Journal of Hydrology (2006) **331**, 293–299

available at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/jhydrol

Parameter estimation of an ARMA model for river flow forecasting using goal programming

Kourosh Mohammadi [a,*], H.R. Eslami [b], Rene Kahawita [c]

[a] *Tarbiat Modarres University, Irrigation and Drainage Engineering, P.O. Box 14115-336 Tehran, Iran*
[b] *Jamab Consulting Engineers Company, Tehran, Iran*
[c] *Ecole Polytechnique de Montreal, Montreal, Que., Canada*

Received 17 March 2005; received in revised form 14 May 2006; accepted 16 May 2006

KEYWORDS
Statistical models;
River flow forecasting;
Goal programming;
Auto regressive moving average

Summary River flow forecasting constitutes one of the most important applications in hydrology. Several methods have been developed for this purpose and one of the most famous techniques is the Auto regressive moving average (ARMA) model. In the research reported here, the goal was to minimize the error for a specific season of the year as well as for the complete series. Goal programming (GP) was used to estimate the ARMA model parameters. Shaloo Bridge station on the Karun River with 68 years of observed stream flow data was selected to evaluate the performance of the proposed method. The results when compared with the usual method of maximum likelihood estimation were favorable with respect to the new proposed algorithm.

Introduction

In long or short-term river operation, river flow estimation is an important parameter. One of the common methods employed is based on using past observed data and forecasting river discharge in the future or using time series analysis. The field of time series analysis has changed in the last decade due to progress and acquisition of new knowledge in non-linear dynamics (Sprave, 1994). Nevertheless, there are still applications where the accurate estimation of linear processes such as auto regressive moving average (ARMA) models are sufficient especially when they are used for linear time series analysis (Hwarng, 2001). However, the methods for this class of models were developed more than 20 years ago, with the restrictions of the then currently available computer resources. Therefore, it is necessary to test the new approaches in applying the ARMA models in time series analysis.

In recent years, artificial neural networks have been investigated to substitute the ARMA models in estimating time series data. Abrahart and See (2000) compared ARMA models to artificial neural network (ANN) for forecasting river flow data for two contrasting catchments. The relative performance between the ANN and ARMA forecasts were quite similar at each station using common data inputs. Application of ARMA models in short-term rainfall prediction

* Corresponding author. Tel.: +98 21 2537156; fax: +98 21 4196524.
E-mail addresses: kouroshm@modares.ac.ir, kourosh.mohammadi@gmail.com (K. Mohammadi).

0022-1694/$ - see front matter
doi:10.1016/j.jhydrol.2006.05.017

for real-time flood forecasting was investigated by Toth et al. (2000). They used three models including ARMA, ANNs and nearest-neighbour approaches. Hwarng (2001) compared an ARMA(p, q) model with an ANN to forecast time series. He presented a summary of other researchers' work and concluded that ANNs are not better than traditional ARMA models in performance if there is no non-linearity in the data.

There have been several attempts to estimate the parameters of ARMA models by different researchers (Ljung and Box, 1979; Ansely, 1979; Gardner et al., 1980; Pearlman, 1980; Melard, 1984; Azrak and Melard, 1998). Most of these methods are based on the Kalman filter which needs a great amount of computation. Monte-Carlo experiments have shown that for ARMA models with relatively short lengths of data, e.g. 50 or 100 observations, exact maximum likelihood estimation is far superior to conditional maximum likelihood estimation or least-squares estimation (Azrak and Melard, 1998). Recently, artificial neural network techniques have been used to estimate ARMA parameters. Chenoweth et al. (2000) showed that an ANN is not able to estimate the order of ARMA models accurately when the number of data points is less than 100.

Goal programming has been used successfully in several different fields for multi criteria decision making. In the 1960s the idea was presented by Charnes and Cooper (1961) by minimizing the sum of the absolute goal deviations. Since then, summaries and reviews of goal programming have been published by several authors, such as Ijiri (1965), Ignizio (1981), and Hillier and Lieberman (1990). Two main drawbacks in goal programming are the mathematical expression of goals and constraints and optimizing all goals simultaneously (Arikan and Gungor, 2001). Advances in computers have assisted in solving the second problem and it is now possible to compute large matrices and obtain the answers in a short time.

