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Type synthesis of unified planar–spatial mechanisms by systematic linkage and topology matrix-graph technique

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Abstract

A systematic linkage technique and a topology matrix-graph approach are put forward for type synthesis of unified planar–spatial mechanism. By using the systematic linkage technique, numerous associated linkages of unified planar–spatial mechanism are created. The equivalent relations between the actual mechanism and the associated linkage are determined, and the formulas for calculating the number of DOF, the complexity, and acceptable associated linkage are derived. By using the topology matrix-graph approach, corresponding topology matrices are constructed, and some topology embryonic graphs and their isomeric topology embryonic graphs of the unified planar–spatial mechanism are constituted. Finally, the numerous topology graphs for type synthesis of unified planar–spatial mechanism are derived. The results show that the two approaches are simple and effective for type synthesis of unified planar–spatial mechanisms.

Keywords: Type synthesis; Planar mechanism; Spatial mechanism; Associated linkage

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1. Introduction

In the type synthesis of mechanism, the technique of the linkage system has been used to analyze, create, and synthesize various planar mechanisms [1–3]. The topology graph is a simple and effective tool for type synthesis of various mechanisms [4], and the topology embryonic graph is a basis for deriving topology graph [5]. Johnson presented the technique of the linkage system for synthesizing the planar mechanisms by creating diversified acceptable planar mechanisms [1]. Tingli studied type synthesis of planar mechanism by using a topology graph of the linkage and the close-conjunction matrix [4]. Shubo studied topology graph theory [5]. McCarthy conducted a synthesis of planar RR and spatial CC chains [6]. Hunt et al. conducted type synthesis of some 3-, 4-, 5-, 6-DOF parallel spatial manipulators [7–11]. Rao studied loop-based detection of isomorphism among chains and inversions and the type of freedom in a multiple degree of freedom chain for a planar mechanism [12]. Linda studied the type synthesis of planar mechanisms by a graph grammar approach [13]. Dar-Zen conducted a study on the topological synthesis of fractionated geared differential mechanisms [14]. Liu Chuanhe presented a variable evolution method of planar pin-jointed kinematic chains based on the fractal method of mechanism evolution [15]. Jin-Kui Chu determined isomorphism among kinetic chains [16]. Carlo analyzed kinematotropic properties and pair connectivities in single-loop spatial mechanisms [17].

However, ways to use the systematic linkage technique for type synthesis of unified planar and spatial mechanisms have not been developed yet. In the light of acceptable associated linkages with different DOF, the questions of how to derive the topology embryonic graph and the topology graph of an associated linkage for synthesizing unified planar–spatial mechanism have not been solved.

For this reason, the type synthesis of unified planar–spatial mechanism is studied by using the systematic linkage technique and topology matrix-graph approach. The following problems are solved in this paper: (1) Determine the relationships between unified planar–spatial mechanism and its associated linkage. (2) Derive the acceptable combinations of basic links with different DOF. (3) Create the topology matrix of acceptable associated linkage. (4) Solve the isomeric topology embryonic graphs. (5) Constitute the acceptable topology graph of unified planar and spatial mechanism.

2. Relations between the mechanism and its associated linkage

An associated linkage of the unified planar and spatial mechanism is an effective tool for type synthesis of mechanism. Johnson had defined two equivalent conditions between the planar mechanism and its associated linkage as below [1].

(1) The DOF of planar mechanism must be the same as that of its associated linkage.

(2) In any associated linkage of planar mechanism, it is permissible to replace any two binary links connected by a one rotational joint with a revolute joint R, a prismatic joint P, because, for each of R and P, there is one-DOF for relative motion, as shown in Fig. 1a.

A unified planar and spatial mechanism generally includes various types of basic kinematic pairs (such as cam pair, gear pair, and helical pair, revolute joint R, prismatic joint P, spherical joint S, universal joint U, cylinder pair C, and plane joint E) and various types of parts (such as

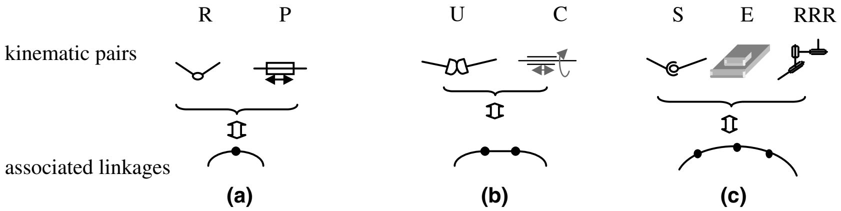


Fig. 1. Equivalent relations between the kinematic pairs of mechanism and associated linkage.

link, cam, gear, frame, and so on). However, the associated linkage of the planar and spatial mechanism include many rotational joints with one-DOF and various types of basic links, such as binary, ternary, quaternary, pentagonal, and hexagonal links, and so on. Here, a rotational joint with one-DOF and a binary link in the associated linkage are designated as a dot and a curve, respectively.

In order to conduct type synthesis of unified planar–spatial mechanisms, based on the two equivalent conditions for the planar mechanism above and the consideration of the relative DOF of various types of kinematic pairs, two additional equivalent conditions between the unified planar–spatial mechanism and its associated linkage are explained as follows.

(3) In any associated linkage of unified planar–spatial mechanism, it is permissible to replace any three binary links connected in series by two rotational joints with a universal joint U or a cylinder joint C, because, for each of U, C, there is two-DOF for relative motion, as shown in Fig. 1b.

(4) In any associated linkage of unified planar–spatial mechanism, it is permissible to replace any four binary links connected in series by three rotational joints with a spherical joint S or a plan joint E, because, for each of S, E, there is three-DOF for relative motion, as shown in Fig. 1c.

3. Derivation of the DOF equation of the associated linkage

The DOF of the mechanism with one link fixed can be calculated by Grubler's equation [2,3]:

$$F = \lambda(l - n - 1) + \sum_{i=1}^{n} f_i \tag{1}$$

where F is the number of DOF for the mechanism; λ is the degree of freedom of space within which the mechanism operates for planar motions $\lambda = 3$, and for spatial motions $\lambda = 6$; l is the number of actual links, including the frame, and n is the number of kinematic pairs for connecting two actual links; f_i is the relative DOF of the ith kinematic pair in the mechanism.

