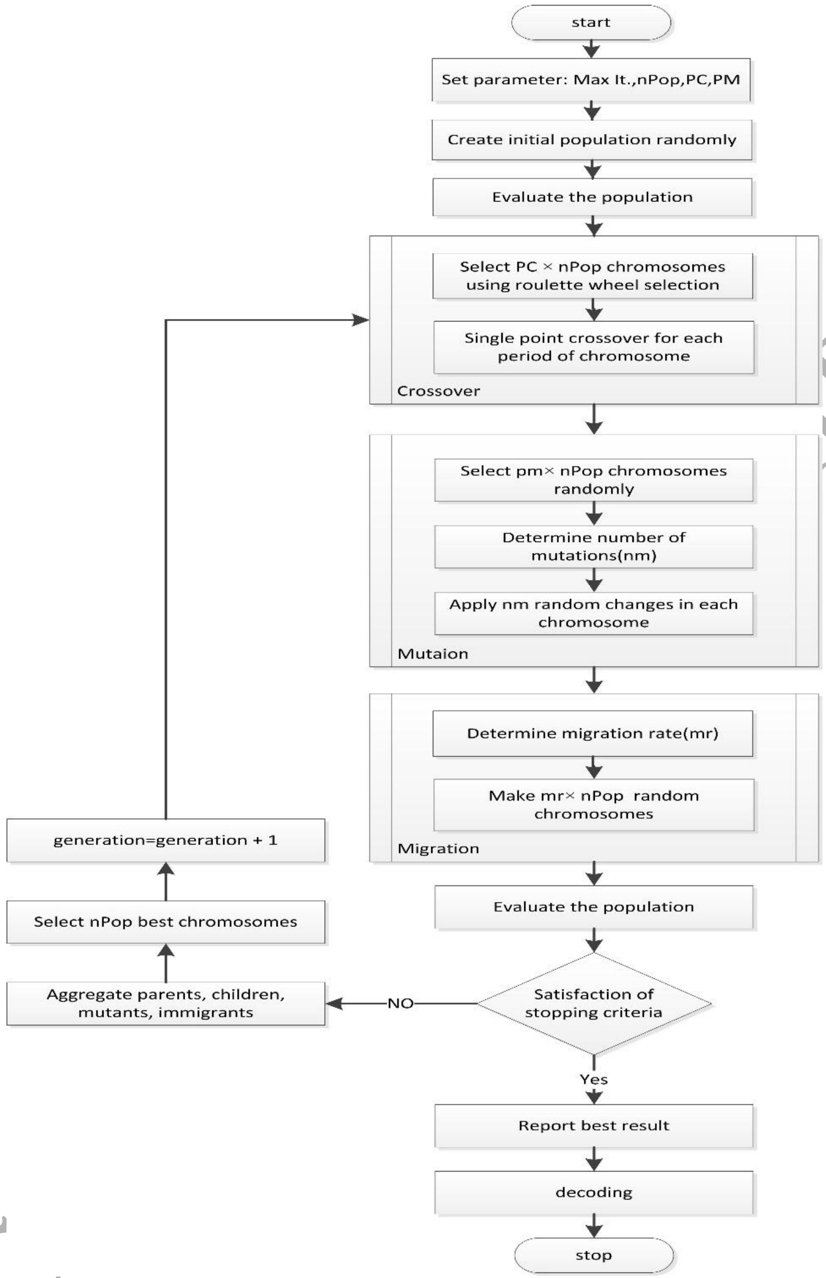

Figure 4. Flow chart of the proposed GA

3. Numerical experiments

In this section, numerical examples are conducted to evaluate the developed mathematical model and the proposed GA. To discuss the main outcomes of the mathematical model, a real world case study is presented in Section 3.1. Also, to analyze efficiency of the proposed GA, the results are compared with the original GA and ICA. The proposed GA differs from the original GA in the immigration operator, dynamic mutation, and the stopping criteria. To analyze the effects of these features, the results of proposed GA are compared with the results of the original GA. Parameters calibration and the obtained results are presented in Section 3.2, and the computational results are provided in Section 3.3.

Proposed GA, original GA and ICA are coded and implemented in MATLAB R2011b running on a system with 4 GB of RAM and core i5 CPU. Furthermore, the optimal solutions are obtained by GAMS 22.2 using CPLEX solver.