The purpose of this research is to apply the goal programming technique in applied hydrology. An attempt has been made to estimate the ARMA model parameters in order to forecast river flow. The objective of the study is to minimize the estimation error in forecasted time series within a specified season rather than for the whole series during a year. The main reason for this point of view is the suitability of time series analyses for the specified season. Some seasons such as spring and fall in this case study could not be modeled using time series analysis. The average percentage error for the whole series was 24.2% but for the period of October–January it was about 36%. The main reason for this was the heavy rainfalls and floods in these seasons which made it difficult to find a good pattern in time series. Summer season was the best season for forecasting using an ARMA model in the study area with an average error of 16%.

Study area

Karun River watershed located in the south-west of Iran with an area of 23,250 km^2 is part of the Persian Gulf watershed. The elevation varies between zero near sea level to more than 4000 m in the mountains. There are 83 river flow gauges on the Karun River and its branches. The longest data set belongs to Ahwaz station near Ahwaz city from 1950 and Fig. 1 shows the river flow histogram in that station (Jamab, 1999). There are two dams constructed on the river and the first station above the upper dam, Karun 1, is the Shaloo bridge station. Since the data at this station are not affected by the dam, they were selected for this research. Monthly river flow data in this station from 1933 to 2001 were used for evaluating the proposed method. The monthly average river flow is 826.2 cubic meters per month. The minimum and maximum monthly average flows are 334.81 and 1763.18 in September and March, respectively.

Traditional ARMA process estimation

The ARMA(p, q) is one of the most traditional techniques in time series analysis. The assumed model is of the form

$$x_t = \phi_0 + \phi_1 x_{t-1} + \cdots + \phi_p x_{t-p} + z_t - \theta_1 z_{t-1} - \cdots - \theta_q z_{t-q} \tag{1}$$

where p is the order of the autoregressive part, q is the order of the moving average part, $\phi_1, \ldots, \phi_p$ are the autoregressive parameters, ϕ_0 is a constant offset, and $\theta_1, \ldots, \theta_q$ are the moving average parameters. z_t denotes the series of errors. The time series x_t should be stationary. The first step is to determine the model orders p and q. It thus becomes necessary to calculate the autocorrelation function and partial autocorrelation function. The plot of these two functions provides hints with respect to the model orders. Nevertheless, this method is only useful for low model orders and does not provide a reliable tool for model identification. If there is seasonality in the data, but no trend, then the data can be modeled as:

$$x_t = s_t + Y_t \tag{2}$$

where Y_t is a stationary process. The seasonality component is such that

$$s_t = s_{t-h} \tag{3}$$

where h denotes the length of the period (Bogacka, 2004) and

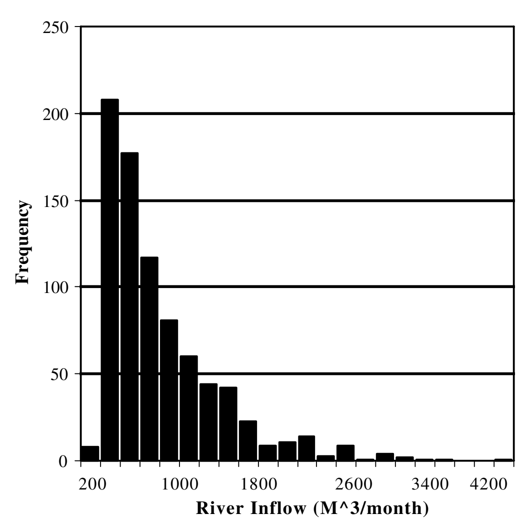

Figure 1 Histogram of Karun River inflow in Shaloo Bridge station.

$$\sum_{k=1}^{h} s_k = 0 \tag{4}$$

A seasonal ARMA model, denoted by ARMA$(P, Q)_h$ is of the form

$$x_t = \Phi_1 B^h x_{t-1} + \cdots + \Phi_P B^{Ph} x_{t-P} + z_t - \Theta_1 B^h z_{t-1} - \cdots - \Theta_Q B^{Qh} z_{t-Q} \tag{5}$$

where $\Phi(B^h)$ and $\Theta(B^h)$ are, respectively, the seasonal AR and MA operators, with seasonal period of length h.

