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## Application of fuzzy TOPSIS in evaluating sustainable transportation systems

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## ABSTRACT

Sustainable transportation systems are the need of modern times. There has been an unexpected growth in the number of transportation activities over years and the trend is expected to continue in the coming years. This has obviously associated environmental costs like air pollution, noise, etc. which is degrading the quality of life in modern cities. To cope us this crisis, municipal administrations are investing in sustainable transportation systems that are not only efficient, robust and economical but also friendly towards environment. The challenge before the transportation decision makers is how to evaluate and select such sustainable transportation systems. In this paper, we present a multicriteria decision making approach for selecting sustainability transportation systems under partial or incomplete information (uncertainty). The proposed approach comprises of three steps. In step 1, we identify the criteria for sustainability assessment of transportation. In step 2, experts provide linguistic ratings to the potential alternatives against the selected criteria. Fuzzy TOPSIS is used to generate aggregate scores for sustainability assessment and selection of best alternative. In step 3, sensitivity analysis is performed to determine the influence of criteria weights on the decision making process. A numerical illustration is provided to demonstrate the applicability of the approach.

The strength of the proposed work is its practical applicability and the ability to generate good quality solutions under uncertainty.

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## 1. Introduction

Sustainability is an important subject for modern transportation decision makers. There have been numerous discussions on how sustainability can be accurately defined and measured. The Brundtland Commission (United Nations World Commission on Environment & Development, 1987) defines sustainability as development that meets the needs of the present without compromising the ability of future generations to meet their own needs. World Bank (1996), Loo (2002) and Schipper (2003) use the “triple bottom line” of economic, environmental, and social equity to define sustainability. Using these definitions, sustainable transportation can be considered as one that is able to meet today's transportation needs without compromising the ability of future generations to meet their transportation needs (Black, 1996; Richardson, 2005). Examples of sustainable transportation are energy efficient vehicles, vehicle with clean fuels like biodiesel, electricity, etc., carsharing, park-and-ride, etc. The center for sustainable

transportation (1997) defines sustainable transportation system as one that:

- Allows the basic access needs of individuals and societies to be met safely and in a manner consistent with human and ecosystem health, and with equity within and between generations.
- Is affordable, operates efficiently, offers choice of transport mode and supports a vibrant economy.
- Limits emissions and waste within the planet's ability to absorb them, minimizes consumption of non-renewable resources, limits consumption of renewable resources to the sustainable yield level, reuses and recycles its components, and minimizes the use of land and the production of noise.

In order to identify, compare and select sustainable transportation system, efficient decision making approaches are required. The commonly used approaches for sustainability evaluation can be classified into the following categories:

1. Life cycle analysis (LCA): Originally developed for industrial processes, the use of LCA (Goedkoop, 2000; Guine, 2002) to evaluate the environmental impact of transport system is growing. Its central concept is to combine, in a small number of

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- criteria, the polluting emissions and resources used during the life course of a product. This method has been the subject of considerable efforts to standardize the impacts assessment and the results interpretation. However, it does not take into consideration for example social aspects, so this method does not relate to our application.
2. Cost-benefit analysis (CBA) and cost-effectiveness analysis (CEA): The cost-benefit analysis is based on taking into account the monetary equivalent of all the positive and negative impacts of the company project. When the advantages of a project are not quantifiable in a monetary way or when the realization degree of the result to reach is given, cost effectiveness analysis is used. This type of study aims to minimize the costs necessary to the achievement of a given objective and not for maximizing the advantages. With CBA and CEA approaches (Kunreuther, Grossi, Seeber, & Smyth, 2003), it is extremely difficult to estimate directly external and social costs (e.g. air pollution, noise pollution, accidents, congestions and fuel costs). For example, air pollution costs, related to transportation, are difficult to estimate because several other economy sectors also generate pollution. In this case the environmental costs should be apportioned according to each sector emission. Applications of cost-benefit analyses for sustainable transportation can be found in El-Diraby, Abdulhai, and Pramod (2005) and Jonsson (2008).
  3. Environmental impact assessment (EIA): The aim of this method is to assess the environmental impacts of a new localized pollution source, such as an industry or highway, and its surroundings (Bond, Curran, Kirkpatrick, & Lee, 2001; Fischer, Wood, & Jones, 2002; Jay & Handley, 2001; Wood, 2002). Applied to transport, EIA has been used to study the environmental impact of some practices. This method is standardized and consists of several stages from the recording of the emissions to decision-making by the authorities. The three aspects of sustainability namely environmental, economic and social are taken into account, as one tries to evaluate the impact of a new activity on the environment, the population and the attractiveness of the neighborhood of the site.
  4. Optimization models: A mathematical optimization model consists of an objective function and a set of constraints in the form of a system of equations or inequalities. In the context of sustainable transportation, an optimization model attempts to find an optimal solution under the constraints of the social, economic and environmental objectives. Linear programming is commonly used. An application of dynamic optimization approach for sustainable urban transport development can be found in Zuidgeest (2005).
  5. System dynamics models: System dynamics is used to model complex systems. In system dynamic models, relationships between the system elements are demonstrated through stocks, flows and a feedback mechanism over time. These models can design and evaluate a cause and effect relationship within an integrated sustainable transportation system (Tao & Hung, 2003). Richardson (2005) presents frameworks for sustainability analysis of passenger and freight transport using influence diagrams and root cause analysis.
  6. Assessment indicator models: The assessment indicator models use indicators to assess sustainability of transportation systems. Tao and Hung (2003) classify them in three categories namely composite index models, multi-level index models and multi-dimension matrix models. The output of a composite index model is a single index representing degree of satisfying economical, social and environmental objectives (Maoh & Kanaroglou, 2009). For example, ecological footprint (Browne, O'Regan, & Moles, 2008), green gross national product, etc. However, a universal and single composite index of sustainable transportation is difficult to obtain (Phillis & Andriantiatsaholainaina, 2001). In multilevel index model, a series of indicators representing different goals and hierarchies are used. In multi-dimensional matrix model, interaction among different indicators is defined using logic architectures. The Pressure-State-Response types of models were the first of this type of models to be introduced in 1990s. Later, Driving-Force-State-Response (DSR) and Driving-Force, Pressure-State-Impact-Response (DPSIR), Driving Force-Pressure-State-Exposure-Effect-Action (DPSEEA) (Waheed, Khan, & Veitch, 2009) frameworks were developed.
  7. Data analysis: This category of models involves use of statistical data and application of data analysis techniques like surveys, hypothesis testing, structural equation modeling etc. to investigate sustainable transportation systems. Ülengin, Kabak, Önsel, Ülengin, and Aktaş (2010) present a problem-structuring model for analyzing transportation-environment relationships. Mohan (1999) use statistical data to illustrate the need of non-motorized modes of traffic and associated infrastructure design for designing sustainable transport system for urban areas.
  8. Multi-Criteria Decision Analysis (MCDA) methods: A wide set of MCDA methods exist: *Multi-Attribute Value Function Theory (MAVT)*, *Multi Attribute Utility Function Theory (MAUT)*, *Analytic Hierarchy Process (AHP)* and *ELECTRE methods*, etc. Decision analysis provides both tools and a framework for the integration of tools from other disciplines to create an overall strategy for decision support. Several applications have been developed especially for management environment (Beinat, 2001; Chang & Chen, 1991; Chen, Tzeng, & Liu, 2003; Dempster, 1968; Spiekermann & Wegener, 2004; Wellar, 2000) by selecting one alternative among several. Generally, in a multi-criteria problem there is no solution optimizing all the criteria at the same time therefore compromise solutions must be found. However, we should note that when different conflicting evaluation criteria are taken into consideration, a multi-criteria problem is mathematically ill defined (Simos, 1990). Decision analysis looks at the paradigm in which an individual decision maker (or decision group) contemplates a choice of action in an uncertain environment. In MCDA methods, the selection is facilitated by evaluating each choice on the criteria set. The criteria must be measurable; even if the measurement is performed only at the nominal scale (yes/no; present/absent) and their outcomes must be measured for every decision alternative. Criterion outcomes provide the basis for choices comparison and consequently facilitate the selection of one satisfactory choice. Criterion outcomes of decision alternatives are collected in a table (called decision matrix or decision table) comprising of a set of columns and rows. MCDA methods (Delgado, Verdegay, & Vila, 1992; Herrera, 1993), especially AHP and ELECTRE, are generally used for ranking several alternatives under criteria set, by using one expert or several experts and by using a numeric same scale or linguistic terms. Yedla and Shrestha (2003) use AHP to evaluate six sustainable transportation modes. Tsamboulas and Mikroudis (2000) present a multi-criteria evaluation framework of environmental impacts and costs of transport initiatives. Awasthi and Omrani (2009) present an AHP and belief theory based approach for evaluating sustainable transportation solutions.
- Recently, other methods combining MCDA and Artificial Intelligence have been explored to develop enhanced methodologies for knowledge based decision support system. By combining MCDA with fuzzy logic theory (Zadeh, 1965; Zadeh, 1986), new methods have been developed like, the most useful method, fuzzy AHP