Based on the four equivalent conditions above, from Eq. (1), a formula for calculating the DOF of the associated linkage of unified planar–spatial mechanism is derived below,

$$F = \lambda(L - N - 1) + N \tag{2}$$

where F is the number of DOF of the associated linkage, which is the same as that of unified planar–spatial mechanism; L is the number of basic links including the frame, L is usually larger than that of actual mechanism; N is the number of rotational joints in the associated linkage, which is the sum of relative DOF of all the kinematic pairs in the mechanism. By comparing Eq. (1) with Eq. (2), it is known that $l - n = L - N$.

4. The acceptable number of links and complexity of associated linkage

The associated linkage is made up of various types of basic links, which are connected by the rotational joints with one-DOF. The element number of the basic link determines the type of basic link designated as binary, ternary, quaternary, pentagonal, or hexagonal links [1]. All the elements of basic links must be of parallel rotational joints in the associated linkage of the planar mechanism, and must be of non-parallel rotational joints in the associated linkage of the spatial mechanism. The number of the basic links in the associated linkage are designated as B, T, Q, P, and H for the binary, ternary, quaternary, pentagonal, and hexagonal links, respectively. Thus, the number of basic link L is the sum of the types of basic links for any associated linkage of unified planar and spatial mechanism. It can be calculated as follows:

$$L = B + T + Q + P + H + \cdots \tag{3}$$

Since each binary link contributes two elements of two rotational joints, and each ternary link contributes three elements of three rotational joints, and so on, with two elements required for each rotational joint, the following equation for calculating the number of rotational joints N is obtained below.

$$N = (2B + 3T + 4Q + 5P + 6H + \cdots)/2 \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (2), the following modified version of Grubler's equation for the associated linkage of unified planar–spatial mechanism with one link fixed is obtained.

$$F = \lambda(B + T + Q + P + H + \cdots) - \frac{(\lambda - 1)}{2}(2B + 3T + 4Q + 5P + 6H + \cdots) - \lambda \tag{5}$$

The derivation implies that Eq. (5) would contain additional terms if links of greater complexity than the hexagonal type were included, but this would be getting into a range of impracticality foreign to the intended simplicity of design.

By subtracting Eq. (5) from Eq. (3), the following equation (6) is obtained.

$$T + 2Q + 3P + 4H = 2(L - F - \lambda)/(\lambda - 1) \tag{6}$$

When $\lambda = 3$ for planar linkages, Eq. (2) gives

$$N = \frac{3}{2}L - \frac{F + 3}{2} \tag{7}$$

Since the value of N and L must be positive integer, and the value of F must be an integer, positive or negative, from an examination of Eq. (7) the following two conclusions are drawn as follows:

1. If F is an odd number ($-1, +1, +3$, etc.), L must be an even number.
2. If F is an even number ($0, +2, +4$, etc.), L must be an odd number.

In the definitions of the different terms, the number of binary, ternary, quaternary, pentagonal, and hexagonal links must be positive integers or 0. Thus, the result from the left side expression ($T + 2Q + 3P + 4H$) of Eq. (6) must be a positive integer or 0. Hence the result from the right side expression ($L - F - 3$) of Eq. (6) must also be a positive integer or 0. Therefore, based on an examination of Eq. (7) and two conclusions above, the third conclusion is drawn as below:

3. The expression $L - F - 3$ must be a positive even number or 0.

From the third conclusion and Eq. (7), the following equation can be derived for planar linkages.

$$T + 2Q + 3P + 4H = 2J, \quad J = 0, 1, 2, \ldots$$

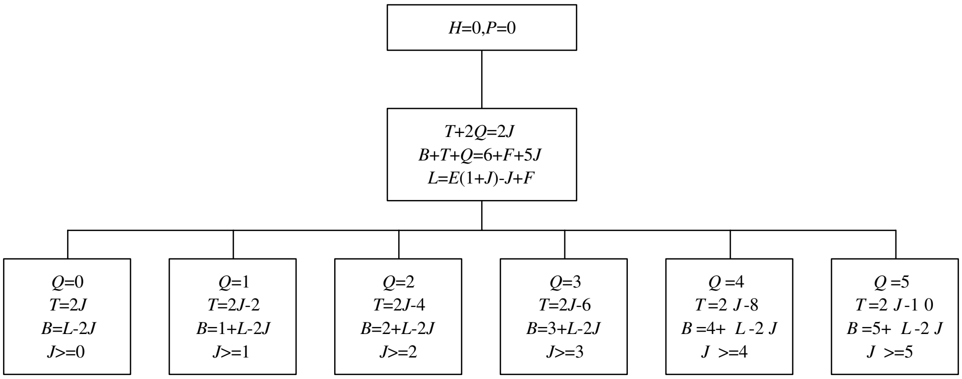


Fig. 2. Decision tree for progressive simultaneous solutions of all acceptable associated linkages from Eqs. (11) and (12) only in the case of $H = 0$, $P = 0$.

Let $H = 0$ and $P = 0$, and a set of Eq. (12) is produced as follows:

$$T + 2Q = 2J, \quad B + T + Q = J \tag{12}$$

Then, if $Q = 0$, 1, 2, 3, 4, 5, respectively, the six acceptable associated linkages could be derived from the set of Eq. (12). The progressive simultaneous solutions of these equations are facilitated by using a decision tree, as shown in Fig. 2. Since T in these equations of the decision tree must be a positive integer or 0, the values of J are limited ($J \geqslant 0, 1, 2, 3, 4, 5$) for the cases of $T = 2(J - i)$ ($i = 1, 2, 3, 4, 5$), respectively. For this reason, the limited values of J are given for each case of the acceptable associated linkages in each leaf frame of the decision tree, as shown in Fig. 2.

Similarly, from Eq. (9), all acceptable associated linkages can be derived in the cases of $H = 0$, $P = 1, 2, 3$; $H = 1$, $P = 0, 1$; and $H = 2$, $P = 0$; respectively. The decision trees for progressive simultaneous solutions of these equations are shown in Fig. 3. However, in the cases of $H = 0$, $P = 4, 5, \ldots$; $H = 1$, $P = 2, 3, \ldots$; and $H = 2$, $P = 1, 2, \ldots$, etc., the acceptable associated linkages correspond to $J > 5$, and it is, hence, not necessary to discuss them in detail here.