3.1 Case study

It is estimated that $40-100 billion is paid annually to keep Iranians, supplied with cheap energy, water, fuel, and basic food [37]. Thereupon the government has devised a multistep plan, namely the targeted subsidiary plan, to cut the subsidies. During the first phase of the plan in 2010, fuel prices had a 400% enhancement, upon which a noticeable increase in the transportation costs happened.

The investigated case is a cargo delivery firm which serves in 11 provinces. The problem is to design a hub network under the described price increases. Initially the plan was implemented in 5 pilot provinces and after 6 months, it was held nationwide. The first period (P_1) refers to the full subsidy prices before starting the plan. Intensity of transportation (tons per day) and unit transportation cost of the first period, are presented in the upper and lower diagonal of Table 2 respectively. During the second period (P_2), subsidies are cut in 5 provinces, highlighted provinces in Figure 5 (b), and finally the third period (P_3) represents free prices. Transportation costs for the second and third period are provided in the upper and lower diagonal of Table 3 respectively. Referring to the World Bank statistics, inflation rate has moved from 10.1% in the first period to 20.6% in the second and third period [38], upon which the related costs are inflated in Table 4. Also due to the possibility of selling surplus land and reusing movable facilities, it is assumed that in a closed hub, 60% of the costs are retrievable (closure benefits). The discount factor (α) and covering cost (*fr*) are assumed to be 0.4 and 10 dollars per kilometer respectively.

GAMS 22.2 is applied to solve the problem. Table 5 presents the established hubs, their covering radius and the ordinary nodes allocated to each one. The total costs of the designed network is $6774117 from which $4892841 is the total transportation costs and $1820576 is the hub established cost. Also the designed network is represented in Figure 5. Considering the dynamic nature of the problem, covering radius of a hub might be changed in each period. According to Table 5, the covering radius of Shiraz is 1100 kilometers in the first period (the distance between Shiraz and Zahedan) while its covering radius in the second and third period is 659 kilometers (the distance between Shiraz and Ahwaz).

Table 2: Intensity of transportation and unit transport costs for P_1

	Isfahan	Shiraz	Yazd	Kerman	Tehran	Bushehr	Bandar Abbas	Mashhad	Zahedan	Tabriz	Ahwaz
Isfahan	0	110	65	60	130	55	75	80	40	70	65
Shiraz	131	0	100	110	150	85	70	60	45	60	75
Yazd	82	116	0	100	110	40	55	60	40	40	35
Kerman	180	156	98	0	130	60	90	85	80	50	35
Tehran	120	252	185	283	0	70	50	130	40	120	85
Bushehr	158	83	198	239	335	0	55	45	30	35	65
Bandar Abbas	266	169	179	132	364	201	0	60	65	40	50

Mashhad	333	375	251	242	244	449	375	0	70	35	30
Zahedan	325	300	243	144	427	383	200	259	0	25	30
Tabriz	283	415	348	446	163	425	527	407	591	0	65
Ahwaz	203	180	295	335	238	132	349	482	480	293	0

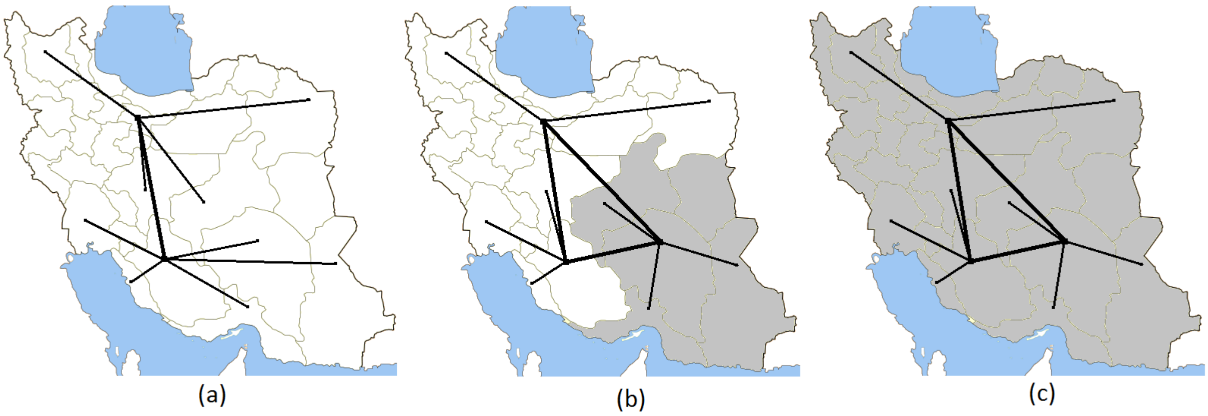

Figure 5. A graphical view of the designed hub network: (a) the first period, (b) the second period, and (c) the third period