The second step is to estimate the model parameters. Two current methods used for this problem are least-squares and maximum likelihood estimation. Since there are various reasons to keep the model order as low as possible, information criteria may be introduced to combine the need for a good fit with the principle of parsimony. These criteria (e.g. Bayesian Information Criterion-BIC, Akaike's Information Criterion-AIC) join the residual variance on the one hand and the method orders on the other. The analyst's aim is then to minimize such a criterion.

In order to find the number of AR and MA parameters, autocorrelation functions (ACF) and partial autocorrelation functions (PACF) were calculated (Figs. 2 and 3, respectively). Based on these graphs, the value of ACF and PACF are relatively high for lag 1 and 2; therefore, 2 AR and 2 MA parameters were considered to be a good model for the Karun river flow at the Shaloo Bridge station. The ARMA model parameters were computed using MiniTAB under the Windows operating system.

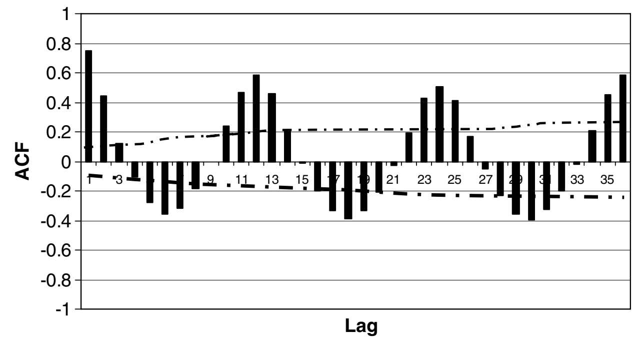

Figure 2 Autocorrelation function for Karun River inflow at Shaloo Bridge station.

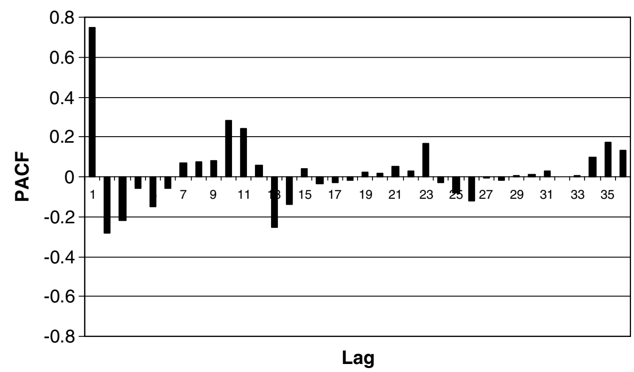

Figure 3 Partial autocorrelation function for Karun River inflow at Shaloo Bridge station.

Estimating ARMA model parameters using goal programming

ARMA model parameters computed in the previous section using maximum likelihood were then refined using an optimization method. Goal programming (GP) is a method that allows several objectives or goals to be attained simultaneously. In this method, the deviation from the goal is measured and after representing the objective function mathematically, a solution which minimizes the weighted sum of the goal deviations is searched. Because of the ACF and PACF results, a maximum of 2 AR and MA parameters have been considered for analysis and all combinations of model parameters from ARMA(1,0) to ARMA(2,2) were tested. The proposed objective function is as follows:

Table 1 Summary of parameter estimation results for the complete time series in the calibration phase