(Simos, 1990), fuzzy comprehensive assessment (Lu, Lo, & Hu, 1999; Yang & Yang, 1998). In addition, some approaches using the framework of evidence theory with MCDA methods have been proposed. In fact, Beynon (2002) and Beynon et al. proposed AHP with Dempster–Shafer (DS) Theory (Dempster, 1968; Smets & Kennes, 1994). Also, an evidential reasoning (ER) approach has been developed for dealing with a complex decision problems in management (Simos, 1990; Xu, Yang, & Wang, 2005). This type of approach, using an evidential reasoning, was already used in several applications (Denoeux & Smets, 2006), but it was not extensively applied to achieve an evaluation of impacts in transport field. In our knowledge, only one research team has recently published papers for environmental analysis which use the evidence theory and multi-criteria analysis (Xu et al., 2005; Yang, Wang, Xu, & Chin, 2006). But in this method, only expert opinions are taken into account. The final solution could be to couple or to adapt different approaches.

In this paper, we present a multicriteria decision making approach for evaluation and selection of sustainable transportation systems under uncertain (fuzzy) environment. The rest of the paper is organized as follows: In Section 2 and 3, we present the preliminaries of fuzzy set theory and fuzzy TOPSIS. In Section 4, we present the multicriteria decision making approach for sustainability assessment of transportation systems based on fuzzy TOPSIS. In Section 5, we present a numerical illustration and finally, in Section 6 we provide the conclusions and steps for future work.

## 2. Preliminaries of fuzzy set theory

Fuzzy set theory is used to model vagueness and uncertainty in decision making processes arising due to lack of complete information (Zadeh, 1965). The fuzzy set theory uses linguistic terms to represent decision maker preferences. For example, the probability that it will rain on Monday can be represented in linguistic terms as high, very high, low, etc. In this paper, we will use fuzzy set theory to model the sustainable transportation decision making process since several model parameters cannot be analytically determined and require expert judgments. For example, mobility, equity, competency, quality of service, etc. Some related definitions of fuzzy set theory adapted from (Buckley, 1985; Dubois & Prade, 1982; Kaufmann & Gupta, 1991; Klir & Yuan, 1995; Pedrycz, 1994; Zadeh, 1965; Zimmermann, 2001) are presented as follows.

**Definition 1.** A fuzzy set  $\tilde{a}$  in a universe of discourse  $X$  is characterized by a membership function  $\mu_{\tilde{a}}(x)$  that maps each element  $x$  in  $X$  to a real number in the interval  $[0, 1]$ . The function value  $\mu_{\tilde{a}}(x)$  is termed the grade of membership of  $x$  in  $\tilde{a}$  (Kaufmann and Gupta). The nearer the value of  $\mu_{\tilde{a}}(x)$  to unity, the higher the grade of membership of  $x$  in  $\tilde{a}$ .

**Definition 2.** A triangular fuzzy number is represented as a triplet  $\tilde{a} = (a_1, a_2, a_3)$ . Fig. 1 presents a triangular fuzzy number  $\tilde{a}$ .

Due to their conceptual and computation simplicity, triangular fuzzy numbers are very commonly used in practical applications (Klir & Yuan, 1995; Pedrycz, 1994; Yeh & Deng, 2004). The membership function  $\mu_{\tilde{a}}(x)$  of triangular fuzzy number  $\tilde{a}$  is given by:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x \leq a_1, \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3, \\ 0, & x > a_3 \end{cases} \quad (1)$$

where  $a_1, a_2, a_3$  are real numbers and  $a_1 < a_2 < a_3$ . The value of  $x$  at  $a_2$  gives the maximal grade of  $\mu_{\tilde{a}}(x)$ , i.e.,  $\mu_{\tilde{a}}(x) = 1$ ; it is the most probable value of the evaluation data. The value of  $x$  at  $a_1$  gives

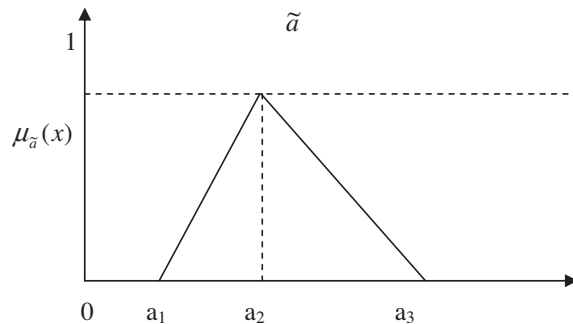


Fig. 1. Triangular fuzzy number  $\tilde{a}$ .

the minimal grade of  $\mu_{\tilde{a}}(x)$ , i.e.,  $\mu_{\tilde{a}}(x) = 0$ ; it is the least probable value of the evaluation data. Constants  $a_1$  and  $a_3$  are the lower and upper bounds of the available area for the evaluation data. These constants reflect the fuzziness of the evaluation data (Liang, 1999). The narrower the interval  $[a_1, a_3]$ , the lower is the fuzziness of the evaluation data.

**Property 1.** Given two fuzzy triangular numbers  $\tilde{a} = (a_1, a_2, a_3)$  and  $\tilde{b} = (b_1, b_2, b_3)$ , the main operations are expressed as follows:

- (1) Addition of two triangular fuzzy numbers  
 $\tilde{a}(+) \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3), \quad a_1 \geq 0, b_1 \geq 0$
- (2) Multiplication of two triangular fuzzy numbers  
 $\tilde{a}(\times) \tilde{b} = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3), \quad a_1 \geq 0, b_1 \geq 0$
- (3) Subtraction of two triangular fuzzy numbers  
 $\tilde{a}(-) \tilde{b} = (a_1 - b_1, a_2 - b_2, a_3 - b_3), \quad a_1 \geq 0, b_1 \geq 0$
- (4) Division of two triangular fuzzy numbers  
 $\tilde{a}(/) \tilde{b} = (a_1/b_1, a_2/b_2, a_3/b_3), \quad a_1 \geq 0, b_1 \geq 0$
- (5) Inverse of a triangular fuzzy number  
 $\tilde{a}^{-1} = (1/a_1, 1/a_2, 1/a_3), \quad a_1 \geq 0$
- (6) Symmetric image  
 $\tilde{a} = (-a_1, -a_2, -a_3), \quad a_1 \geq 0$

**Property 2.** Given any real number  $k$  and a triangular fuzzy number  $\tilde{a}$ , the operations of the two numbers are given by:

- (1) Multiplication of a triangular fuzzy number by a constant  
 $k * \tilde{a} = (ka_1, ka_2, ka_3), \quad a_1 \geq 0, k \geq 0$
- (2) Division of a triangular fuzzy number by a constant  
 $k/\tilde{a} = (k/a_1, k/a_2, k/a_3), \quad a_1 \geq 0, k \geq 0$
- (3) Division of a constant by a triangular fuzzy number  
 $\tilde{a}/k = (a_1/k, a_2/k, a_3/k), \quad a_1 \geq 0, k \geq 0$

The proofs of these operations are straightforward and hence omitted.