Table 3: Transportation costs for P_2 and P_3

	Isfahan	Shiraz	Yazd	Kerman	Tehran	Bushehr	Bandar Abbas	Mashhad	Zahedan	Tabriz	Ahwaz
Isfahan	0	131	139	303	120	158	399	333	526	283	203
Shiraz	262	0	174	311	252	83	353	375	580	415	180
Yazd	164	232	0	197	341	297	358	377	485	522	383
Kerman	361	311	197	0	495	358	235	364	260	670	503
Tehran	239	504	369	566	0	335	546	244	641	163	238
Bushehr	316	166	396	477	670	0	302	539	574	425	132
Bandar Abbas	532	388	358	265	728	403	0	562	400	633	523
Mashhad	667	749	503	485	488	899	749	0	389	407	482
Zahedan	649	600	485	289	855	766	400	519	0	886	720
Tabriz	566	831	696	893	327	851	1054	814	1181	0	293
Ahwaz	406	359	590	671	477	265	697	964	959	586	0

Table 4: Periodic hub establishment cost, closure cost and closure benefits

Cities	Hub establishment costs			Closure costs			Closure benefits		
	P_1	P_2	P_3	P_1	P_2	P_3	P_1	P_2	P_3
Isfahan	1525218	1679265	2025194	610087	671706	810078	915131	1007559	1215116
Shiraz	491094	540694	652077	196437	216278	260831	294656	324416	391246

Yazd	752764	828793	999524	301105	331517	399810	451658	497276	599714
Kerman	463818	510664	615861	185527	204266	246344	278291	306398	369517
Tehran	818818	901519	1087232	327527	360608	434893	491291	540911	652339
Bushehr	546364	601547	725466	218546	240619	290186	327819	360928	435279
bandar Abbas	636364	700636	844967	254545	280255	337987	381818	420382	506980
Mashhad	765455	842766	1016376	306182	337106	406550	459273	505660	609825
Zahedan	446727	491847	593167	178691	196739	237267	268036	295108	355900
Tabriz	709091	780709	941535	283636	312284	376614	425455	468425	564921
Ahwaz	538188	592545	714610	215275	237018	285844	322913	355527	428766

Table 5. The designed hub network

Periods	Established hubs	Covering radius (Km)	Allocated cities
First period	Tehran	894	Mashhad, Tabriz, Isfahan, Yazd
	Shiraz	1100	Ahwaz, Bushehr, Bandar Abbas, Kerman, Zahedan
Second period	Tehran	894	Mashhad, Tabriz
	Kerman	529	Zahedan, Yazd, Bandar Abbas
	Shiraz	659	Ahwaz, Bushehr, Isfahan
Third period	Tehran	894	Mashhad, Tabriz
	Kerman	529	Zahedan, Yazd, Bandar Abbas
	Shiraz	659	Ahwaz, Bushehr, Isfahan

3.2 Parameters setting

Parameter calibration plays an important role in the efficiency of metaheuristic algorithms. In this article Taguchi method is applied for parameters tuning. To reduce the number of experiments, this method proposes a fractional factorial experiment instead of full factorial experiments. Taguchi method divides the factors in to signal (or controllable) and noise factors. The method tries to minimize the effect of noise and determines the optimal level of the signal factors [39]. For a comprehensive study on the Taguchi method, the readers are referred to works of Peace [40] and Taguchi et al. [41].

Based upon the previous experiments, each parameter (or factor) is assigned three levels. Table 6 presents the parameters, their abbreviations, and the intended levels. The parameters are analyzed with an L_9 design and each experiment is run 5 times. Considering the minimization nature of the objective function, a lower response level is more desirable. Accordingly the signal-to-noise (S/N) ratio is calculated as follows.

$$S/N = -10\log\left(\frac{1}{n}\sum_{i=1}^{n} y_i^2\right) \tag{19}$$

Where y_i is the response in the ith replication and n is the number of replications in experiments. L_9 orthogonal array and the obtained responses are provided in Tables 7 – 9. Mean S/N ratios for each algorithm are presented in Figure 6. Notably, a larger value of the S/N ratio is more desirable. Referring to Figure 5, in the proposed GA, parameters Maxit, N_P, P_m, and P_c should be set at levels 3, 2, 1, and 3 respectively. Selected parameters for each of the algorithms are shown in Table 10.

