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Robust and optimal control of shimmy vibration in aircraft nose landing gear

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ABSTRACT

Shimmy vibration of aircraft nose landing gear is damped and controlled using a nonlinear control which is optimal and robust against parametric uncertainties and external disturbances. Shimmy vibration is the lateral and torsional vibrations in the wheel of the aircraft that is self-excited and causes instability in high speed performances which can damage the landing gear of the aircraft, its fuselage and even may result in hurting the passengers. Thus, control and damping of this vibration are extremely important. In this paper a robust optimal controller is designed by integrating sliding mode control (SMC) together with State-Dependent Riccati Equation (SDRE) to prevent the shimmy vibrations in aircraft nose landing gear. The SDRE compensator controls the nonlinear system in an optimal way while the sliding mode controller guarantees its stability against uncertainties and disturbances. The proposed controller can effectively suppress the shimmy vibration of the landing gear with variable taxiing velocity and wheel caster length. To verify the optimal performance and robustness of the proposed controller, vibration response of the system is simulated by MATLAB software and its performance and efficiency are verified using comparative analysis. Considerable improvement can be seen in the performance of the closed loop system since not only the vibrations are effectively damped but also the consumption of energy is minimized.

1. Introduction

Shimmy vibration is a very important common phenomenon in the landing gear system during either the take off or landing of an aircraft. The required energy of this type of vibration is provided from the kinetic energy of the forward motion of the aircraft [1]. In the landing gear of a taxiing aircraft, shimmy is a state of self-excited oscillations, caused by the dynamic reaction forces between elastic tires and the ground. In fact, shimmy is a combined oscillatory motion of the landing gear in torsional, lateral and longitudinal directions, caused by the interaction between the dynamics of the tire and the landing gear, with a frequency range of 10–30 Hz [2]. Though it can occur in both the nose and the main landing gear, however the shimmy of the nose wheel is more serious and common [3]. Shimmy is an unstable phenomenon and it is affected by certain combination of parameters such as mass, damping coefficient, geometrical quantities, taxiing speed, excitation forces and nonlinearities such as friction and free play. Shimmy not only leads to instabilities which degrade comfort, but also it can affect the pilot's visibility and cause more dangerous results such as loss of control, excessive tire wear, failure of mechanical components or even collapse of the landing gear as a whole.

The first efforts toward decreasing the destructive effects of these vibrations were passive. In order to suppress the shimmy motion, a shimmy damper was used in Boeing 737 and Airbus A-320 aircrafts as a conventional preventive measure [4]. However, as mentioned in [5], shimmy damping requirements often conflict with good high-speed directional control; furthermore, once the landing gear design is completed, the structural parameters for shimmy suppression cannot be changed. Current shimmy suppressing methods are shimmy damper and structural damping. The main disadvantages found in the current shimmy dampers include the need for frequent maintenance and also increasing the temperature which causes the hydraulic fluid to expand and leak through the seals thereby reducing the damping efficiency of the device. Hence, when external disturbances or uncertain parameters arise in the landing gear system, no further action can be taken. In some operational situations, such as worn parts, severe climate, and rough runway, an active control strategy can be effective for shimmy vibration control. With the advent of high-speed and highly reliable microprocessors used in the controller implementation, the idea of active control of landing gears has gained new momentum and is unavoidable.