**Property 3.** Given two triangular fuzzy numbers  $(\tilde{a}, \tilde{b})$  and any real number  $k$ , the commutative operations of these two numbers are expressed as follows:

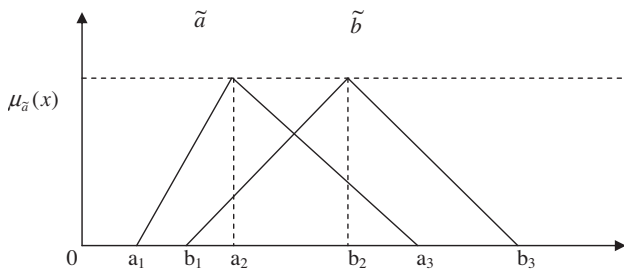


Fig. 2. Two triangular fuzzy numbers.

Table 1  
Linguistic terms for alternative ratings.

Linguistic term	Membership function
Very poor (VP)	(1, 1, 3)
Poor (P)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Good (G)	(5, 7, 9)
Very good (VG)	(7, 9, 9)

Table 2  
Linguistic terms for criteria ratings.

Linguistic term	Membership function
Very low	(1, 1, 3)
Low	(1, 3, 5)
Medium	(3, 5, 7)
High	(5, 7, 9)
Very high	(7, 9, 9)

$$\tilde{a}(+) \tilde{b} = \tilde{b}(+) \tilde{a}, \quad a_1 \geq 0, b_1 \geq 0, k \geq 0$$

$$\tilde{a}(\times) \tilde{b} = \tilde{b}(\times) \tilde{a}, \quad a_1 \geq 0, b_1 \geq 0, k \geq 0$$

$$\tilde{a}(-) \tilde{b} = \tilde{b}(-) \tilde{a}, \quad a_1 \geq 0, b_1 \geq 0, k \geq 0$$

$$K * \tilde{a} = \tilde{a} * k, \quad a_1 \geq 0, b_1 \geq 0, k \geq 0$$

The proofs of these operations are straightforward and hence omitted.

**Property 4.** Let  $\tilde{a} = (a_1, a_2, a_3)$  and  $\tilde{b} = (b_1, b_2, b_3)$  be two triangular fuzzy numbers (Fig. 2).

The distance between them using the vertex method is given by:

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3} [(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]}$$

### 2.1. Linguistic variables and fuzzy set theory

In fuzzy set theory, conversion scales are applied to transform the linguistic terms into fuzzy numbers. In this paper, we will use a scale of 1–9 to rate the criteria and the alternatives. Table 1 presents the linguistic variables and fuzzy ratings used for the alternatives and Table 2 presents the linguistic variables and fuzzy ratings used for the criteria.

### 3. Fuzzy TOPSIS

The fuzzy TOPSIS approach involves fuzzy assessments of criteria and alternatives in TOPSIS (Hwang & Yoon, 1981). The TOPSIS approach chooses alternative that is closest to the positive ideal solution and farthest from the negative ideal solution. A positive

ideal solution is composed of the best performance values for each criterion whereas the negative ideal solution consists of the worst performance values. The various steps of fuzzy TOPSIS are presented as follows:

**Step 1:** Assignment of ratings to the criteria and the alternatives.

Let us assume there are  $J$  possible candidates called  $A = \{A_1, A_2, \dots, A_j\}$  which are to be evaluated against  $n$  criteria,  $C = \{C_1, C_2, \dots, C_j\}$ . The criteria weights are denoted by  $w_i$  ( $i = 1, 2, \dots, m$ ). The performance ratings of each decision maker  $D_k$  ( $k = 1, 2, \dots, K$ ) for each alternative  $A_j$  ( $j = 1, 2, \dots, n$ ) with respect to criteria  $C_i$  ( $i = 1, 2, \dots, m$ ) are denoted by  $\tilde{R}_k = \tilde{x}_{ijk}$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n; k = 1, 2, \dots, K$ ) with membership function  $\mu_{\tilde{R}_k}(x)$ .

**Step 2:** Compute aggregate fuzzy ratings for the criteria and the alternatives.

If the fuzzy ratings of all decision makers is described as triangular fuzzy number  $\tilde{R}_k = (a_k, b_k, c_k)$ ,  $k = 1, 2, \dots, K$ , then the aggregated fuzzy rating is given by  $\tilde{R} = (a, b, c)$ ,  $k = 1, 2, \dots, K$  where;

$$a = \min_k \{a_k\}, \quad b = \frac{1}{K} \sum_{k=1}^K b_k, \quad c = \max_k \{c_k\}$$

If the fuzzy rating and importance weight of the  $k$ th decision maker are  $\tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk})$  and  $\tilde{w}_{ijk} = (w_{jk1}, w_{jk2}, w_{jk3})$ ,  $i = 1, 2, \dots, m$ ,  $j = 1, 2, \dots, n$  respectively, then the aggregated fuzzy ratings ( $\tilde{x}_{ij}$ ) of alternatives with respect to each criteria are given by  $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$  where

$$a_{ij} = \min_k \{a_{ijk}\}, \quad b_{ij} = \frac{1}{K} \sum_{k=1}^K b_{ijk}, \quad c_{ij} = \max_k \{c_{ijk}\} \quad (2)$$

The aggregated fuzzy weights ( $\tilde{w}_{ij}$ ) of each criterion are calculated as  $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$  where

$$w_{j1} = \min_k \{w_{jk1}\}, \quad w_{j2} = \frac{1}{K} \sum_{k=1}^K w_{jk2}, \quad w_{j3} = \max_k \{w_{jk3}\} \quad (3)$$

**Step 3:** Compute the fuzzy decision matrix.

The fuzzy decision matrix for the alternatives ( $\tilde{D}$ ) and the criteria ( $\tilde{W}$ ) is constructed as follows:

$$\tilde{D} = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ A_1 & \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ A_2 & \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ A_3 & \dots & \dots & \dots & \dots \\ A_4 & \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{matrix}, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (4)$$

$$\tilde{W} = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n) \quad (5)$$

**Step 4:** Normalize the fuzzy decision matrix.

The raw data are normalized using linear scale transformation to bring the various criteria scales into a comparable scale. The normalized fuzzy decision matrix  $\tilde{R}$  is given by:

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n}, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (6)$$

where

$$\tilde{r}_{ij} = \left( \frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*} \right) \quad \text{and} \quad c_j^* = \max_i c_{ij} \quad (\text{benefit criteria}) \quad (7)$$

$$\tilde{r}_{ij} = \left( \frac{a_j^-}{c_{ij}^-}, \frac{a_j^-}{b_{ij}^-}, \frac{a_j^-}{a_{ij}^-} \right) \quad \text{and} \quad a_j^- = \min_i a_{ij} \quad (\text{cost criteria}) \quad (8)$$

**Step 5:** Compute the weighted normalized matrix.

The weighted normalized matrix  $\tilde{V}$  for criteria is computed by multiplying the weights ( $\tilde{w}_j$ ) of evaluation criteria with the normalized fuzzy decision matrix  $\tilde{r}_{ij}$



$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n}, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad \text{where } \tilde{v}_{ij} = \tilde{r}_{ij}(\cdot) \tilde{w}_j \quad (9)$$

**Step 6:** Compute the fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS).

The FPIS and FNIS of the alternatives is computed as follows:

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*) \quad \text{where } \tilde{v}_j^* = \max_i \{v_{ij3}\}, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (10)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-) \quad \text{where } \tilde{v}_j^- = \min_i \{v_{ij1}\}, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n \quad (11)$$

**Step 7:** Compute the distance of each alternative from FPIS and FNIS.

The distance ( $d_i^+, d_i^-$ ) of each weighted alternative  $i = 1, 2, \dots, m$  from the FPIS and the FNIS is computed as follows:

$$d_i^+ = \sum_{j=1}^n d_v(\tilde{v}_{ij}, \tilde{v}_j^*), \quad i = 1, 2, \dots, m \quad (12)$$

$$d_i^- = \sum_{j=1}^n d_v(\tilde{v}_{ij}, \tilde{v}_j^-), \quad i = 1, 2, \dots, m \quad (13)$$

where  $d_v(\tilde{a}, \tilde{b})$  is the distance measurement between two fuzzy numbers  $\tilde{a}$  and  $\tilde{b}$ .