Model (none seasonal)(seasonal)	Method	Mean absolute error			SD
		Maximum	Average	Minimum	
Without constant offset					
ARMA (1,0)(1,0)	Maximum likelihood	233.51	20.87	0.04	20.08
	GP	74.02	20.40	0.03	16.76
ARMA (2,0)(2,0)	Maximum likelihood	246.31	32.19	0.01	26.34
	GP	75.01	19.58	0.15	16.78
With constant offset					
ARMA (1,0)(1,0)	Maximum likelihood	253.82	34.08	0.02	23.03
	GP	73.05	22.28	0.03	17.21
ARMA (2,0)(2,0)	Maximum likelihood	182.25	24.23	0.13	22.98
	GP	75.04	17.28	0.01	16.09
ARMA (1,1)(0,0)	Maximum likelihood	226.73	26.27	0.09	23.88
	GP	85.01	21.24	0.02	17.73
ARMA (2,2)(0,0)	Maximum likelihood	226.73	26.27	0.09	23.88
	GP	94.99	25.81	0.18	18.61

$$\min \sum_{i=1}^{NT} U_m \times EP_i + V_m \times EN_i \quad m = 1, \ldots, 12 \tag{6}$$

and constraints are:

$$\begin{aligned} &AR_1 \times X_{i-1} + AR_2 \times X_{i-2} + \cdots + AR_n \times X_{i-n} \\ &\quad + MA_1 \times R_{i-1} + MA_2 \times R_{i-2} + \cdots + MA_n \times R_{i-n} \\ &\quad + SAR_1 \times X_{i-12} + SAR_2 \times X_{i-24} + \cdots + SAR_n \times X_{i-n\times 12} \\ &\quad + SMA_1 \times R_{i-12} + SMA_2 \times R_{i-24} + \cdots + SMA_n \times R_{i-n\times 12} \\ &\quad + C_m + EP_i - EN_i < (1 + \mathrm{div}) \times X_i \end{aligned}$$

$$\begin{aligned} &AR_1 \times X_{i-1} + AR_2 \times X_{i-2} + \cdots + AR_n \times X_{i-n} \\ &\quad + MA_1 \times R_{i-1} + MA_2 \times R_{i-2} + \cdots + MA_n \times R_{i-n} \\ &\quad + SAR_1 \times X_{i-12} + SAR_2 \times X_{i-24} + \cdots + SAR_n \times X_{i-n\times 12} \\ &\quad + SMA_1 \times R_{i-12} + SMA_2 \times R_{i-24} + \cdots + SMA_n \times R_{i-n\times 12} \\ &\quad + C_m + EP_i - EN_i > (1 - \mathrm{div}) \times X_i \end{aligned}$$

$$0 \leqslant EP_i \leqslant \mathrm{Ediv} \times X_i$$
$$0 \leqslant |EN_i| \leqslant \mathrm{Ediv} \times X_i$$