Each leaf frame in all the decision trees corresponds to each case of all acceptable associated linkages in Figs. 2 and 3. Based on the unified equations in each leaf frame, for $J = 0, 1, \ldots, 5$; $\lambda = 3, 6$; and different DOF, the number of various types of basic links (b, t, q, p, h) is obtained for each acceptable associated linkage, as shown in Table 1. The acceptable number of links L for $J \leqslant 5$ is shown in Table 1.

From the resulting derivation shown in Table 1 and the equations in the decision trees in Figs. 2 and 3, some important conclusions are obtained.

1. In each type of acceptable associated linkage of unified planar–spatial mechanism, different DOF can only cause a change in the number of binary link, but does not influence upon the other types of basic links. The number of binary links for each acceptable associated linkage increases or decreases along with the increase or decrease of value of DOF.
2. If both a planar associated linkage and a spatial associated linkage with the same complexity J and DOF include the same group of basic links (such as ternary, quaternary, pentagonal, or hexagonal links), then the difference between the number of binary links in the planar associated linkage and that in the spatial associated linkage is $3(1 + J)$.

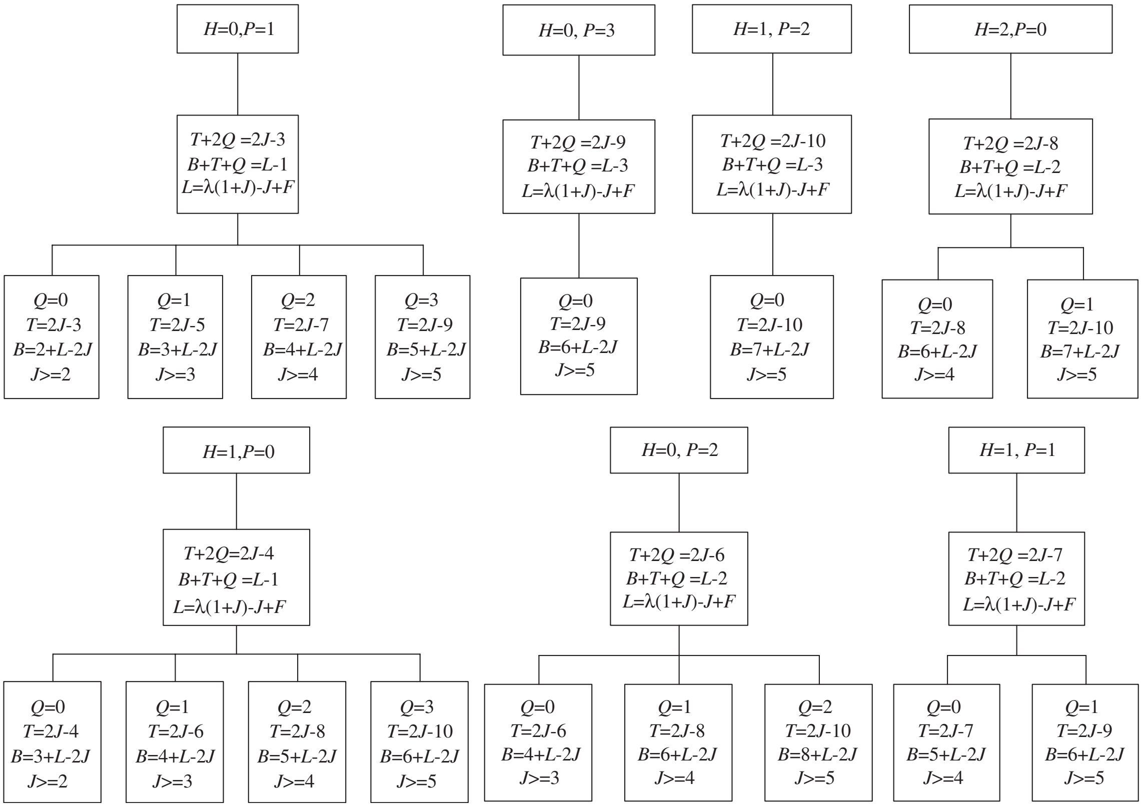


Fig. 3. The decision trees for progressive simultaneous solutions of all acceptable associated linkages from Eqs. (11) and (12) in the cases of $H = 0$, $P = 1$, 2, 3; $H = 1$, $P = 0$, 1, 2 and $H = 2$, $P = 0$, respectively.

3. In the case of the lowest complexity of the associated linkage ($J = 0$), the simplest acceptable associated linkages for different DOF only include the binary links, and the number of binary links is $B = 3 + F$ for a planar mechanism, and $B = 6 + F$ for a spatial mechanism.
4. In the case of $J = 1$, the acceptable associated linkages include the binary, ternary, and quaternary links for different DOF (F values), and the acceptable number of links is $L = 2\lambda + F - 1$. It implies that in the case of $J = 1$, the pentagonal and hexagonal links are not needed for type synthesis of the unified planar and spatial mechanisms for different DOF (F values).

6. Topology matrix

The topology matrix is an effective tool for constituting topology embryonic graph. Before constituting it, all binary links must be removed from the associated linkage, and the four symbols (h, p, q, t) are used to designate hexagonal, pentagonal, quaternary, ternary basic links in

Table 1
The possible associated linkages with different DOF (F value) for complexity $J = 0, 1, 2, 3, 4, 5$ and the acceptable number of links