**Step 8:** Compute the closeness coefficient ( $CC_i$ ) of each alternative.

The closeness coefficient  $CC_i$  represents the distances to the fuzzy positive ideal solution ( $A^*$ ) and the fuzzy negative ideal solution ( $A^-$ ) simultaneously. The closeness coefficient of each alternative is calculated as:

$$CC_i = \frac{d_i^-}{d_i^- + d_i^+}, \quad i = 1, 2, \dots, m \quad (14)$$

**Step 9:** Rank the alternatives.

In step 9, the different alternatives are ranked according to the closeness coefficient ( $CC_i$ ) in decreasing order. The best alternative is closest to the FPIS and farthest from the FNIS.

#### 4. Evaluating sustainable transportation systems under uncertainty

The proposed framework for evaluation of sustainable transportation systems under uncertainty consists of three steps.

1. Selection of evaluation criteria.
2. Evaluation and selection of best alternative using selected criteria.
3. Conduct sensitivity analysis to determine the influence of criteria weights on decision making.

These steps are presented in detail as follows.

##### 4.1. Criteria selection

The first step involves selection of criteria for evaluating sustainability of transportation systems. The criteria were identified from literature review (Basler, 1998; Black, Paez, & Suthanya 2002; Jeon, Amekudzi, & Guensler, 2008; Kanaroglou & Buliung 2008; Litman, 2009; Morse, McNamara, Acholo, & Okwoli, 2001), discussion with transportation experts and our practical experience with city transportation projects (ECOSYMPA and SUCCESS) in La Rochelle, France. The final list contains 24 criteria. These criteria are shown in Table 3.

It can be seen in Table 3, that criterion C1, C5–C12 are the cost (C) category criteria that is, the lower the value, the more sustainable the alternative (or transportation system). The remaining criteria are benefit (B) type criteria, that is, the higher the value, the more sustainable the transportation system.

##### 4.2. Alternatives evaluation and selection using fuzzy TOPSIS

The second step involves allocation of linguistic ratings to the 24 criteria and the potential alternatives for each of the criteria by the decision makers or experts. The criteria ratings are provided from Table 2 and the alternative ratings for each of the criteria from Table 1. The linguistics terms are then transformed to fuzzy

**Table 3**  
Criteria for sustainability evaluation of transportation systems.

Criteria	Definition	Category
Operating costs (C1)	Costs to operator for running the transportation service	C
Safety (C2)	Safety offered by the transportation system	B
Security (C3)	Security from theft, vandalism offered by the transportation system	B
Reliability (C4)	Ability to perform the promised service dependably and accurately	B
Air pollutants (C5)	Air pollutants from the transportation system	C
Noise (C6)	Noise from the transportation system	C
GHG emissions (C7)	GHG emissions from the transportation system	C
Usage of fossil fuels (C8)	Usage of fossil fuels like petrol, diesel	C
Travel costs (C9)	Costs for travel between any given stations	C
Waste from road transport (C10)	Waste from road transport: number of end-of-life vehicles, number of used tires	C
Energy consumption (C11)	Energy consumption by the transportation system	C
Land usage (C12)	Land space used for running the transportation service	C
Accessibility (C13)	Access to residential areas, activity areas and other transportation modes	B
Benefits to economy (C14)	Benefits to economy from the transportation mode e.g. labor employment, resource usage	B
Competency (C15)	State of the art technology, equipment and infrastructure employed by the transportation mode	B
Equity (C16)	Equity across genders, age groups, handicapped people	B
Possibility of expansion (C17)	Ability to expand the service if required	B
Mobility (C18)	Ability to service over the transportation area	B
Productivity (C19)	Ability to achieve performance targets	B
Occupancy rate (C20)	Capacity utilization of transportation mode	B
Share in public transit (C21)	Contribution to public transport	B
Convenience to use (C22)	Convenience in using the transportation service	B
Quality of service (C23)	Quality of service provided by the transportation staff	B
Tangibles (C24)	Physical facilities, equipment, and appearance of personnel	B

C (cost) – The lower the better.  
 B (Benefit) – The higher the better.

triangular numbers. Then, fuzzy TOPSIS (Section 3) is applied to aggregate the criteria and the alternative ratings to generate an overall score for assessing the sustainability performance of the alternatives (urban transportation systems). The alternative with the highest score is selected as the best alternative for sustainable transportation and recommended for implementation in the city.

4.3. Sensitivity analysis

The step 3 involves conducting the sensitivity analysis. Sensitivity analysis addresses the question, “How sensitive is the overall decision to small changes in the individual weights assigned during the pair-wise comparison process?” This question can be answered by varying slightly the values of the weights and observing the effects on the decision. This is useful in situations where uncertainties exist in the definition of the importance of different factors. In our case, we will conduct sensitivity analysis in order to see the importance of criteria weights in selecting the best alternative among the available alternatives (sustainable transportation systems).

5. Numerical illustration

Let us assume that a city transportation group is interested in implementing a sustainable transportation system in the city. The alternatives available to the city are Carsharing (C), Ridesharing (R) and Park-n-ride (PR). We have chosen these three systems because of our familiarity and practical experience with them during the project SUCCESS in La Rochelle, France. Carsharing is an alternative for private car use. The carsharing vehicles are available for customers use at different stations just like a private car. Users reserve the cars by telephone, internet or kiosk and get access to the vehicle at the requested time and location. After usage, the vehicle is returned to the same or different station from where it was picked up. Carsharing is very popular in Europe, North America, Asia, etc. as one of the sustainable modes of transportation.

In ridesharing, several users share a common car for a ride, thereby reducing the number of trips that were to take place if each of

Table 4  
Linguistic assessments for the 24 criteria.

Criteria	Decision makers			Aggregate fuzzy ratings
	D1	D2	D3	
C1	VH	H	H	(5, 7.67, 9)
C2	VH	VH	H	(5, 8.33, 9)
C3	VH	H	H	(5, 7.67, 9)
C4	VH	VH	H	(5, 8.33, 9)
C5	H	VH	H	(5, 7.67, 9)
C6	H	H	H	(5, 7, 9)
C7	H	VH	VH	(5, 8.33, 9)
C8	H	H	VH	(5, 7.67, 9)
C9	VH	VH	VH	(7, 9, 9)
C10	VH	VH	VH	(7, 9, 9)
C11	VH	VH	VH	(7, 9, 9)
C12	H	H	VH	(5, 7.67, 9)
C13	H	VH	H	(5, 7.67, 9)
C14	H	H	H	(5, 7, 9)
C15	VH	H	H	(5, 7.67, 9)
C16	VH	H	VH	(5, 8.33, 9)
C17	VH	VH	H	(5, 8.33, 9)
C18	VH	VH	H	(5, 8.33, 9)
C19	H	H	H	(5, 7, 9)
C20	VH	VH	VH	(7, 9, 9)
C21	H	H	VH	(5, 7.67, 9)
C22	VH	VH	VH	(7, 9, 9)
C23	VH	H	VH	(5, 8.33, 9)
C24	H	VH	VH	(5, 8.33, 9)

them was riding individually. Ridesharing is very common in college campuses and universities. Users can see the available rides between a given origin–destination online through the ridesharing organization’s website and choose the one that suits them most. The difference between carsharing and ridesharing is that in carsharing the ownership of the vehicles belongs to the carsharing organization whereas in ridesharing the vehicle belongs to the users.