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Nomenclature

$[A]$	state matrix	M_4	tire damping moment from tread width .... N m
$A_{21,\ldots,33}$	elements of state matrix	M_5	input moment
a	contact patch length .... m	M_z	tire aligning moment .... N m
$[B]$	input matrix	$L(x(t))$	state feedback gain matrix of observer
c	torsional stiffness of strut .... N m/rad	$P(x)$	transformation matrix
C_{F_α}	tire side force derivative 1/rad	$P(x)_{21,22,23}$	elements of transformation matrix
C_{M_α}	tire aligning moment derivative................ m/rad	$Q(x), R(x)$	weighting matrices
$d(x,t)$	external disturbance .... N m	$s(x(t),t)$	integral sliding surface
e_{eff}	effective caster length .... m	t	time .... s
e	wheel caster length................................. m	U	Lyapunov function
F_y	tire side force .... N	u	input in dynamic system
F_z	vertical force .... N	u_{opt}	optimal input in dynamic system
I_z	moment of inertia of strut about z-axis .... kg m^2	V	wheel forward velocity .... m/s
$J(x)$	index (cost) function	x	state of dynamic system
$K(x)$	state feedback gain matrix of controller	y	output of dynamic system
K_e	external moment constant	y_1	lateral shift of leading tire contact point........... m
K	torsional damping of strut (viscous friction moment coefficient)............................... N m/(rad/s)	α	slip angle or deflection angle of tire............... rad
		α_g	limiting slip angle for aligning moment........... deg
K	constant of tread width tire moment........ N m^2/rad	γ	appropriate positive constant
M	moment.. N m	δ	limiting slip angle for tire side force .... deg
M_1	spring moment .... N m	σ	relaxation length of tire deflection.................. m
M_2	damping moment................................. N m	ψ	yaw angle of landing gear.......................... rad
M_3	total tire moment about z-axis................... N m	φ	rake angle .... rad

Recently, some active damping solutions have been investigated for general oscillatory systems and different cases have been studied. First, simple controllers such as PD were developed to damp oscillation of linear second order type systems [6]. Afterward, modern control theories such as optimal control, adaptive control, robust control, fuzzy and neural network based controllers were used to design active damping systems for more complex oscillatory cases. Active control concept presents a possibility of stable control on the vibrating response of the aircraft landing gear. Lately, the active or semi-active vertical vibration control for landing gear has attracted the attention of researchers and has shown some advantages. Although the concept of active landing gear is not new, no aircraft production is yet equipped by such a system, as reported in [7]. Furthermore, there is scant research on developing the control strategy that can deal with time-varying parameters and the uncertainty of the landing gear. These studies are mostly conducted for the vertical suspension systems so far. A common PID was designed for elimination of aircraft landing gear vertical vibration in dissertation [8] and the active control is compared with semi-active control on aircraft suspension system. NASA in [9], started from a simplified model of main landing gear of aircraft and implemented an external servo-hydraulic system for active control in vertical damping. Thus it can be said that landing gear active and semi-active control is restricted to vertical damping and active suspension [8,10,11] while the extension of active concept to landing gear shimmy control is proved to be possible and is considered in this paper. The only nonpassive effort which is made for nonlinear dynamic model of shimmy of aircraft landing gears is related to [12] where semi-active control is proposed for a multi-body aircraft simulation model based on three different control laws.

Investigating the mentioned previous researches also shows that no study is done yet to control the shimmy vibration in an optimal way. Considering the importance of achieving the highest accuracy and stability using the least consumed energy encourages us to design and implement a proper closed loop optimal control system to decrease the unwanted shimmy vibration of the landing gear using the minimum input force.

Some research can be mentioned which control the shimmy vibration in an optimal way in recent years but all of them use linearized approximation of the landing gear dynamics. In [13] model predictive control (MPC) or receding horizon control (RHC) is proposed as an active shimmy suppression strategy for the linearized state space of the system. Kothare et al. [14] consider robust model predictive control (RMPC) methods for a linear parameter varying (LPV) system that has both probabilistic uncertainty and time-varying parameters. In [13], an attempt is made to apply the proposed robust model predictive control strategy to suppress the shimmy during the taxiing and landing of an aircraft with linearized model. Pouly et al. [15] present a controller based on feedback linearization method and an indirect fuzzy adaptive controller is described to perform an active damping of the nose landing gear shimmy phenomenon.

It can be seen that, in the mentioned literature, optimal control of shimmy is done based on the linearized model of the plant. A shimmy vibration in aircraft nose landing gear has a highly nonlinear dynamics for which linear control design is far from adequate.