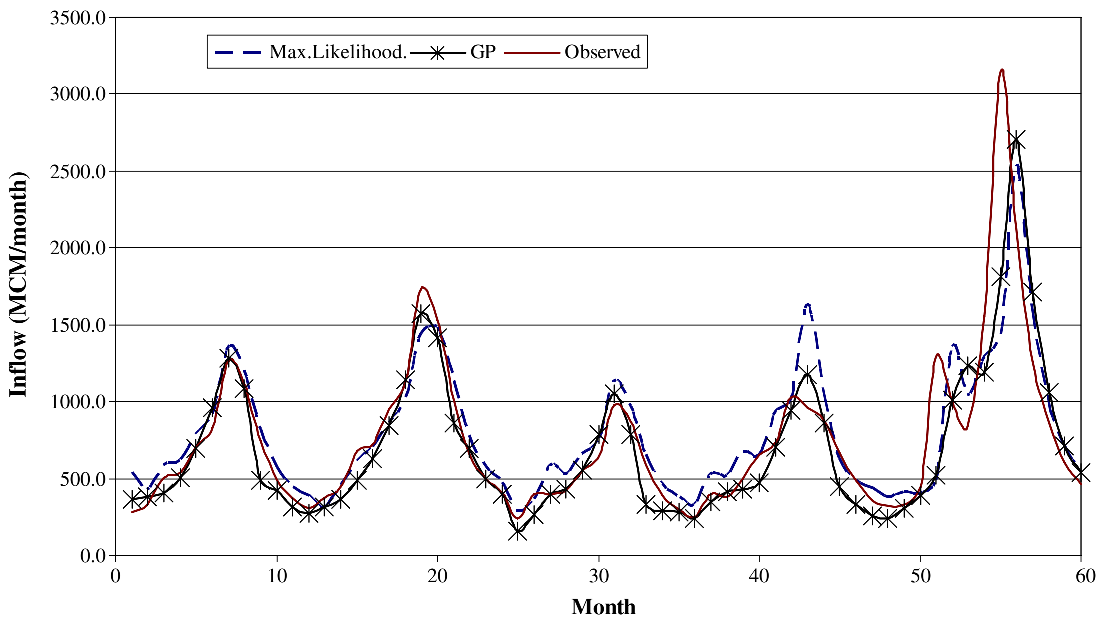

Figure 4 Comparison between different methods and observed data for calibration period.

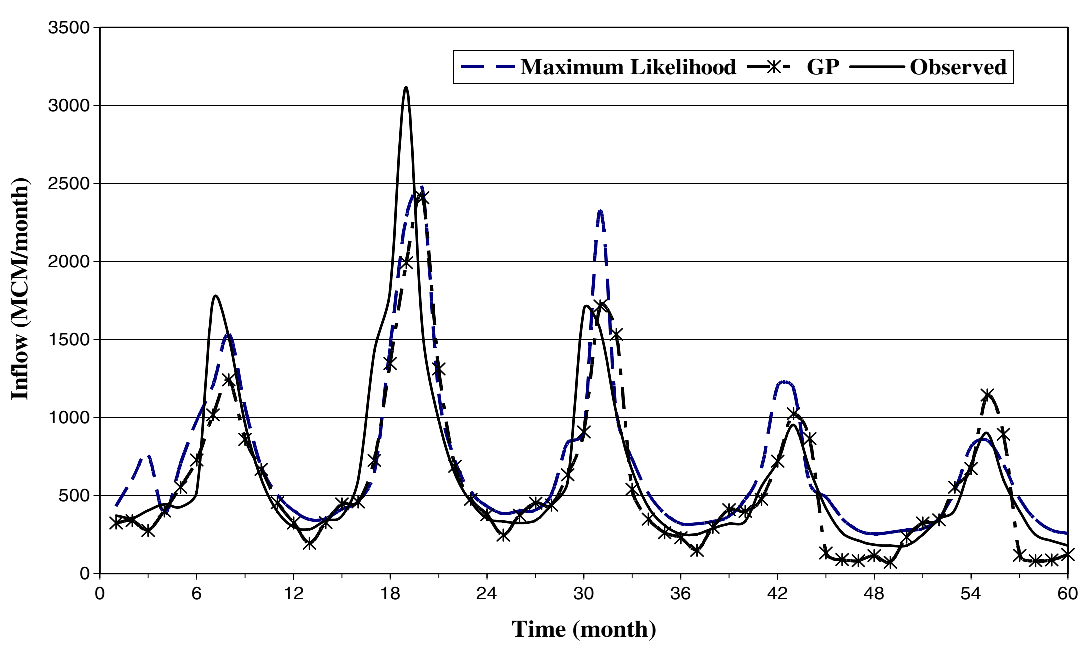

Figure 5 Comparison between different methods and observed data for verification period.

where $AR_1, \ldots, AR_n$ are non-seasonal autoregressive parameters, $SAR_1, \ldots, SAR_n$ are seasonal autoregressive parameters, $MA_1, \ldots, MA_n$ and $SMA_1, \ldots, SMA_n$ are non-seasonal and seasonal moving average parameters, U_m and V_m are the coefficients for monthly deviation from the actual value, EP is the positive relative error, EN is the negative relative error, Ediv is the relative error range for maximum possible error, div is the relative error for forecasting, X is the number of data points in the time series data, R is the residual series, C_m represents the constants in the model, and NT is the number of data in the time series.

The FORTRAN 99 programming language was used to develop the optimization program with two objectives (goals) being considered in the model: minimizing the deviation in the whole series and minimizing the deviation in the specific season which was from October to January.