Park-and-ride involves parking of private cars by users in reserved parking areas outside the city centers and use of public transport to get access to the city centers. Park-and-ride is very popular in UK, America, etc. and is an effective means of reducing congestion inside city centers arising due to private vehicles.

Table 5  
Linguistic assessments for the three alternatives.

Criteria	Alternatives								
	A1			A2			A3		
	D1	D2	D3	D1	D2	D3	D1	D2	D3
Operating costs (C1)	VL	VH	H	L	VH	M	M	VL	VH
Safety (C2)	VH	L	M	VL	M	VH	H	VL	VL
Security (C3)	L	VL	H	VH	H	M	VH	VH	VH
Reliability (C4)	VL	VH	VH	VL	M	VL	VH	VH	H
Air pollutants (C5)	VH	VL	VL	L	L	M	M	H	H
Noise (C6)	M	M	M	VL	M	H	H	VL	L
GHG emissions (C7)	H	L	H	M	VH	VH	L	VL	M
Usage of fossil fuels (C8)	H	VL	H	VL	M	VH	H	L	M
Travel costs (C9)	M	L	L	VH	VL	M	VL	VL	VH
Waste from road transport (C10)	M	VL	VH	L	M	H	M	VH	VH
Energy consumption (C11)	L	VH	H	H	VH	M	VH	VL	H
Land usage (C12)	VL	VL	VL	VH	M	H	VH	VL	VL
Accessibility (C13)	VL	L	H	VL	VH	H	H	VL	L
Benefits to economy (C14)	M	VL	H	H	M	VL	M	M	VL
Competency (C15)	L	M	M	VH	VL	H	VH	M	H
Equity (C16)	M	L	VL	H	VL	L	VH	VL	VL
Possibility of expansion (C17)	M	H	M	M	VL	M	VL	L	L
Mobility (C18)	L	L	H	H	VH	VH	VH	VL	VH
Productivity (C19)	VH	M	M	L	L	VH	VH	VL	L
Occupancy rate (C20)	H	VL	VL	L	L	H	M	H	VL
Share in public transit (C21)	M	M	L	VH	VH	M	M	H	L
Convenience to use (C22)	M	M	VL	L	VH	H	H	VL	VH
Quality of service (C23)	VH	VL	L	M	L	M	M	H	M
Tangibles (C24)	L	M	L	M	VH	VH	VL	L	H

**Table 6**  
Aggregate fuzzy criteria weights.

Criteria	Weight
C1	(5, 7.67, 9)
C2	(5, 8.33, 9)
C3	(5, 7.67, 9)
C4	(5, 8.33, 9)
C5	(5, 7.67, 9)
C6	(5, 7, 9)
C7	(5, 8.33, 9)
C8	(5, 7.67, 9)
C9	(7, 9, 9)
C10	(7, 9, 9)
C11	(7, 9, 9)
C12	(5, 7.67, 9)
C13	(5, 7.67, 9)
C14	(5, 7, 9)
C15	(5, 7.67, 9)
C16	(5, 8.33, 9)
C17	(5, 8.33, 9)
C18	(5, 8.33, 9)
C19	(5, 7, 9)
C20	(7, 9, 9)
C21	(5, 7.67, 9)
C22	(7, 9, 9)
C23	(5, 8.33, 9)
C24	(5, 8.33, 9)

**Table 7**  
Aggregate fuzzy decision matrix.

Criteria	Alternatives		
	A1 (R)	A2 (C)	A3 (PR)
C1	(1, 5.67, 9)	(1, 5.67, 9)	(1, 5, 9)
C2	(1, 5.67, 9)	(1, 5, 9)	(1, 3, 9)
C3	(1, 3.67, 9)	(3, 7, 9)	(7, 9, 9)
C4	(1, 6.33, 9)	(1, 2.33, 7)	(5, 8.33, 9)
C5	(1, 3.67, 9)	(1, 3.67, 7)	(3, 6.33, 9)
C6	(3, 5, 7)	(1, 4.33, 9)	(1, 3.67, 9)
C7	(1, 5.67, 9)	(3, 7.67, 9)	(1, 3, 7)
C8	(1, 5, 9)	(1, 5, 9)	(1, 5, 9)
C9	(1, 3.67, 7)	(1, 5, 9)	(1, 3.67, 9)
C10	(1, 5, 9)	(1, 5, 9)	(3, 7.67, 9)
C11	(1, 6.33, 9)	(3, 7, 9)	(1, 5.67, 9)
C12	(1, 1, 3)	(3, 7, 9)	(1, 3.67, 9)
C13	(1, 3.67, 9)	(1, 5.67, 9)	(1, 3.67, 9)
C14	(1, 4.33, 9)	(1, 4.33, 9)	(1, 3.67, 7)
C15	(1, 4.33, 7)	(1, 5.67, 9)	(3, 7, 9)
C16	(1, 3, 7)	(1, 3.67, 9)	(1, 3.67, 9)
C17	(3, 5.67, 9)	(1, 3.67, 7)	(1, 2.33, 5)
C18	(1, 4.33, 9)	(5, 8.33, 9)	(1, 6.33, 9)
C19	(3, 6.33, 9)	(1, 5, 9)	(1, 4.33, 9)
C20	(1, 3, 9)	(1, 4.33, 9)	(1, 4.33, 9)
C21	(1, 4.33, 7)	(3, 7.67, 9)	(1, 5, 9)
C22	(1, 3.67, 7)	(1, 6.33, 9)	(1, 5.67, 9)
C23	(1, 4.33, 9)	(1, 4.33, 7)	(3, 5.67, 9)
C24	(1, 3.67, 7)	(3, 7.67, 9)	(1, 3.67, 9)

Let us assume that a committee of three decision makers D1, D2 and D3 is formed to select the sustainable transportation system for the city. The criteria used for evaluation are same as presented in Table 3. The committee used linguistic assessments (Tables 1 and 2) to rate the 24 criteria (Table 3) and the three alternatives (Carsharing (A1), Ridesharing (A2), Park-and-ride (A3)). The results are shown in Tables 4 and 5 respectively.

Then, the aggregated fuzzy weights ( $\tilde{w}_{ij}$ ) of each criterion is calculated using Eq. (3). For example, for criteria C1 "Operating costs", the aggregated fuzzy weight is given by  $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$  where:

$$w_{j1} = \min_k(7, 5, 5), \quad w_{j2} = \frac{1}{3} \sum_{k=1}^3 (9 + 7 + 7),$$

$$w_{j3} = \max_k(9, 9, 9) = \tilde{w}_j = (5, 7.67, 9)$$

Likewise, we compute the aggregate weights for the remaining 23 criteria. The aggregate weights of the 23 criteria are presented in Table 6.

Then, the aggregate fuzzy weights of the alternatives are computed using Eq. (2). For example, the aggregate rating for alternative A1 (Carsharing) for criteria C1 (Operating Costs) using the rating given by the three decision makers is computed as follows:

$$a_{ij} = \min_k(1, 7, 5), \quad b_{ij} = \frac{1}{3} \sum_{k=1}^3 (1 + 9 + 7),$$

$$c_{ij} = \max_k(3, 9, 9) = (1, 5.667, 9)$$

Likewise, the aggregate ratings for the three alternatives (A1, A2, A3) with respect to the 23 criteria are computed. The aggregate fuzzy decision matrix for the alternatives is presented in Table 7.