In this paper in order to control the nonlinear state space of the shimmy in an optimal way, the concept of SDRE is employed. State-Dependent Riccati Equation (SDRE) techniques are rapidly emerging as general design and synthesis methods of nonlinear feedback controllers and estimators for a broad class of nonlinear regulator problems. In essence, the SDRE approach involves mimicking standard of linear quadratic regulator (LQR) formulation for linear systems [16]. In this paper a nonlinear optimal controller is developed for this challenging plant based on the powerful closed loop optimizer tool of SDRE. However the vibrating nature of the studied plant obliges us to guarantee the stability of the system using an additional robust controller. That's why sliding mode controller is also employed here to neutralize the destructive effects of shimmy uncertainties and external disturbances. As a result the unwanted shimmy vibration of the aircraft landing gear is optimally damped in this paper using the proposed nonlinear optimal and robust controller based on integration of optimal SDRE and robust sliding mode controlling strategies. This control strategy not

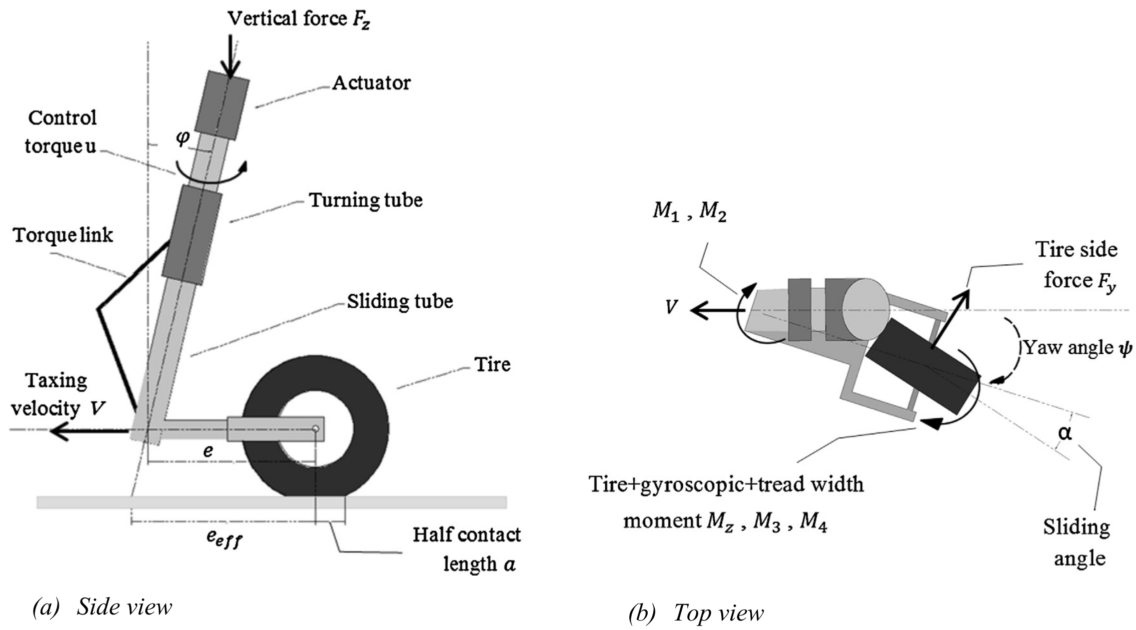

Fig. 1. Schematic model of a nose landing gear of an aircraft.

only causes optimal performance of the system but also guarantees the robustness of the shimmy response.