J	B		T	Q	P	H	No.
	$\lambda = 3$	$\lambda = 6$					
0	$3 + F$	$6 + F$	0	0	0	0	0.1
1	$3 + F$	$9 + F$	2	0	0	0	1.1
	$4 + F$	$10 + F$	0	1	0	0	1.2
2	$3 + F$	$12 + F$	4	0	0	0	2.1
	$4 + F$	$13 + F$	2	1	0	0	2.2
	$5 + F$	$14 + F$	0	2	0	0	2.3
	$5 + F$	$14 + F$	1	0	1	0	2.4
	$6 + F$	$15 + F$	0	0	0	1	2.4
3	$3 + F$	$15 + F$	6	0	0	0	3.1
	$4 + F$	$16 + F$	4	1	0	0	3.2
	$5 + F$	$17 + F$	2	2	0	0	3.3
	$6 + F$	$18 + F$	0	3	0	0	3.4
	$5 + F$	$17 + F$	3	0	1	0	3.5
	$6 + F$	$18 + F$	1	1	1	0	3.6
	$7 + F$	$19 + F$	0	0	2	0	3.7
	$6 + F$	$18 + F$	2	0	0	1	3.8
	$8 + F$	$20 + F$	0	1	0	1	3.9
4	$3 + F$	$18 + F$	8	0	0	0	4.1
	$4 + F$	$19 + F$	6	1	0	0	4.2
	$5 + F$	$20 + F$	4	2	0	0	4.3
	$6 + F$	$21 + F$	2	3	0	0	4.4
	$7 + F$	$22 + F$	0	4	0	0	4.5
	$6 + F$	$21 + F$	4	0	0	1	4.6
	$7 + F$	$22 + F$	2	1	0	1	4.7
	$8 + F$	$23 + F$	0	2	0	1	4.8
	$8 + F$	$23 + F$	1	0	1	1	4.9
	$9 + F$	$24 + F$	0	0	0	2	4.10
	$5 + F$	$20 + F$	5	0	1	0	4.11
	$6 + F$	$21 + F$	3	1	1	0	4.12
	$7 + F$	$22 + F$	1	2	1	0	4.13
	$7 + F$	$22 + F$	2	0	2	0	4.14
	$9 + F$	$24 + F$	0	1	2	0	4.15
5	$3 + F$	$21 + F$	10	0	0	0	5.1
	$4 + F$	$22 + F$	8	1	0	0	5.2
	$5 + F$	$23 + F$	6	2	0	0	5.3
	$6 + F$	$24 + F$	4	3	0	0	5.4
	$7 + F$	$25 + F$	2	4	0	0	5.5
	$8 + F$	$26 + F$	0	5	0	0	5.6
	$5 + F$	$23 + F$	7	0	1	0	5.7
	$6 + F$	$24 + F$	5	1	1	0	5.8
	$7 + F$	$25 + F$	3	2	1	0	5.9
	$8 + F$	$26 + F$	1	3	1	0	5.10
	$7 + F$	$25 + F$	4	0	2	0	5.11
	$9 + F$	$27 + F$	2	1	2	0	5.12
	$11 + F$	$29 + F$	0	2	2	0	5.13

Table 1 (*continued*)

J	B		T	Q	P	H	No.
	$\lambda = 3$	$\lambda = 6$					
	9 + F	27 + F	1	0	3	0	5.14
	7 + F	24 + F	6	0	0	1	5.15
	7 + F	25 + F	4	1	0	1	5.16
	8 + F	26 + F	2	2	0	1	5.17
	9 + F	27 + F	0	3	0	1	5.18
	8 + F	26 + F	3	0	1	1	5.19
	9 + F	27 + F	1	1	1	1	5.20
	9 + F	27 + F	2	0	0	2	5.21
	10 + F	28 + F	0	1	0	2	5.22
	10 + F	28 + F	0	0	2	1	5.23

Acceptable number of basic links

$L = \lambda J + \lambda – J + F$

J	$\lambda = 3$ for planar	$\lambda = 6$ for spatial
0	3 + F	6 + F
1	5 + F	11 + F
2	7 + F	16 + F
3	9 + F	21 + F
4	11 + F	26 + F
5	13 + F	31 + F

the associated linkage, respectively. These symbols of basic links are lined in a row from left to right according to the decreasing order of their element number at the tope of topology matrix. Each symbol in the tope row aligns with each column of the topology matrix. Next, all symbols in the tope row are rotated about a diagonal of the topology matrix to the left column. Each symbol in this column aligns with each row of the topology matrix. Next, the symbols of the similar basic links are put together. For instance, the symbols of several similar hexagonal basic links are put together and represented by "$h\ldots$ " or "$hhh\ldots$". Therefore, a formal topology matrix A is expressed as Eq. (13), and a_{ij} is an item in the ith row and the jth column in A.

$$A = [a_{ij}]_{n\times n} = \begin{array}{c} \\ h \\ \cdots \\ p \\ \cdots \\ q \\ \cdots \\ t \\ \cdots \end{array} \begin{array}{c} \begin{array}{cccccccc} h\ldots & & p\ldots & & q\ldots & & t\ldots & \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1i} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2i} & \cdots & a_{2j} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ii} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{j1} & a_{j2} & \cdots & a_{ji} & \cdots & a_{jj} & \cdots & a_{jn} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{ni} & \cdots & a_{nj} & \cdots & a_{nn} \end{pmatrix} \end{array} \tag{13}$$

6.1. The characteristics of the topology matrix

Some important characteristics of the topology matrix A are explained as follows.

1. The value of item a_{ij} in A is equal to the number of the path between a basic link aligning with the jth column and another basic link aligning with the ith row. Each path is constituted by a group of series connected binary links.
2. The topology matrix A is a symmetry matrix with n rows and n columns. Since any type of basic link does not allow connecting itself by a path, the value of each item at the diagonal is 0. Therefore, the number of the path between a basic link aligning with the ith row and the same basic link aligning with the ith column is 0, that is $a_{ii} = 0$, $i = 1,2,\ldots,n$.
3. The sum of the values of all items in A is equal to the sum of the products by the number of various basic links and their element numbers. Since both ends of each path must be connected to two other basic links, the sum of the values of all items is equal to twice of the sum of the path number m. Thus, a relevant formula is

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} = 3T + 4Q + 5P + 6H = 2m \tag{14}$$

4. Both the ith row and the ith column in A correspond to the same symbol of the basic link, and the sum of the items in the ith row is equal to the sum of the items in the ith column. When both the ith row and the ith column correspond to the symbol t, q, p or h of a basic link, respectively, the sums of the items in both the ith row and the ith column are 3, 4, 5, or 6, respectively. A relevant formula is

$$\sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ji} = \begin{cases} 3 \Rightarrow T \\ 4 \Rightarrow Q \\ 5 \Rightarrow P \\ 6 \Rightarrow H \end{cases} \tag{15}$$

7. Relation and theory of topology matrix and topology embryonic graph

In fact, a topology graph of the mechanism is the simplification of the associated linkage. A topology embryonic graph is simplification of the associated linkage without binary link. Thus, numerous topology graphs can be derived from one topology embryonic graph. In the topology graph, a dot is used to designate a basic link, and a path (curve) is used to designate a group of serially binary links connected by rotational joints. In order to simplify the type synthesis of mechanism, each serially connected binary links in the associated linkage is replaced by a path, and other type of basic links (such as hexagonal, pentagonal, quaternary, and ternary basic links) are replaced by dots that simultaneously connect 6, 5, 4 and 3 paths, respectively. Thus a topology embryonic graph is formed.