In the next step, we perform normalization of the fuzzy decision matrix of alternatives using Eqs. (6)–(8). For example, the normalized rating for alternative A1 for criteria C1 (Operating costs) is given by:

$$a_j^- = \min_i(1, 1, 1) = 1$$

$$\tilde{r}_{ij} = \left( \frac{1}{9}, \frac{1}{5.667}, \frac{1}{9} \right) = (0.11, 0.176, 1)$$

The normalized value of alternative A1 for criteria C2 (Safety) is given by,

$$c_j^+ = \max_i(9, 9, 9) = 9$$

$$\tilde{r}_{ij} = \left( \frac{1}{9}, \frac{5.667}{9}, \frac{9}{9} \right) = (0.11, 0.629, 1)$$

Likewise, we compute the normalized values of the alternatives for the remaining criteria. The normalized fuzzy decision matrix for the three alternatives is presented in Table 8.

Then, the fuzzy weighted decision matrix for the three alternatives is constructed using Eq. (9). The  $\tilde{r}_{ij}$  values from Table 8 and  $\tilde{w}_j$  values from Table 6 are used to compute the fuzzy weighted

**Table 8**  
Normalized fuzzy decision matrix for alternatives.

Criteria	Alternatives		
	A1 (R)	A2 (C)	A3 (PR)
C1	(0.11, 0.176, 1)	(0.11, 0.176, 1)	(0.11, 0.2, 1)
C2	(0.11, 0.629, 1)	(0.11, 0.56, 1)	(0.11, 0.33, 1)
C3	(0.11, 0.407, 1)	(0.33, 0.78, 1)	(0.78, 1, 1)
C4	(0.11, 0.703, 1)	(0.11, 0.259, 0.78)	(0.56, 0.925, 1)
C5	(0.11, 0.272, 1)	(0.142, 0.27, 1)	(0.11, 0.157, 0.33)
C6	(0.14, 0.2, 0.33)	(0.11, 0.23, 1)	(0.11, 0.272, 1)
C7	(0.11, 0.176, 1)	(0.11, 0.13, 0.33)	(0.14, 0.33, 1)
C8	(0.11, 0.2, 1)	(0.11, 0.2, 1)	(0.11, 0.2, 1)
C9	(0.14, 0.27, 1)	(0.11, 0.2, 1)	(0.11, 0.272, 1)
C10	(0.11, 0.2, 1)	(0.11, 0.2, 1)	(0.11, 0.13, 0.33)
C11	(0.11, 0.157, 1)	(0.11, 0.142, 0.33)	(0.11, 0.176, 1)
C12	(0.33, 1, 1)	(0.11, 0.142, 0.33)	(0.11, 0.272, 1)
C13	(0.11, 0.40, 1)	(0.11, 0.629, 1)	(0.11, 0.407, 1)
C14	(0.11, 0.48, 1)	(0.11, 0.48, 1)	(0.11, 0.407, 0.78)
C15	(0.11, 0.48, 0.78)	(0.11, 0.629, 1)	(0.33, 0.78, 1)
C16	(0.11, 0.33, 0.78)	(0.11, 0.407, 1)	(0.11, 0.407, 1)
C17	(0.33, 0.62, 1)	(0.11, 0.407, 0.78)	(0.11, 0.259, 0.56)
C18	(0.11, 0.48, 1)	(0.56, 0.925, 1)	(0.11, 0.703, 1)
C19	(0.33, 0.703, 1)	(0.11, 0.56, 1)	(0.11, 0.48, 1)
C20	(0.11, 0.33, 1)	(0.11, 0.48, 1)	(0.11, 0.48, 1)
C21	(0.11, 0.48, 0.78)	(0.33, 0.85, 1)	(0.11, 0.56, 1)
C22	(0.11, 0.407, 0.78)	(0.11, 0.703, 1)	(0.11, 0.62, 1)
C23	(0.11, 0.48, 1)	(0.11, 0.48, 0.78)	(0.33, 0.62, 1)
C24	(0.11, 0.407, 0.778)	(0.33, 0.85, 1)	(0.11, 0.407, 1)

**Table 9**  
 Weighted normalized alternatives, FPIS and FNIS.

Criteria	Alternatives			FNIS ( $A^-$ )	FPIS ( $A^*$ )
	A1	A2	A3		
C1	(0.55, 1.35, 9)	(0.55, 1.35, 9)	(0.55, 1.53, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C2	(0.55, 5.24, 9)	(0.55, 4.62, 9)	(0.55, 2.77, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C3	(0.55, 3.12, 9)	(1.66, 5.96, 9)	(3.88, 7.66, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C4	(0.55, 5.86, 9)	(0.55, 2.16, 7)	(2.77, 7.71, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C5	(0.55, 2.09, 9)	(0.71, 2.09, 9)	(0.55, 1.21, 3)	(0.55, 0.55, 0.55)	(9, 9, 9)
C6	(0.71, 1.4, 3)	(0.55, 1.61, 9)	(0.55, 1.90, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C7	(0.55, 1.47, 9)	(0.55, 1.08, 3)	(0.71, 2.77, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C8	(0.55, 1.53, 9)	(0.55, 1.53, 9)	(0.55, 1.53, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C9	(1, 2.45, 9)	(0.77, 1.8, 9)	(0.77, 2.45, 9)	(0.77, 0.77, 0.77)	(9, 9, 9)
C10	(0.77, 1.8, 9)	(0.77, 1.8, 9)	(0.77, 1.17, 3)	(0.77, 0.77, 0.77)	(9, 9, 9)
C11	(0.77, 1.42, 9)	(0.77, 1.28, 3)	(0.77, 1.58, 9)	(0.77, 0.77, 0.77)	(9, 9, 9)
C12	(1.66, 7.66, 9)	(0.55, 1.09, 3)	(0.55, 2.09, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C13	(0.55, 3.12, 9)	(0.55, 4.82, 9)	(0.55, 3.12, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C14	(0.55, 3.37, 9)	(0.55, 3.37, 9)	(0.55, 2.85, 7)	(0.55, 0.55, 0.55)	(9, 9, 9)
C15	(0.55, 3.69, 7)	(0.55, 4.82, 9)	(1.66, 5.96, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C16	(0.55, 2.77, 7)	(0.55, 3.39, 9)	(0.55, 3.39, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C17	(1.66, 5.24, 9)	(0.55, 3.39, 7)	(0.55, 2.16, 5)	(0.55, 0.55, 0.55)	(9, 9, 9)
C18	(0.55, 4.01, 9)	(2.77, 7.71, 9)	(0.55, 5.86, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C19	(1.66, 4.92, 9)	(0.55, 3.88, 9)	(0.55, 3.37, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C20	(0.77, 3, 9)	(0.77, 4.33, 9)	(0.77, 4.33, 9)	(0.77, 0.77, 0.77)	(9, 9, 9)
C21	(0.55, 3.69, 7)	(1.66, 6.53, 9)	(0.55, 4.25, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C22	(0.77, 3.66, 7)	(0.77, 6.33, 9)	(0.77, 5.66, 9)	(0.77, 0.77, 0.77)	(9, 9, 9)
C23	(0.55, 4.01, 9)	(0.55, 4.01, 7)	(1.66, 5.24, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)
C24	(0.55, 3.39, 7)	(1.66, 7.09, 9)	(0.55, 3.395, 9)	(0.55, 0.55, 0.55)	(9, 9, 9)

decision matrix for the alternatives. For example, for alternative A1, the fuzzy weight for criteria C1 (Operating costs) is given by:

$$\tilde{w}_{ij} = (0.11, 0.176, 1)(\cdot)(5, 7.66, 9) = (0.55, 1.35, 9)$$

Likewise, we compute the fuzzy weights of the three alternatives for the remaining criteria (Table 9). Then, the fuzzy positive ideal solution ( $A^*$ ) and the fuzzy negative ideal solutions ( $A^-$ ) are computed using Eqs. (10), (11) for the three alternatives. For example, for criteria C1 (Congestion),  $A^- = (0.55, 0.55, 0.55)$  and

$A^* = (9, 9, 9)$ . Similar computations are performed for the remaining criteria. The results are presented in last two columns of Table 9.