Table 6: GA and ICA parameters

Algorithm	Parameters	Factor levels		
		1	2	3
Proposed GA and original GA	Maximum number of iterations (Maxit)	150	200	250
	Population size (N_P)	100	200	300
	Mutation rate (P_m)	0.10	0.15	0.20
	Crossover rate (P_c)	0.70	0.80	0.85
ICA	Number of imperialists (N_I)	5	9	13
	Number of colonies (N_C)	100	150	200
	Revolution rate (P_R)	0.05	0.10	0.15
	Assimilation rate (P_A)	0.70	0.75	0.80

Table 7: L_9 design and experimental results of the proposed GA

Experiment	Factor				y_1	y_2	y_3	y_4	y_5
	Maxit	N_P	P_m	P_c					
1	150	100	0.1	0.70	786930	821359	756482	746721	772194
2	150	200	0.15	0.80	764110	763232	720482	793213	828401
3	150	300	0.20	0.85	829630	825843	825362	803415	741022
4	200	100	0.15	0.85	795927	742226	773972	843124	734212
5	200	200	0.20	0.70	790622	752289	842173	790231	831212
6	200	300	0.10	0.80	819595	763389	762131	734819	799121
7	250	100	0.20	0.80	741352	849531	805271	832105	791203
8	250	200	0.10	0.85	744732	815280	793119	740336	732261
9	250	300	0.15	0.70	733221	774992	773310	809921	769671

Table 8: L_9 design and experimental results of original GA

Experiment	Factor				y_1	y_2	y_3	y_4	y_5
	Maxit	N_P	P_m	P_c					
1	150	100	0.1	0.70	846527	815371	802415	778101	782748
2	150	200	0.15	0.80	794862	774251	793102	742101	843640
3	150	300	0.20	0.85	869516	791756	813310	788289	804374
4	200	100	0.15	0.85	795116	795312	832844	766190	783027
5	200	200	0.20	0.70	790481	805379	835491	800918	750294
6	200	300	0.10	0.80	819595	753321	861034	780215	784532
7	250	100	0.20	0.80	851785	868316	794721	818634	763386
8	250	200	0.10	0.85	814353	826642	790306	794972	864362
9	250	300	0.15	0.85	793965	752913	762316	824402	805385

Table 9: L_9 design and experimental results of ICA

Experiment	Factor				y_1	y_2	y_3	y_4	y_5
	N_I	N_C	P_R	P_A					
1	5	100	0.05	0.70	763271	795482	774216	804372	794352
2	5	150	0.10	0.75	785329	845531	746525	785582	804532
3	5	200	0.15	0.80	774236	793412	852542	766302	814912
4	9	100	0.10	0.80	846423	831236	809241	778901	823282
5	9	150	0.15	0.70	735872	852352	788715	821026	763251
6	9	200	0.05	0.75	773478	774372	769482	809423	793193
7	13	100	0.15	0.75	795456	740231	743626	802320	763182
8	13	150	0.05	0.80	805386	860193	737641	744336	753419
9	13	200	0.10	0.70	748756	798442	794821	821763	748093

Table 10: Tuned parameters for the proposed GA, original GA, and ICA

Parameter	Original GA	Proposed GA	Parameter	ICA
Maxit	200	250	N_I	13
N_P	300	200	N_C	200
P_m	0.15	0.10	P_R	0.15
P_c	0.7	0.85	P_A	0.80