7.1. Determination of homogenous topology embryonic graph

In the topology embryonic graphs, some homogenous graphs may exist, and they must be determined to avoid synthesizing similar type mechanisms [4,5]. Consequently, two rules for determining the homogenous topology embryonic graph of associated linkage are put forward and explained as below.

Rule 1: If the dot and the path in the two topology embryonic graphs remain in accordance with one by one, then the two topology embryonic graphs are a homogenous one.

Rule 2: In the topology matrix A, if a basic link aligning with the ith row is similar to the one aligning with the jth row, when all the items of the ith row are simultaneously exchanged with the corresponding items of the jth row, and all the items of the ith column are simultaneously exchanged with the corresponding items of the jth column, then a new matrix A_1 is derived, and the topology embryonic graph of A_1 is identical to that of A.

In fact, the result of the rule 2 is equivalent to that of exchanging the two similar basic links in the topology embryonic graphs. The essential topology embryonic graph of the mechanism, however, has not been changed. For example, a planar or spatial mechanism with DOF F can be synthesized by using No. 3.2 associated linkage ($Q = 1$, $T = 4$ and $B = 4 + F$) or ($Q = 1$, $T = 4$ and $B = 16 + F$) in Table 1, respectively. When all the binary links are removed, only one quaternary link q and four ternary links (t_1, t_2, t_3, t_4) remain in the associated linkage. From Eq. (13), a topology matrix A and its relative topology embryonic graph are derived, as shown in Fig. 4a. In the four ternary links (t_1, t_2, t_3, t_4) of A, when all the items of the second row are simultaneously exchanged with the corresponding items of the third row, and all the items of the second column are simultaneously exchanged with the corresponding items of the third column, a new topology matrix A_1 and its relative topology embryonic graph are derived, as shown in Fig. 4b. Obviously, the new topology embryonic graph of A_1 is the same as the original one of A.

Based on the topology matrix shown in Fig. 4a, if one quaternary link q and four ternary links (t_1, t_3, t_2, t_4) are connected by eight paths in a different order, such as (q, t_1, t_2, t_3, t_4), (q, t_2, t_1, t_3, t_4), (q, t_3, t_2, t_1, t_4), (q, t_4, t_2, t_3, t_1), (q, t_1, t_3, t_2, t_4), (q, t_1, t_4, t_3, t_2) and (q, t_1, t_2, t_4, t_3), seven different topology matrices could be obtained, and the topology embryonic graph corresponding to each topology matrix is an isomeric embryonic graph. Therefore, from Eqs. (13)–(15), several different

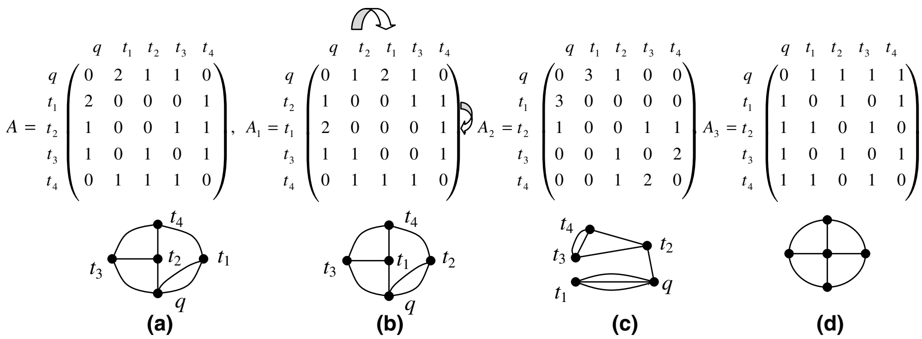


Fig. 4. The four topology matrixes and their topology embryonic graphs for the five basic links $Q = 1$, $T = 4$.

topology matrices and their topology embryonic graphs could be derived from one associated linkage. These topology embryonic graphs could also be determined whether they are the isomeric embryonic graph or not.

Based on the two rules above, the processes of solving many the same topology embryonic graphs could be avoided by using a topology matrix. On the other hand, from some known planar and spatial mechanisms, some new topology matrices and their topology embryonic graphs could be derived by using the reverse synthesis approach [1].

7.2. Determination of three different types of topology embryonic graphs

The topology embryonic graphs can be classified into open, closed, and unreasonable topology embryonic graphs in the light of their characteristics. If an unreasonable topology embryonic graph is used for type synthesis of mechanism, then the new mechanism may include some redundant parts, and this mechanism must be avoided. In order to identify different type of topology embryonic graph, some definitions are given and explained, respectively.

Definition 1. In a topology embryonic graph, if all paths from a dot can only be connected with another dot, then the first dot is an end dot. For instance, dot t_1 is an end dot, as shown in Fig. 4c. A topology embryonic graph with an end dot is defined as an open topology embryonic graph.

Definition 2. A topology embryonic graph without any end dot is a closed topology embryonic graph.

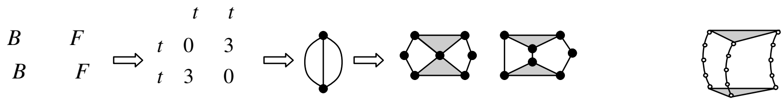
Definition 3. If a topology embryonic graph can be divided into two island graphs directly or by removing a path from it, then this topology embryonic graph is an unreasonable one.

Definition 1 can be explained from the characteristic 4 of topology matrix A. When an item's value in A is equal to the element number of a basic link aligning this item's row, all the other item's values in this item's row must be 0. This implies that all paths drawing from a basic link in the item's row (an end dot) are connected with another basic link in the item's column (another dot). Therefore, the topology embryonic graph derived from this matrix is an open one.