Then, we compute the distance  $d_v(\cdot)$  of each alternative from the fuzzy positive ideal matrix ( $A^*$ ) and fuzzy negative ideal matrix ( $A^-$ ) using Eqs. (12) and (13). For example, for alternative A1 and criteria C1, the distances  $d_v(A_1, A^*)$  and  $d_v(A_1, A^-)$  are computed as follows:

$$d_v(A_1, A^*) = \sqrt{\frac{1}{3}[(0.556 - 0.556)^2 + (1.353 - 0.556)^2 + (9 - 0.556)^2]} = 4.89$$

$$d_v(A_1, A^-) = \sqrt{\frac{1}{3}[(0.556 - 9)^2 + (0.556 - 9)^2 + (0.556 - 9)^2]} = 6.57$$

Likewise, we compute the distances for the remaining criteria for the three alternatives. The results are shown in Table 10.

Then, we compute the distances  $d_i^+$  and  $d_i^-$  using Eqs. (12) and (13). For example, for alternative A1 and criteria C1, the distances  $d_i^+$  and  $d_i^-$  are given by:

$$d_i^+ = \sqrt{\frac{1}{3}[(0.556 - 0.556)^2 + (1.353 - 0.556)^2 + (9 - 0.556)^2]} + \sqrt{\frac{1}{3}[(0.556 - 0.556)^2 + (5.247 - 0.556)^2 + (9 - 0.556)^2]} + \dots + \sqrt{\frac{1}{3}[(0.556 - 0.556)^2 + (3.395 - 0.556)^2 + (7 - 0.556)^2]} = 115.39$$

**Table 10**  
 Distance  $d_v(A_i, A^*)$  and  $d_v(A_i, A^-)$  for alternatives.

Criteria	$d^-$			$d^+$		
	A1	A2	A3	A1	A2	A3
C1	4.89	4.89	4.90	6.57	6.57	6.50
C2	5.57	5.41	5.03	5.33	5.48	6.05
C3	5.09	5.82	6.65	5.93	4.58	3.04
C4	5.75	3.83	6.51	5.19	6.37	3.66
C5	4.95	4.95	1.46	6.29	6.22	7.47
C6	1.49	4.91	4.93	7.35	6.47	6.36
C7	4.90	1.44	5.03	6.52	7.52	5.97
C8	4.90	4.90	4.90	6.50	6.50	6.50
C9	4.84	4.78	4.84	5.96	6.30	6.06
C10	4.78	4.78	1.30	6.30	6.30	7.40
C11	4.75	1.31	4.76	6.45	7.37	6.38
C12	6.40	1.44	4.95	4.30	7.51	6.29
C13	5.09	5.46	5.09	5.93	5.43	5.93
C14	5.13	5.13	3.94	5.85	5.85	6.13
C15	4.13	5.46	5.82	5.87	5.43	4.58
C16	3.93	5.14	5.14	6.16	5.84	5.84
C17	5.61	4.06	2.72	4.75	5.96	6.68
C18	5.26	6.51	5.75	5.65	3.66	5.19
C19	5.52	5.23	5.13	4.84	5.69	5.85
C20	4.91	5.16	5.16	5.87	5.45	5.45
C21	4.13	6.00	5.32	5.87	4.46	5.58
C22	3.95	5.72	5.52	5.77	4.98	5.11
C23	5.26	4.22	5.61	5.65	5.77	4.75
C24	4.06	6.19	5.14	5.96	4.37	5.84

**Table 11**  
 Closeness coefficient ( $CC_i$ ) of the three alternatives.

	A1	A2	A3
$d_i^-$	115.39	112.83	115.70
$d_i^+$	140.96	140.20	138.76
$CC_i$	0.549	0.554	0.545



**Table 12**  
 Experiments for sensitivity analysis.

S. no.	Definition	Overall score ( $CC_i$ )			Ranking
		A1 (C)	A2 (R)	A3 (PR)	
Expt. 1	$W_{C1-C24} = (1, 1, 3)$	0.59	0.60	0.58	R > C > PR
Expt. 2	$W_{C1-C24} = (1, 3, 5)$	0.569	0.57	0.56	R > C > PR
Expt. 3	$W_{C1-C24} = (3, 5, 7)$	0.562	0.569	0.56	R > C > PR
Expt. 4	$W_{C1-C24} = (5, 7, 9)$	0.558	0.564	0.55	R > C > PR
Expt. 5	$W_{C1-C24} = (7, 9, 9)$	0.541	0.544	0.53	R > C > PR
Expt. 6	$W_{C1} = (7, 9, 9), W_{C2-C24} = (1, 1, 3)$	0.588	0.597	0.58	R > C > PR
Expt. 7	$W_{C2} = (7, 9, 9), W_{C1,C3-C24} = (1, 1, 3)$	0.580	0.590	0.581	R > PR > C
Expt. 8	$W_{C3} = (7, 9, 9), W_{C1-C2,C4-C24} = (1, 1, 3)$	0.584	0.581	0.57	C > R > PR
Expt. 9	$W_{C4} = (7, 9, 9), W_{C1-C3,C5-C24} = (1, 1, 3)$	0.578	0.601	0.56	R > C > PR
Expt. 10	$W_{C5} = (7, 9, 9), W_{C1-C4,C6-C24} = (1, 1, 3)$	0.587	0.595	0.59	R > PR > C
Expt. 11	$W_{C6} = (7, 9, 9), W_{C1-C5,C7-C24} = (1, 1, 3)$	0.604	0.597	0.60	C > PR > R
Expt. 12	$W_{C7} = (7, 9, 9), W_{C1-C6,C8-C24} = (1, 1, 3)$	0.588	0.615	0.58	R > C > PR
Expt. 13	$W_{C8} = (7, 9, 9), W_{C1-C7,C9-C24} = (1, 1, 3)$	0.588	0.59	0.58	R > C > PR
Expt. 14	$W_{C9} = (7, 9, 9), W_{C1-C8,C10-C24} = (1, 1, 3)$	0.586	0.59	0.58	R > C > PR
Expt. 15	$W_{C10} = (7, 9, 9), W_{C1-C9,C11-C24} = (1, 1, 3)$	0.588	0.597	0.59	R > PR > C
Expt. 16	$W_{C11} = (7, 9, 9), W_{C1-C10,C12-C24} = (1, 1, 3)$	0.589	0.61	0.58	R > C > PR
Expt. 17	$W_{C12} = (7, 9, 9), W_{C1-C11,C13-C24} = (1, 1, 3)$	0.568	0.615	0.57	R > PR > C
Expt. 18	$W_{C13} = (7, 9, 9), W_{C1-C12,C14-C24} = (1, 1, 3)$	0.584	0.588	0.58	R > C > PR
Expt. 19	$W_{C14} = (7, 9, 9), W_{C1-C13,C15-C24} = (1, 1, 3)$	0.583	0.592	0.58	R > C > PR
Expt. 20	$W_{C15} = (7, 9, 9), W_{C1-C14,C16-C24} = (1, 1, 3)$	0.587	0.588	0.58	R > C > PR
Expt. 21	$W_{C16} = (7, 9, 9), W_{C1-C15,C17-C24} = (1, 1, 3)$	0.59	0.593	0.58	R > C > PR
Expt. 22	$W_{C17} = (7, 9, 9), W_{C1-C16,C18-C24} = (1, 1, 3)$	0.576	0.598	0.58	R > PR > C
Expt. 23	$W_{C18} = (7, 9, 9), W_{C1-C17,C19-C24} = (1, 1, 3)$	0.583	0.57	0.58	C > PR > R
Expt. 24	$W_{C19} = (7, 9, 9), W_{C1-C18,C20-C24} = (1, 1, 3)$	0.574	0.590	0.57	R > PR > C
Expt. 25	$W_{C20} = (7, 9, 9), W_{C1-C19,C21-C24} = (1, 1, 3)$	0.586	0.592	0.58	R > C > PR
Expt. 26	$W_{C21} = (7, 9, 9), W_{C1-C20,C22-C24} = (1, 1, 3)$	0.587	0.5802	0.58	C > PR > R
Expt. 27	$W_{C22} = (7, 9, 9), W_{C1-C21,C23-C24} = (1, 1, 3)$	0.589	0.587	0.58	C > PR > R
Expt. 28	$W_{C23} = (7, 9, 9), W_{C1-C22,C24} = (1, 1, 3)$	0.583	0.59	0.57	R > C > PR
Expt. 29	$W_{C24} = (7, 9, 9), W_{C1-C23} = (1, 1, 3)$	0.589	0.58	0.587	C > PR > R
Expt. 30	$W_{C1,C5-C12} = (7, 9, 9), W_{C2-C4,C13-C24} = (1, 1, 3)$	0.57	0.61	0.58	R > PR > C
Expt. 31	$W_{C1,C5-C12} = (1, 1, 3), W_{C2-C4,C13-C24} = (7, 9, 9)$	0.53	0.51	0.52	C > PR > R