The organization of this paper is as follows: Section 2 describes a typical landing gear dynamic with nonzero rake angle and shows the model for shimmy vibration of an aircraft nose landing gear. Section 3 formulates the optimal problem incorporated with sliding mode control. First the optimal control problem based on LQR and SDRE methods are explained. Afterwards the robustness of the system is fulfilled by attaching a sliding mode controller which results in Robust Optimal Sliding-Mode Controller (ROSMC). Finally a simulation study is performed in Section 4 to confirm the controller performance. The simulation results verify the effectiveness and robustness of the proposed controller. LQR and SDRE are compared. Also the optimality and robustness of the system is proved by conducting some comparative studies. The correctness, efficiency and even superiority of the proposed strategy over traditional cases are verified by comparing the results with literatures. It will be shown that not only the vibrations are actively damped, but also the consumption of energy is optimized using the proposed method.

2. Modeling of the nose landing gear shimmy

A landing gear assembly is shown in Fig. 1. The nonlinear shimmy dynamic equations are developed. The following equations (1)–(8) describe the torsional dynamics of the landing gear that is derived using Newton–Euler method for rotational motion of the landing gear [12].

$$I_z\ddot{\psi} = M_1 + M_2 + M_3 + M_4 + M_5 \tag{1}$$

where:

$$M_1 + M_2 = c\psi + K\dot{\psi} \tag{2}$$

$$M_3 = M_z - e_{eff}F_y \tag{3}$$

$$e_{eff} = e\cos(\varphi) + \tan(\varphi)\big(\mathcal{R} + e\sin(\varphi)\big) \tag{4}$$

$$F_y = \begin{cases} C_{F_\alpha}\alpha F_z, & \alpha \le \delta \\ C_{F_\alpha}\delta F_z\,\mathrm{sign}(\alpha), & \alpha \ge \delta \end{cases} \tag{5}$$

$$M_z = \begin{cases} C_{M_\alpha}\frac{\alpha_g}{180}\sin(\frac{180}{\alpha_g}\alpha)F_z, & |\alpha| \le \alpha_g \\ 0, & |\alpha| \ge \alpha_g \end{cases} \tag{6}$$

$$M_4 = \frac{k}{V}\cos(\varphi)\dot{\psi} \tag{7}$$

$$M_5 = K_e u \tag{8}$$

Torsional dynamics of the landing gear is described by equation (1) where I_z is the moment of inertia about z-axis, M_1 is a linear spring torque provided by the turning tube and the torque link, M_2 is a damping moment combined of viscous friction in the bearings of the oil pneumatic shock absorber and shimmy damper. M_3 is composed of aligning torque M_z about the tire's center and cornering force F_y. e_{eff} is the effective caster length of landing gear. The damping moment M_4 due to tread width depends on velocity and yaw rate. M_5 is the control moment provided by the active controller.

The total mass of the aircraft's fuselage is assumed to be a single lumped mass that exerts a vertical force F_z on the landing gear.

The lateral deflection of the tire is described by the model of an elastic string proposed by Von Schlippe [7]. Using this approximation, the kinematic relation between the lateral shift y_1 of the leading contact point of the tire and the yaw angle of the wheel ψ is established in equation (9). $\dot{\psi}$ is the yaw rate of the wheel.

$$\dot{y}_1 + \frac{V}{\sigma}y_1 = V\cos(\varphi)\psi + (e_{eff} - a)\cos(\varphi)\dot{\psi} \tag{9}$$

Additionally, slip angle α can be approximately given by

$$\alpha \approx \tan(\alpha) = \frac{y_1}{\sigma} \tag{10}$$

In the above equations, c, K, k, C_{M_α}, C_{F_α}, are experimentally measured constants as detailed in Table 1.

3. Control of nose landing gear

Here two strategies of optimization are presented. The first one is a linear closed loop optimizer tool which is designed according to the linearized model of the system and the second one is a nonlinear optimal controller compatible to the original nonlinear plant of the system. The results of these two approaches are compared and analyzed and the superiority of the latter case is shown.

Table 1
System parameters and their values [16].