Definition 2 can be explained by the characteristic 3 of A. When an item's value in A is less than the element number of a basic link aligning this item's row, some other item's value in this item's row must not be 0. This implies that all paths extending from a basic link in this item's row (a dot) are connected with some other basic links in this item's column (some dots). Therefore, the topology embryonic graph derived from this matrix is a closed one.

Definition 3 can be explained by the characteristic 4 of topology matrix A. When an item's value in A is larger than the element number of a basic link in this item's row, Eq. (15) of topology matrix A cannot be satisfied. This implies that the topology embryonic graph derived from this matrix is an unreasonable one.

In summary, if an item's value in topology matrix A is equal to, less than or larger than the element number of a basic link aligning this item's row (or column), respectively, the topology embryonic graph derived from A is an open, closed or unreasonable topology embryonic graph, respectively.



7.3. The simplest open topology embryonic graph

The simplest open topology embryonic graph only includes two end dots and some paths for connecting the two end dots. Obviously, the configuration of the simplest open topology embryonic graphs must be the simplest one, therefore, they are most useful for type synthesis of various mechanisms.

From an open topology embryonic graph of No. 1.1 associated linkage ($T = 2$, $B = 3 + F$) (see Fig. 5a), a Watt planar mechanism with one-DOF (see Fig. 5d), a Stephenson planar mechanism with one-DOF [1,2] (see Fig. 5e), and a planar parallel mechanism with 3-DOF (see Fig. 5f) [10,11] can be synthesized by using the simplest 2×2 topology matrix (see Fig. 5b).

From an open topology embryonic graph of No. 1.1 associated linkage ($T = 2$, $B = 6 + F = 9$) (see Fig. 5a), some types of the spatial 3-RPS, 3-UPU, 3-UUP, 3-RSP, 3-RPRU, and 3-RRPR parallel mechanisms with 3-DOF [9–12] (see Fig. 5g) can be synthesized by using the simplest 2×2 topology matrix (see Fig. 5b).

From an open topology embryonic graph of No. 4.10 associated linkage ($P = 2$, $B = 24 + F = 30$) (see Fig. 5h), some types of the spatial 6-SPU, 6-SUP, 6-URPU, and 3/6-SPS parallel mechanisms with 6-DOF [10] (see Fig. 5k) can be synthesized by using the simplest 2×2 topology matrix (see Fig. 5h).

Similarly, from an open topology embryonic graph of No. 2.3 associated linkage ($Q = 2$, $B = 14 + F = 18$), some novel types of the spatial 2UPU/2UPS, 2UUP/2USP, 2RPS/2UPS parallel mechanisms with 4-DOF can be synthesized by using the simplest 2×2 topology matrix. From an open topology embryonic graph of No. 3.7 associated linkage ($P = 2$, $B = 19 + F = 24$), some types of spatial parallel mechanisms with 5-DOF can be synthesized by using the simplest 2×2 topology matrix.

8. Solving an isomeric topology embryonic graph

In the light of each acceptable associated linkage in Table 1, there may be many types of topology matrices. Thus, how to derive all topology matrices and their corresponding isomeric topology embryonic graphs from each acceptable associated linkage is a key problem to solve. The process of deriving isomeric topology embryonic graphs is:

1. Remove all the binary links from each acceptable associated linkage.
2. Constitute the basic topology matrix of each associated linkage without any binary link.
3. By using symbol "/", separate different symbols of basic links and their corresponding digital items in the first row of the topology matrix.
4. Based on Eq. (15), the sum of all digital items in the first row must be equal to the element number of the basic link in the first row. Rearrange all digital items in the first row, and avoid using any item repeatedly.
5. List all non-repeated digital groups in the first row.
6. From each non-repeated digital group, derive its topology matrix and topology embryonic graph.
7. Based on the basic characteristics of the topology matrix, if possible, exchange the items in another row or column and derive a new isomeric topology embryonic graph.
8. Repeat steps 5–7, till all isomeric topology embryonic graphs are derived.
9. Delete unreasonable isomeric topology embryonic graphs.

For example, from the No. 4.4 associated linkage $Q = 3$, $T = 2$, $B = 6 + F$ for planar mechanism or $Q = 3$, $T = 2$, $B = 21 + F$ for spatial mechanism with different DOF in Table 1, their isomeric topology embryonic graphs can be derived as below.

1. Remove all binary links from the No. 4.4 associated linkage, thus the three quaternary links (q_1, q_2, q_3) and the two ternary links (t_1, t_2) remain in associated linkage.
2. After division by "/", the order of the symbols of the basic links at tope row of topology matrix A is qqq/tt. All items in the first row of A are also divided into two parts. Based on Eq. (15), 14 types of non-repeated digital groups are arranged as follows:

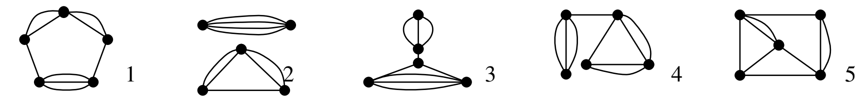
   (000/22) (000/31) (001/30) (001/21) (001/30) (001/21) (011/11)
   (011/20) (002/20) (02/10) (022/00) (003/10) (031/00) (004/00)

3. Based on rule 2 and the non-repeated digital groups above, analyze and determine possible repeated digital groups.

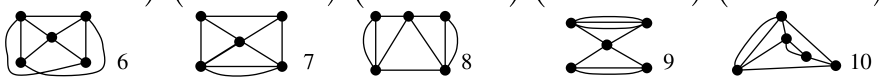
For instance, both (000/31) and (000/13) are repeated digital groups, since both 1 and 3 in the different digital groups locate in the same type of subgroup (ternary links tt). However, (000/31), (003/10) and (031/00) are non-repeat digital groups, since 3 and 1 in the different digital groups locate in different types of subgroup (quaternary qqq and ternary link tt), respectively. Although both (000/22) and (004/00) are non-repeated digital groups, two different analysis matrices could be derived from them. After an exchange of a suitable row and column in each matrix, the two same matrices could be obtained, and only one of the two same matrices retain. Similarly, another two same matrices could be obtained from digital groups (000/22) and (031/00), respectively, as shown in graph 1 and in graph 18 of Fig. 6. In the other case, from (011/1), (01/20), (002/20), (012/10), and (022/00), other five different analysis matrices and their relative topology embryonic graphs could be derived.