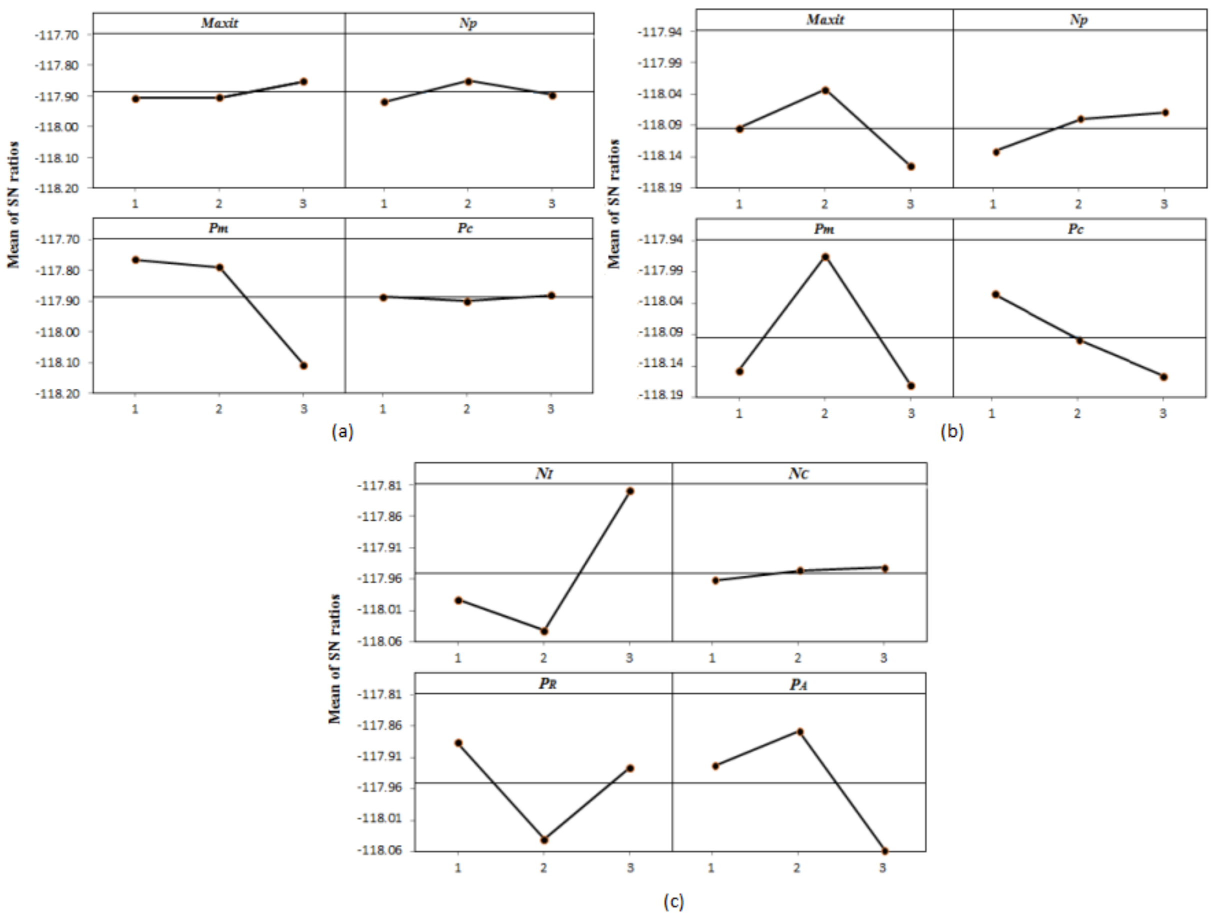

Figure 6. Mean S/N ratios for the proposed GA (a), original GA (b), and ICA (c)

3.3 Computational results

The results of proposed GA are compared with the results of the mathematical model, original GA, and ICA. For this purpose, some numerical examples are solved. In the designed problems, fixed hub establishment cost, ranges uniformly in [7000, 12000] and the closure benefits have a uniform distribution in [4000, 7000]. Costs and benefits might change periodically with rate $1 + \alpha$, in which α has a uniform distribution in [-0.05, 0.15]. Closure costs are uniformly distributed in [1000, 4000]. Covering costs are proportionate to the covering radius. Distances have uniform distribution in [33, 99], and the covering cost equates to the distance multiplied by 10. Closure costs change periodically with the rate $1 + \gamma$, in which γ is uniformly distributed in [-0.2, 0.8]. Numerical experiments have 5, 10, 15, 20, 25, 30, 40, 60, 80, and 100 nodes. The discount factor (α) equates to 0.3, 0.6 and the number of periods (t) is 2, 3 and 4. In Tables 11 and 12, each sample problem is denoted as "a – b – c" where "a" is the number of periods, "b" is the number of nodes, and "c" is the discount factor.

Objective value and computational time for the proposed mixed integer model and the GA are presented in Table 11. Also, the table determines the total established hubs (TEH), total closed hubs (TCH) and number of facility movements (NM) during the planning horizon. As shown in Table11, in the experiments with 5 nodes, mathematical model performs faster than the proposed GA; however, by increasing the problem size, its computational time has an exponential growth. Figure 7 compares the growth in computational time of the mathematical model with the proposed GA. It is obvious from the table that along with increasing number of periods (t), computational time increases in both GAMS and GA solutions. Referring to Table 11, discount factor α does not have a meaningful effect on the computational time, whereas the objective value increases with increasing the discount factor in most cases. Also, according to Table 12 the proposed GA is superior to both the original GA and the ICA in all of the experiments.