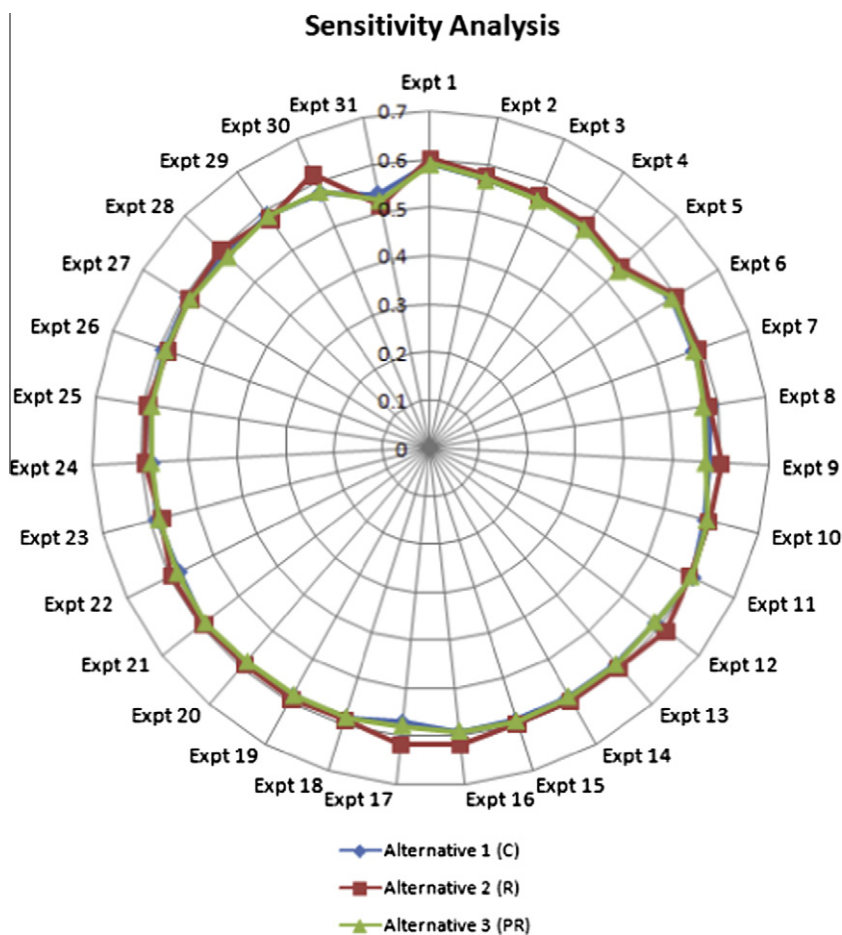


Fig. 3. Results of sensitivity analysis.

$$\begin{aligned}
 d_i^- &= \sqrt{\frac{1}{3}[(0.556 - 9)^2 + (1.353 - 9)^2 + (9 - 9)^2]} \\
 &+ \sqrt{\frac{1}{3}[(0.566 - 9)^2 + (5.247 - 9)^2 + (9 - 9)^2]} + \dots \\
 &+ \sqrt{\frac{1}{3}[(0.556 - 9)^2 + (3.395 - 9)^2 + (7 - 9)^2]} \\
 &= 140.96
 \end{aligned}$$

Likewise, using distances  $d_i^+$  and  $d_i^-$  (Eq. (14)), we compute the closeness coefficient ( $CC_i$ ) of the three alternatives. For example, for alternative A1, the closeness coefficient is given by:

$$CC_i = d_i^- / (d_i^- + d_i^+) = 140.96 / (140.96 + 115.39) = 0.549$$

Likewise,  $CC_i$  for the other two alternatives are computed. The final results are shown in Table 11.

By comparing the  $CC_i$  values of the three alternatives (Table 11), we find that  $A2(C) > A1(R) > A3(PR)$ . Therefore, alternative A2 (R or ridesharing) is recommended as sustainable transportation for the city.

### 5.1. Sensitivity analysis

To investigate the impact of criteria weights (denoted by  $W_{C_i}$  for criteria  $C_i$  where  $i = 1, 2, \dots, n$ ) on the selection of sustainable transportation, we conduct the sensitivity analysis. Thirty-one experiments were conducted. The details of the 31 experiments are presented in Table 12.

It can be seen in Table 12, that in the first five experiments, weights of all criteria are set equal to (1, 1, 3), (1, 3, 5), (3, 5, 7), (5, 7, 9) and (7, 9, 9) respectively. In experiments 6–29, the weight of one criteria is set as highest (7, 9, 9) one by one and the remaining criteria are set to the lowest value (1, 1, 3). The goal is to see which criteria is most important in influencing the decision making process. For example, in experiment 6, the criteria C1 has the highest weight = (7, 9, 9) whereas the remaining criteria have weight = (1, 1, 3). In experiment 30, the weights of all the “Cost” category criteria are set as highest that is, criteria C1, C5–C12 = (7, 9, 9) while the weight of remaining criteria are set as lowest = (1, 1, 3). In experiment 31, we do the inverse by setting the weights of “Cost” category criteria (C1, C5–C12) as lowest = (1, 1, 3) while the weights of other criteria as highest = (7, 9, 9). The results of the sensitivity analysis are presented in Fig. 3.

It can be seen from Table 12 and Fig. 3 that out of 31 experiments, alternative A2 (Ridesharing) has score highest in 24 experiments. In the remaining experiments (experiment numbers 8, 11, 23, 26–27, 29 and 31), the alternative A1 (Carsharing) has emerged as the winner. Therefore, for this city, alternative A1 (Ridesharing) is recommended as the most sustainable alternative for implementation.

## 6. Conclusion

In this paper, we present a multi-criteria decision making approach for sustainability assessment of urban transportation systems under fuzzy environment. The proposed approach comprises of three steps. In step 1, the criteria for evaluating sustainability of urban transportation systems are identified. These criteria are Operating costs, Safety, Security, Reliability, Air Pollutants, Noise, GHG emissions, Usage of fossil fuels, Travel costs, Waste from road transport, Energy consumption, Land usage, Accessibility, Benefits to economy, Competency, Equity, Possibility of expansion, Mobility, Productivity, Occupancy rate, Share in public transit, Convenience to use, Quality of service, and Tangibles. In step 2, the experts provide linguistic ratings to the criteria and the

alternatives. Fuzzy TOPSIS is used to aggregate the ratings and generate an overall performance score for measuring sustainability of each alternative. The alternative with the highest score is selected as the best sustainable transportation system and recommended for implementation in the city. In the third and the last step, we perform sensitivity analysis to determine the influence of criteria weights on the decision making process.

The strength of our approach is the ability to perform sustainability assessment of transportation systems under partial or incomplete information. The proposed approach can be practically applied by cities in evaluation and selection of sustainable transportation systems. Since the decision making process is sensitive to the number of participants involved and their expertise with the subject, they should be carefully chosen.

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