Based on these non-repeated digital groups and characteristics of topology matrix-graph, 18 different topology matrices and their relative topology embryonic graphs are derived as follows. Based on Definition 3, it is known that the graphs 2, 3, 4 are unreasonable topology embryonic graphs.

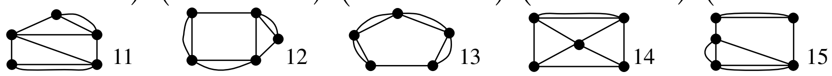
$$\begin{pmatrix}0&0&0&2&2\\0&0&3&0&1\\0&3&0&0&0\\2&0&0&0&0\\2&1&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&2&2\\0&0&4&0&0\\0&4&0&0&0\\2&0&0&0&1\\2&0&0&1&0\end{pmatrix}\begin{pmatrix}0&0&0&3&1\\0&0&3&0&1\\0&3&0&0&1\\3&0&0&0&0\\1&1&1&0&0\end{pmatrix}\begin{pmatrix}0&0&1&3&0\\0&0&2&0&2\\1&2&0&0&1\\3&0&0&0&0\\0&2&1&0&0\end{pmatrix}\begin{pmatrix}0&0&1&2&1\\0&0&2&1&1\\1&2&0&0&1\\2&1&0&0&0\\1&1&1&0&0\end{pmatrix}$$



$$\begin{pmatrix}0&1&1&1&1\\1&0&1&1&1\\1&1&0&1&1\\1&1&1&0&1\\1&1&1&1&0\end{pmatrix}\begin{pmatrix}0&1&1&1&1\\1&0&1&0&2\\1&1&0&2&0\\1&0&2&0&0\\1&2&0&0&0\end{pmatrix}\begin{pmatrix}0&1&1&1&1\\1&0&2&1&0\\1&2&0&0&1\\1&1&0&0&1\\1&0&1&1&0\end{pmatrix}\begin{pmatrix}0&1&1&1&1\\1&0&3&0&0\\1&3&0&0&0\\1&0&0&0&2\\1&0&0&2&0\end{pmatrix}\begin{pmatrix}0&1&1&2&0\\1&0&2&0&1\\1&2&0&0&1\\2&0&0&0&1\\0&1&1&1&0\end{pmatrix}$$



$$\begin{pmatrix}0&1&1&2&0\\1&0&1&0&2\\1&1&0&1&1\\2&0&1&0&1\\0&2&1&1&0\end{pmatrix}\begin{pmatrix}0&0&2&2&0\\0&0&1&1&2\\2&1&0&0&1\\2&1&0&0&0\\0&2&1&0&0\end{pmatrix}\begin{pmatrix}0&0&2&2&0\\0&0&2&0&2\\2&2&0&0&0\\2&0&0&0&1\\0&2&0&1&0\end{pmatrix}\begin{pmatrix}0&1&2&1&0\\1&0&1&1&1\\2&1&0&0&1\\1&1&0&0&1\\0&1&1&1&0\end{pmatrix}\begin{pmatrix}0&1&2&1&0\\1&0&2&0&1\\2&2&0&0&0\\1&0&0&0&2\\0&1&0&2&0\end{pmatrix}$$



$$\begin{pmatrix}0&2&2&0&0\\2&0&0&1&1\\2&0&0&1&1\\0&1&1&0&1\\0&1&1&1&0\end{pmatrix}\begin{pmatrix}0&2&2&0&0\\2&0&1&0&1\\2&1&0&1&0\\0&0&1&0&2\\0&1&0&2&0\end{pmatrix}\begin{pmatrix}0&0&3&1&0\\0&0&0&2&2\\3&0&0&0&1\\1&2&0&0&0\\0&2&1&0&0\end{pmatrix}\begin{pmatrix}0&1&3&0&0\\1&0&0&2&1\\3&0&0&0&1\\0&2&0&0&1\\0&1&1&1&0\end{pmatrix}$$

Fig. 6. The 18 different topology matrices and their isomeric topology embryonic graphs derived from No. 4.4 associated linkage ($Q = 3$, $T = 2$, $B = 21 + F$) for spatial mechanism in Table 1.

9. Solving topology graphs from a topology embryonic graph

A topology embryonic graph cannot be employed directly in the type synthesis of the mechanism until all removed binary links are rearranged systematically into every path in the topology embryonic graph. Consequently, many different topology graphs of the isomeric mechanism need to be derived from the topology embryonic graph. There are many possible ways to rearrange all removed binary links into this topology embryonic graph. How to rearrange all the removed binary links is the second key issue for solving the topology graph. The process of solving a topology graph is explained below

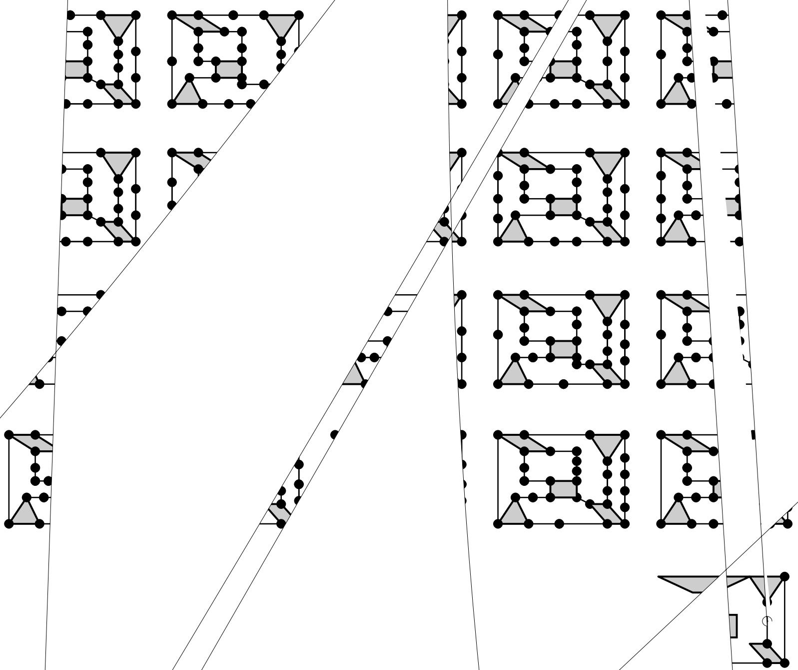
1. In the light of each topology embryonic graph, a table of digital groups is constituted, as shown in Table 2. All items in the first row of the table retain the same as those at tope of the topology matrix in Eq. (13), and every symbol of the basic link has a subscript