Furthermore, the proposed GA provides high-quality solutions. For problems in which the optimal solution was found in a reasonable time, the proposed GA attains the optimum solution in all cases (Table 11). The proposed GA is compared with the original GA and ICA in Table 12. The relative gap between the optimal solutions and the solutions obtained by the original GA and ICA is presented in Table 12. For instances with 25 nodes and less, the calculations are:

$$\frac{Obj_{heuristic} - Obj_{optimal\ solution}}{Obj_{optimal\ solution}} \times 100 \tag{20}$$

And for the problems with more than 30 nodes (30, 40, 60, 80 and 100); it is calculated as bellow.

$$\frac{Obj_{heuristic} - Obj_{proposed\ GA}}{Obj_{proposed\ GA}} \times 100 \tag{21}$$

The results confirm the superiority of the proposed GA to the ICA and to the original GA in all instances with regard to the relative gap.

In addition to the computational time, the number of function evaluations (NFE) is a well-known performance indicator for the algorithms. Unlike the original GA, the proposed GA is a dynamic algorithm in which the number of iterations and the number of evaluated chromosomes are not predetermined. The original GA evaluates the fitness function 51300 times in each experiment; however, computational results in Table 12 indicate a smaller NFE (with the average 30448) for the proposed GA in all of the instances.

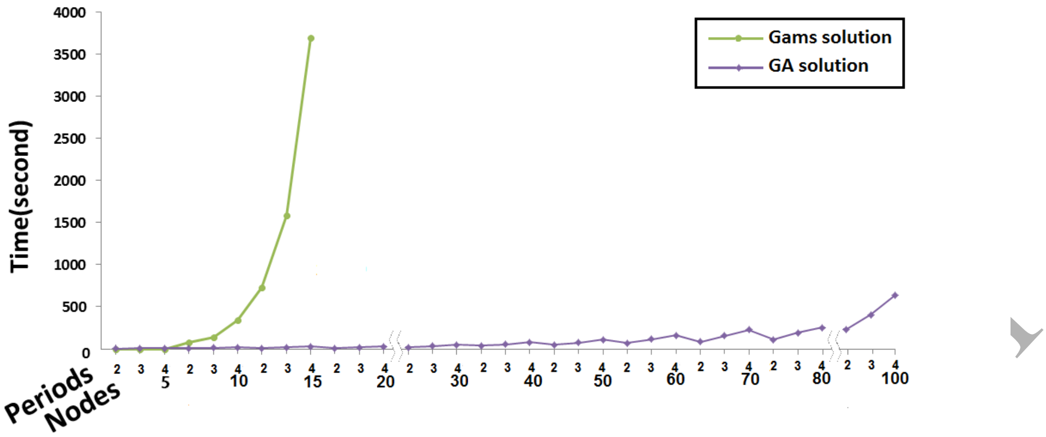

Figure 7. Growth of computational time

Table 11: GAMS output versus the proposed GA

Problem	GAMS					Proposed GA	
	Obj.value	CPU (s)	TEH	TCH	NM	Obj.value	CPU (s)
2 – 5 – 0.3	10232	0.97	1	0	0	10232	11.36
2 – 5 – 0.6	10520	0.97	1	0	0	10520	12.46
3 – 5 – 0.3	19212	1.03	1	0	0	19212	13.21
3 – 5 – 0.6	19212	1.03	1	0	0	19212	12.57
4 – 5 – 0.3	23047	1.52	1	0	0	23047	17.64
4 – 5 – 0.6	23047	1.51	1	0	0	23047	16.81
2 – 10 – 0.3	29249	82.84	2	0	0	29249	14.62
2 – 10 – 0.6	29249	82.31	2	0	0	29249	15.19
3 – 10 – 0.3	41457	150.29	2	1	1	41457	18.26
3 – 10 – 0.6	41457	150.40	2	1	1	41457	19.18
4 – 10 – 0.3	47137	345.95	2	1	0	47137	21.58
4 – 10 – 0.6	51320	346.04	2	1	1	51320	18.37
2 – 15 – 0.3	55751	737.18	3	1	0	55751	14.28
2 – 15 – 0.6	55751	738.28	3	1	0	55751	12.56
3 – 15 – 0.3	80985	1593.28	4	0	0	80985	21.15
3 – 15 – 0.6	84009	1592.03	5	1	1	84009	22.96
4 – 15 – 0.3	108187	3698.35	4	1	1	108187	28.51
4 – 15 – 0.6	107708	3699.50	3	0	0	107708	25.67
2 – 20 – 0.3	67456	5167.73	3	0	0	67456	15.07
2 – 20 – 0.6	91847	5168.09	3	1	1	91847	14.51
3 – 20 – 0.3	123816	9484.21	4	1	1	123816	22.80
3 – 20 – 0.6	138235	9485.34	3	0	0	138235	20.53
4 – 20 – 0.3	149148	14457.29	4	2	1	149148	29.44
4 – 20 – 0.6	164548	14458.63	5	1	1	164548	26.07
2 – 25 – 0.3	115761	24349.66	4	0	0	115761	19.36
2 – 25 – 0.6	157875	24350.88	4	1	1	157875	13.43
3 – 25 – 0.3	177674	36640.75	5	1	1	177674	21.77
3 – 25 – 0.6	235884	36639.48	4	1	0	235884	23.60
4 – 25 – 0.3	218061	47259.89	5	1	0	218061	37.09
4 – 25 – 0.6	264815	47261.63	6	2	1	264815	31.85