Results and discussion

In this work, two methods have been used to identify the ARMA model parameters. These two methods are the maximum likelihood which is used in most commercial software and error minimization using goal programming. The latter has the advantage that one can simultaneously calibrate the model for the whole series as well as for a specific season. In order to evaluate the proposed method, 63 years of data were used to estimate the model parameters and the last five years were selected for model verification. Tables 1 and 3 summarize the different models tested for calibration and verification phases. Both seasonal and non-seasonal ARMA models were tested and results for ARMA(2, 0)(2, 0) with no MA parameter which had the best results are shown in Figs. 4 and 5 for calibration and verification periods,

Table 2 Summary of parameter estimation results for a specific season using the complete series in the calibration phase

Model (none seasonal)(seasonal)	Method	Mean absolute error	
		Maximum	Average
Without constant offset			
ARMA (1, 0)(1, 0)	Maximum likelihood	74.02	24.33
	GP	126.65	32.38
ARMA (2, 0)(2, 0)	Maximum likelihood	75.01	24.61
	GP	94.03	27.57
With constant offset			
ARMA (1, 0)(1, 0)	Maximum likelihood	73.05	25.34
	GP	182.25	29.88
ARMA (2, 0)(2, 0)	Maximum likelihood	75.04	24.75
	GP	226.73	28.71
ARMA (1, 1)(0, 0)	Maximum likelihood	85.01	27.14
	GP	226.73	28.71
ARMA (2, 2)(0, 0)	Maximum likelihood	94.85	35.88
	GP	233.51	29.76

Table 3 Summary of parameter estimation results for the complete time series for the verification period

Model (none seasonal)(seasonal)	Method	Mean absolute error			SD
		Maximum	Average	Minimum	
Without constant offset					
ARMA (1, 0)(1, 0)	Maximum likelihood	78.08	24.94	0.91	18.71
	GP	153.44	47.04	0.86	35.58
ARMA (2, 0)(2, 0)	Maximum likelihood	75.66	21.89	0.30	18.26
	GP	110.83	44.55	1.35	27.81
With constant offset					
ARMA (1, 0)(1, 0)	Maximum likelihood	70.76	26.95	0.23	19.61
	GP	87.57	28.21	0.17	19.86
ARMA (2, 0)(2, 0)	Maximum likelihood	67.25	20.43	0.06	15.01
	GP	104.36	31.46	0.39	19.35
ARMA (1, 1)(0, 0)	Maximum likelihood	79.77	19.27	1.03	16.79
	GP	90.16	19.48	0.25	21.68
ARMA (2, 2)(0, 0)	Maximum likelihood	72.3	21.87	0.26	16.94
	GP	95.70	17.58	0.23	19.95

Table 4 Summary of parameter estimation results for a specific season using the complete series for the verification phase

Model (none seasonal)(seasonal)	Method	Mean absolute error	
		Maximum	Average
Without constant offset			
ARMA (1,0)(1,0)	Maximum likelihood	54.96	23.29
	GP	89.20	36.85
ARMA (2,0)(2,0)	Maximum likelihood	52.58	22.31
	GP	60.07	25.63
With constant offset			
ARMA (1,0)(1,0)	Maximum likelihood	49.09	22.49
	GP	87.57	34.66
ARMA (2,0)(2,0)	Maximum likelihood	53.14	23.42
	GP	104.36	32.65
ARMA (1,1)(0,0)	Maximum likelihood	60.38	23.56
	GP	90.16	32.71
ARMA (2,2)(0,0)	Maximum likelihood	72.30	34.04
	GP	95.70	30.00

respectively. Since it was not possible to show all 63 years of data clearly in one graph, five years were randomly selected from the calibration or training data. Month number 1 in this figure is September. For other models during the calibration period, GP showed better results in general. During the verification period, ARMA(2,2) with GP method performed better than the other models. However, the maximum likelihood approach also could forecast the river flow as well as the GP approach or even better in other types of ARMA models especially for verification data. As is evident in Figs. 4 and 5, the GP method could estimate the high flows better than the maximum likelihood. On the other hand, superior performances for forecasting the low flows were obtained with the maximum likelihood principle.