Table 2
Digital groups for associated linkage ($Q = 3$, $T = 2$, $B = 21 + F$) of spatial mechanism with $F = 1$

No.	q_1				q_2				q_3				t_1			t_2			Sum
1	2	2	3	3	3	3	1	2	1	3	3	3	2	3	3	2	2	3	44
2	2	2	3	3	3	3	2	1	2	3	3	3	2	3	3	2	1	3	44
3	2	2	3	4	4	3	1	2	1	2	3	3	2	3	2	2	2	3	44
4	2	2	3	3	3	3	1	3	1	2	3	3	2	3	2	2	3	3	44
5	2	2	3	3	3	3	2	3	2	2	3	2	2	3	2	2	3	2	44
6	3	1	3	4	4	3	1	1	1	3	3	3	1	3	3	3	1	3	44
7	3	1	3	2	2	3	1	3	1	3	3	3	1	3	3	3	3	3	44
8	3	1	3	3	3	3	1	2	1	3	3	3	1	3	3	3	2	3	44
9	4	1	3	3	3	3	1	1	1	3	3	3	1	3	3	4	1	3	44
10	4	1	3	3	3	3	1	2	1	2	3	3	1	3	2	4	2	3	44
11	2	1	3	3	3	3	1	3	1	3	3	3	1	3	3	2	3	3	44
12	2	1	3	3	3	3	2	2	2	3	3	3	1	3	3	2	2	3	44
13	2	1	3	3	3	3	2	3	2	3	3	2	1	3	3	2	3	2	44
14	2	1	3	3	3	3	2	2	2	3	4	2	1	4	3	2	2	2	44
15	2	1	3	3	3	3	1	2	1	3	4	3	1	4	3	2	2	3	44
16	1	1	3	4	4	3	1	2	1	3	4	3	1	4	3	1	2	3	44
17	1	1	3	4	4	3	2	2	2	3	4	2	1	4	3	1	2	2	44
18	1	1	4	4	4	4	1	2	1	3	4	2	1	4	3	1	2	2	44
19	1	1	4	3	3	4	1	2	1	3	5	2	1	5	3	1	2	2	44
20	1	1	3	3	3	3	1	2	1	3	5	3	1	5	3	1	2	3	44
21	4	0	3	3	3	3	1	2	1	3	3	3	0	3	3	4	2	3	44
22	4	0	3	3	3	3	2	2	2	3	3	2	0	3	3	4	2	2	44
23	3	0	3	3	3	3	2	2	2	3	3	3	0	3	3	3	2	3	44
24	3	0	3	3	3	3	1	2	1	3	4	3	0	4	3	3	2	3	44
25	3	0	3	3	3	3	1	3	1	2	4	3	0	4	2	3	3	3	44

to distinguish it from other similar basic links. From the second row to the last row of the table, each basic link corresponds to a column, and each element of the basic link corresponds to a sub-column of its column. The number of columns indicates the number of basic links without binary link. The number of sub-columns indicates twice of the number of paths in the topology embryonic graph. For example, there are five columns for five basic links $(q_1, q_2, q_3, t_1, t_2)$, and 18 sub-columns for four elements of (q_1, q_2, q_3) and three elements of (t_1, t_2) in Table 2.

2. An item's value in each sub-column of the column equals to the number of series connected binary between the element of a basic link in the same column and the element of a basic link in another column. Since each binary provides two elements for constituting the isomeric topology graph, the sum of item value of each row must be twice of the number of the binary. A digital group in each row corresponds to a type of isomeric topology graph.
3. Divide each topology embryonic graph into several non-repeated closed loops. Based on the third and fourth conclusions in Section 5, the number of links in each closed loop of the topology graph must be larger than or equal to $3 + F$ for the planar mechanism with DOF F and $6 + F$ for the spatial mechanism with DOF F, respectively.



ext, an exam... ill... ology... s from No. 4 ... associated linkage
3, $T = 2$, B ... + ... pology e...oryonic graph 5 (see Fig. 6), numerous
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using digital group approach, many isomeric topology graphs can be derived from the topology embryonic graph. Some important conclusions are obtained as below.

1. In each type of acceptable associated linkage of unified planar–spatial mechanism, different DOF can only cause a change in the number of binary link, but does not influence upon the other types of basic links. The number of binary links for each acceptable associated linkage increases or decreases along with the increase or decrease of value of DOF.
2. If both a planar associated linkage and a spatial associated linkage with the same complexity J and DOF include the same group of basic links (such as ternary, quaternary, pentagonal, or hexagonal links), then the difference between the number of binary links in the planar associated linkage and that in the spatial associated linkage retain $3(1 + J)$.
3. The simplest associated linkage for different DOF only include the binary links, and the number of binary links is $B = 3 + F$ for planar mechanism, and $B = 6 + F$ for spatial mechanism.
4. In the case of $J = 1$, the pentagonal and hexagonal links are not needed for type synthesis of the unified planar and spatial mechanisms for different F values.
5. In the topology matrix A, if a basic link aligning with the ith row is similar to the one aligning with the jth row, when all the items of the ith row are simultaneously exchanged with the corresponding items of the jth row, and all the items of the ith column are simultaneously exchanged with the corresponding items of the j th column, then a new matrix A_1 is derived, and the topology embryonic graph of A_1 is identical to that of A.
6. If an item's value in topology matrix A is equal to, less than or larger than the element number of a basic link in the item's row (or column), respectively, then the topology embryonic graph derived from topology matrix is an open, closed or unreasonable topology embryonic graph, respectively.

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