Table 12: Computational results for the Proposed GA, original GA and ICA

problem	proposed GA			Original GA			ICA		
	Obj.value	CPU(s)	NFE	Obj.value	CPU(s)	relative gap	Obj.value	CPU(s)	relative gap
2-5-0.3	10232	11.36	28096	10232	13.52	0.000	10232	12.44	0.000
2-5-0.6	10520	12.46	30854	10520	15.01	0.000	10520	13.85	0.000
3-5-0.3	19212	13.21	24051	19212	15.57	0.000	19212	14.38	0.000
3-5-0.6	19212	12.57	24051	19212	15.46	0.000	19212	12.85	0.000
4-5-0.3	23047	17.64	24051	23047	19.70	0.000	23047	18.13	0.000
4-5-0.6	23047	16.81	24051	23067	19.50	0.001	23047	19.90	0.000
2-10-0.3	29249	14.62	24051	29249	15.64	0.000	29249	14.37	0.000
2-10-0.6	29249	15.19	24051	29249	15.55	0.000	29351	16.17	0.003
3-10-0.3	41457	18.26	24051	41497	21.56	0.001	41467	19.92	0.000
3-10-0.6	41457	19.18	24432	41457	21.91	0.000	41457	17.56	0.000
4-10-0.3	47137	21.58	25581	47217	29.89	0.002	47137	27.88	0.000
4-10-0.6	51320	18.37	25387	51320	29.40	0.000	51320	26.88	0.000
2-15-0.3	55751	14.28	24241	55794	20.06	0.001	55813	20.43	0.001
2-15-0.6	55751	12.56	24432	55751	19.78	0.000	55751	17.94	0.000
3-15-.0.3	80985	21.15	25771	80985	29.80	0.000	80985	30.26	0.000
3-15-0.6	84009	22.96	29476	84367	36.69	0.004	84213	32.87	0.002
4-15-0.3	108187	28.51	26929	108251	40.09	0.001	108187	40.87	0.000
4-15-0.6	107708	25.67	27706	107812	40.81	0.001	107708	36.82	0.000
2-20-0.3	82997	15.07	25581	83151	21.18	0.002	82997	21.65	0.000
2-20-0.6	91847	14.51	25197	92413	20.54	0.006	91847	20.88	0.000
3-20-0.3	123816	22.80	27317	123816	32.12	0.000	123816	28.69	0.000
3-20-0.6	138235	20.53	25197	138682	29.05	0.003	138912	29.60	0.005
4-20-0.3	149148	29.44	27317	149268	41.77	0.001	149261	42.48	0.001
4-20-0.6	164548	26.07	27906	164641	42.48	0.001	164712	32.74	0.001
2-25-0.3	115761	19.36	27706	115812	27.53	0.000	115825	27.96	0.001
2-25-0.6	157875	13.43	25966	158123	22.20	0.002	157961	19.49	0.001
3-25-0.3	177674	21.77	25771	177691	36.38	0.000	177721	31.54	0.000
3-25-0.6	235884	23.60	25197	236441	33.69	0.002	237215	34.28	0.006
4-25-0.3	218061	37.09	27516	218112	53.53	0.000	218214	53.82	0.001
4-25-0.6	264815	31.85	26352	264815	45.62	0.000	264921	39.76	0.000
2-30-0.3	177775	21.95	26352	177775	31.82	0.000	179456	31.93	0.009
2-30-0.6	194652	21.70	25966	195145	31.47	0.003	194923	31.59	0.001
3-30-0.3	263619	29.28	27906	263862	50.08	0.001	264813	36.76	0.005
3-30-0.6	280087	29.23	28297	282412	49.80	0.008	282681	42.24	0.009
4-30-0.3	346700	38.57	28879	347812	66.50	0.003	346963	56.41	0.001