Parameter	Parameters and their values		
	Description	Value	Units
	Structure parameters		
e	Caster length	0.12	M
C	Torsional stiffness of strut	-1×10^5	$\mathrm{N\,m\,rad^{-1}}$
K	Torsional damping of strut	-45	$\mathrm{N\,m\,s\,rad^{-1}}$
I_z	Moment of inertia of strut	1	$\mathrm{kg\,m^2}$
φ	Rake angle	0.1571	rad
	Tire parameters		
$\mathcal{R}$	Radius of nose wheel	0.362	m
a	Contact patch length	0.1	m
k	Damping coefficient of elastic tyre	-270	$\mathrm{N\,m^2\,rad^{-1}}$
c_{M_α}	Self-aligning coefficient of elastic tyre	-2	m/rad
c_{F_α}	Restoring coefficient of elastic tyre	20	$\mathrm{rad^{-1}}$
σ	Relaxation length	0.3	m
δ	Restoring force limit	0.087	rad
α_g	Self-aligning moment limit	0.1745	rad
	Continuation parameters		
F_z	Vertical force on the gear	9000	N
v	Forward velocity	75	$\mathrm{m\,s^{-1}}$

3.1. Stability of the system

According to equation (1) and torsional dynamics of the landing gear, the Lyapunov function is chosen as equation (11) to represent the relative vibratory energy of the landing gear system:

$$U = \frac{1}{2} c\psi^2 + \frac{1}{2} I_z \dot{\psi}^2 \tag{11}$$

The rate of change of the Lyapunov function is then:

$$\begin{aligned} \dot{U} &= c\psi\dot{\psi} + I_z\dot{\psi}\ddot{\psi} \\ &= 2c\psi\dot{\psi} + (M_z - e_{eff}F_y) + \left(K + \frac{k}{V}\cos(\varphi)\right)\dot{\psi}^2 \end{aligned} \tag{12}$$

For stability of the system, equation (12) must be negative.

3.2. Control design using linear quadratic regulator

The LQR approach for obtaining an optimal solution of the control problem has the following procedure [17,18]. According to [19], F_y and M_z/F_z are linearly proportional to sideslip angle within a small range. Based on this assumption, the nonlinear dynamic system is linearized using Taylor series expansion and rearranged as state-space equations (13). In this series, sine of the angle alpha can be estimated as alpha itself whenever the angle is sufficiently small. So the state can be summarized after this assumption.

$$\begin{aligned} &\dot{x} = Ax + Bu \\ &\Rightarrow \frac{d}{dt}\begin{bmatrix} \psi \\ \dot{\psi} \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}\begin{bmatrix} \psi \\ \dot{\psi} \\ y_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} M_5 \\ &y = Cx \Rightarrow y = [\psi] = [1 \quad 0 \quad 0]\begin{bmatrix} \psi \\ \dot{\psi} \\ y_1 \end{bmatrix} \end{aligned} \tag{13}$$

where $A \in \mathfrak{R}^{n\times n}$ is the dynamic matrix, $B \in \mathfrak{R}^{n\times m}$ is the input matrix, $C \in \mathfrak{R}^{p\times n}$ is the output matrix, $x \in \mathfrak{R}^n$ is the state vector, $u \in \mathfrak{R}^m$ is the control law, $y \in \mathfrak{R}^p$ is the output vector. The LQR requires controllability condition of the linearized system. In the above equation we have:

$$\begin{aligned} &A_{21} = \frac{c}{I_z}, \qquad A_{22} = \frac{(K + \frac{k}{V}\cos(\varphi))}{I_z}, \\ &A_{23} = \frac{(c_{M_\alpha} - e_{eff}c_{F_\alpha})F_z}{I_z\sigma}, \qquad A_{31} = V\cos(\varphi), \\ &A_{32} = (e_{eff} - a)\cos(\varphi), \qquad A_{33} = -\frac{V}{\sigma} \end{aligned} \tag{14}$$

where $u = M_5$ is the torque of the shimmy plant and can be considered as the controlling input of the system. In order to find the optimum value of the input, following Riccati equation needs to be calculated:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{15}$$