In order to test the model for a specific season, October to January was selected. The reason for this selection was the inability of other forecasting methods such as rainfall-runoff, snowmelt models, artificial neural networks and the standard ARMA methods in predicting the river flow with enough accuracy for the case study. All these methods were tested prior to using the ARMA model. Consequently, the reservoir faced operational difficulties during these months. Tables 2 and 4 show the results of the models which have relatively close agreement with the maximum likelihood method. Results show that when the number of AR and MA parameters increase, the GP method has a better performance compared with the maximum likelihood method. ARMA(2,2) in both the calibration and verification stages had better predictive capabilities. To compare the models, the mean absolute error was used as the defining criterion.

Conclusion

Goal programming was used to calculate ARMA model coefficients. Results for the Karun River proved that the method is accurate and efficient enough for this purpose. The main disadvantage of the GP method is the high computational cost especially for a large number of model parameters. This negative aspect may be mitigated somewhat by using efficient optimization algorithms to reduce the computational time. With the rapid advances in computer technology, however, it is expected that significant reductions in this constraint will be realized making it more and more feasible to test these new methods.

Since this technique is in its preliminary phase, the numbers of AR and MA parameters are manually selected by the user. The next step will be the optimizing of these model parameters using integer programming.

References

Abrahart, R.J., See, L., 2000. Comparing neural network and autoregressive moving average techniques for the provision of continuous river flow forecasts in two contrasting catchments. Journal of Hydrological Processes 14, 2157–2172.

Ansely, C.F., 1979. An algorithm for the exact likelihood of a mixed autoregressive moving average process. Biometrika 66, 59–65.

Arikan, F., Gungor, Z., 2001. An application of fuzzy goal programming to a multiobjective project network problem. Journal of Fuzzy Sets and Systems 119, 49–58.

Azrak, R., Melard, G., 1998. The exact quasi-likelihood of time dependent ARMA models. Journal of Statistical Planning and Inference 68, 31–45.

Bogacka, B., 2004. Time Series. Course Handouts. Available from: <http://www.maths.qmul.ac.uk/~bb/TS_MainPage.html>.

Charnes, A., Cooper, W.W., 1961. Management Models and Industrial Application of Linear Programming. John Wiley & Sons, New York, USA.

Chenoweth, T., Hubata, R., St. Louis, R.D., 2000. Automatic ARMA identification using neural networks and the extended samples autocorrelation function: a reevaluation. Journal of Decision Support Systems 29, 21–30.

Gardner, G., Harvey, A.C., Phillips, G.D.A., 1980. Algorithm AS 154: an algorithm for exact maximum likelihood estimation of autoregressive moving average models by means of Kalman filtering. Journal of the Royal Statistical Society Series C – Applied Statistics 29, 311–322.

Hillier, F.S., Lieberman, G.J., 1990. Introduction to Operations Research, fifth ed. McGraw-Hill Inc., New York, USA.

Hwarng, H.B., 2001. Insights into neural network forecasting of time series corresponding to ARMA (p,q) structures. Journal of Omega 29, 273–289.

Ignizio, J., 1981. Linear programming in single and multiple-objective system. Prentice-Hall, Englewoods, Cliffs, NJ, USA.

Ijiri, Y., 1965. Management Goals and Accounting for Control. North-Holland, Amestrdam, The Netherlands.
Jamab, 1999. National Water Master Plan. Jamab Consulting Engineers Company, Iran (in Persian).
Ljung, G., Box, G.E.P., 1979. The likelihood functions of stationary autoregressive moving average models. Biometrika 66, 265—270.
Melard, G., 1984. Algorithm AS197: a fast algorithm for the exact likelihood of autoregressive moving average models. Journal of the Royal Statistical Society Series C — Applied Statistics 33, 104—114.
Pearlman, J.G., 1980. An algorithm for the exact likelihood of a high order autoregressive moving average process. Biometrika 67, 232—233.
Sprave, J., 1994. Linear neighborhood evolution strategy. In: Proceedings of the 3rd Annual Conference on Evolutionary Programming. World Scientific, Singapore, pp. 42—51.
Toth, E., Brath, A., Montanari, A., 2000. Comparison of short-term rainfall prediction models for real-time flood forecasting. Journal of Hydrology 239, 132—147.