problem	proposed GA			Original GA			ICA		
	Obj.value	CPU(s)	NFE	Obj.value	CPU(s)	relative gap	Obj.value	CPU(s)	relative gap
4-30-0.6	392356	44.54	28879	393412	65.02	0.003	396065	64.97	0.009
2-40-0.3	282178	35.73	30457	284721	53.03	0.009	285931	44.18	0.013
2-40-0.6	322915	31.01	27706	324211	45.96	0.004	326201	45.33	0.010
3-40-0.3	422745	50.62	30664	424612	76.08	0.004	426117	74.10	0.008
3-40-0.6	475376	48.16	29871	486449	72.12	0.023	492219	70.49	0.035
4-40-0.3	545107	69.79	32452	556853	104.29	0.022	553014	102.22	0.015
4-40-0.6	665146	52.42	29666	673391	93.82	0.012	672251	76.86	0.011
2-60-0.3	602728	64.23	35097	604214	100.66	0.002	602728	94.58	0.000
2-60-0.6	716929	45.50	30664	720618	86.04	0.005	717351	67.01	0.001
3-60-0.3	874574	82.25	37342	879074	157.15	0.005	876312	121.19	0.002
3-60-0.6	1035040	68.98	32452	1041215	132.30	0.006	1044825	83.66	0.009
4-60-0.3	1027456	111.01	38171	1029317	214.72	0.002	1031031	163.69	0.003
4-60-0.6	1371700	101.40	35916	1394694	198.23	0.017	1412621	149.55	0.030
2-80-0.3	1019915	97.23	35287	1026321	163.35	0.006	1023379	143.90	0.003
2-80-0.6	1205921	89.36	33661	1264438	152.13	0.049	1242688	132.26	0.030
3-80-0.3	1255795	130.54	40071	1282256	282.01	0.021	1315623	193.28	0.048
3-80-0.6	1611591	134.88	41337	1619883	288.00	0.005	1619883	199.73	0.005
4-80-0.3	1432306	168.63	40071	1521743	361.40	0.062	1456117	249.88	0.017
4-80-0.6	1905039	186.51	44086	1924841	401.29	0.010	2025216	276.37	0.063
2-100-0.3	1401816	194.25	41337	1519029	345.69	0.084	1483068	287.98	0.058
2-100-0.6	1776351	196.82	40914	1787881	355.20	0.006	1793144	232.11	0.009
3-100-0.3	1730317	264.67	44086	1820220	606.34	0.052	1798545	392.52	0.039
3-100-0.6	2308048	337.37	43896	2426014	606.80	0.051	2466252	500.41	0.069
4-100-0.3	2129941	525.71	43039	2185312	960.61	0.026	2243671	617.49	0.053
4-100-0.6	2657675	418.39	44086	2718275	779.77	0.023	2732799	620.79	0.028

4. Conclusion

The article proposes a multi-period HSCP with flexible covering radius. There are many real-world applications such as telecommunication network or cargo delivery systems in which the covering radius is a flexible parameter rather than a fixed one. The proposed model assumes a fixed hub establishment and variable covering cost in which the latter is proportional to the area covered by the hub. Furthermore, to exert the changes in problem parameters during the planning horizon, a dynamic model is proposed. Facilities are divided into movable and static, and the savings from released movable facilities as well as the closure costs for static ones are taken in to account. Considering the computational complexity of the problem, an effective GA is proposed to solve it. The proposed GA benefits from dynamic migration and mutation operators which helped the algorithm to avoid trapping in local optima. The algorithm is compared with the original GA and ICA. The results showed that the proposed GA outperforms ICA and original GA in the quality of the obtained solutions. Also the proposed GA is superior to the applied optimization toolbox (GAMS) and original GA in computational time.

The dynamic model proposed here, assumes that the savings resulting from movable facilities are the same for all hubs in a period. An interesting further research direction is to assume that the debated savings are variable for different hubs due to their sizes and other features.

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