where $P \in R^{n\times m}$ is a solution of Riccati equation, Q and R are the positive definite weighting matrices related to the gain of the states $x(t)$ and control effort $u(t)$, respectively. The optimal control problem is to define the control variables that solve a system of equations, while minimizing a specific functional called the performance index. In this paper, the equations of motion of the shimmy problem are rewritten using the optimal control formulation, that is, they are written in terms of the state and control variables. A performance index is also chosen for the problem that minimizes the energy consumed during the shimmy oscillations. Special care is given to define the performance index, writing it in terms of the control variables and in agreement with the formalism adopted. We select the weight matrices of LQR based on the time domain specifications of the system to be controlled. The open loop eigenvalues of the system are found to be -250, $+20.15 \pm 500i$. The positive imaginary part of eigenvalues suggests that the open loop system is unstable in nature and emphasizes the necessity for a feedback controller. Since the given system has only one input, then, we select coefficients for the matrices Q and R as bellow to satisfy the required settling time and overshoot:

$$Q = 100\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad R = \{0.01\} \tag{16}$$

The control input is optimized in this method considering the following index function in which the error is minimized together with the control effort:

$$J = \int_{t_0}^{\infty} (X^T Q X + u^T R u)dt \tag{17}$$

And this objective function is as below for the landing gear system which is the case study of this paper:

$$\begin{aligned} J &= \int_{t_0}^{\infty} (100X^T X + 0.01u^T u)dt \\ &= \int_{t_0}^{\infty} \left(100(\psi^2 + \dot{\psi}^2 + y_1^2) + 0.01M_5^2\right)dt \end{aligned} \tag{18}$$

It can be proved that using the LQR approach the optimal controlling input is as below which is calculated here for the studied shimmy plant:

$$u = -R^{-1}B^T Px = -(P_{21}\psi + P_{22}\dot{\psi} + P_{23}y_1) \tag{19}$$

3.3. Control design via state-dependent Riccati equation

The SDRE nonlinear regulator has the same structure as the Linear Quadratic Regulator (LQR), except that all of the matrices are state-dependent. The SDRE approach for obtaining a suboptimal solution of the control problem has the following procedure [20, 21]. The original nonlinear model of the system should be represented in the following nonlinear state-space form. Direct parameterization is used to convert the nonlinear dynamics of the form

$\dot{x} = f(x) + g(x)$ to the State Dependent Coefficient (SDC) form, as follows:

$$\dot{x} = A(x)x + B(x)u,\ y = C(x)x \tag{20}$$

Applying the above procedure for the nonlinear system of shimmy equations (1)–(8), the state-space equations of the nonlinear model can be written as [19]:

$$\begin{aligned} &x_1 = \psi \to \dot{x_1} = x_2 \\ &x_2 = \dot{\psi} \to \dot{x_2} = \ddot{\psi} = \frac{1}{I_z}\left(cx_1 + Kx_2 + \frac{k}{V}\cos(\varphi)x_2 + M_3 + M_5\right) \\ &x_3 = y_1 \to \dot{x_3} = V\cos(\varphi)x_1 + (e_{eff} - a)\cos(\varphi)x_2 - \frac{V}{\sigma}x_3 \end{aligned} \tag{21}$$

Thus $A(x)$ can be written as:

$$A(x) = \begin{bmatrix} 0 & 1 & 0 \\ A_{21} & A_{22} & A_{23}(x) \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \tag{22}$$

where

$$\begin{aligned} &A_{21} = \frac{c}{I_z}, \qquad A_{22} = \frac{(K + \frac{k}{V}\cos(\varphi))}{I_z}, \\ &A_{23}(x) = \frac{M_3}{I_z x_3} = \frac{M_3}{I_z y_1}, \qquad A_{31} = V\cos(\varphi), \\ &A_{32} = (e_{eff} - a)\cos(\varphi), \qquad A_{33} = -\frac{V}{\sigma} \end{aligned} \tag{23}$$

Here again we have $u = M_5$. In this case the following state-dependent Riccati equation should be solved:

$$A^T(x)P(x) + P(x)A(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0 \tag{24}$$

where $Q(x)$ and $R(x)$ are positive definite state dependent weighting matrices of state $x(t)$ and control effort $u(t)$, respectively. The following values are assumed for the matrices $C(x)$, $Q(x)$ and $R(x)$ of SDRE control:

$$\begin{aligned} &C = (1 \quad 0 \quad 0), \\ &Q(x) = 100\begin{bmatrix} 1 + abs(x_1) & 0 & 0 \\ 0 & 1 + abs(x_2) & 0 \\ 0 & 0 & 1 + abs(x_3) \end{bmatrix}, \\ &R = \{0.01\} \end{aligned} \tag{25}$$

The control input is calculated so that the following state dependent performance index is minimized:

$$J(x) = \frac{1}{2}\int_{t_0}^{\infty}\left(X^T Q(x)X + u^T R(x)u\right)dt \tag{26}$$

The mentioned objective function which is again the summation of error and input can be calculated as below for the landing gear system of the present case study:

$$\begin{aligned} J = &\int_0^{\infty}\left(100\left(\psi^2 + |\psi^3| + \dot{\psi}^2 + |\dot{\psi}^3| + y_1^2 + |y_1^3|\right)\right. \\ &\left. + 0.01\left(K(x)X\right)^2\right)dt \end{aligned} \tag{27}$$

where state dependent coefficient of $K(x)$ is the optimal gain of SDRE. In the multivariable case, there always exist an infinite number of SDC parameterizations. Therefore, the choice of the matrix $A(x)$ isn't unique [20]. The pair $\{A(x), B(x)\}$ is a controllable parameterization of the nonlinear system in a region Ω if $\{A(x), B(x)\}$ is pointwise controllable in the linear sense for all $x \in \Omega$. Therefore, the choice of $A(x)$ must be such that the state dependent controllability matrix $[B(x)A(x)B(x)\ldots A_{n-1}(x)B(x)]$ has full rank [21]. Finally the nonlinear optimal feedback control input for shimmy case results as:

$$\begin{aligned} u_{opt} &= -R^{-1}(x)B^T(x)P(x)x \\ &= -\left(P_{21}(x)\psi + P_{22}(x)\dot{\psi} + P_{23}(x)y_1\right) \end{aligned} \tag{28}$$

$P_{ij}(x)$ is the element of $P \in \Re^{n\times m}$ related to the row i and the column j that can be determined by simultaneous solving of a system of equations related to Riccati equation which can be done here numerically since they are state dependent. By solving the Riccati equation, the transformation matrix P is obtained and the resultant state feedback gain is calculated using the transformation matrix.

3.4. Sliding mode control

During the last two decades, variable structure systems (VSS) and sliding mode control (SMC) have received significant interest and have become well-established research areas with great potential for practical applications. The discontinuous nature of the control action in SMC is claimed to result in outstanding robustness features for both system stabilization and output tracking problems. This good performance also includes insensitivity to parameter variations and rejection of disturbances [22].

Uncertainty for the above mentioned nonlinear state space can be defined as:

$$\dot{x} = F(x) + g(x)u(t) + g(x)d(x,t), \qquad x(0) = x_0 \tag{29}$$

where $d(x,t)$ is an unknown function representing the uncertainties including internal parameter variations, external disturbances and un-modeled dynamics. Another assumption is regarded as follows:

$$d(x,t) \le d_m \tag{30}$$

where d_m is the maximum range of disturbance and it is a positive constant. Integral sliding mode control is an improvement to conventional sliding mode control that uses a nonlinear sliding surface having an integral term. The main idea of integral sliding mode control is to compose two parts of the controller, i.e. the continuous and discontinuous parts. The continuous component is used to control the nominal system while the discontinuous component is used to reject disturbances and to suppress parametric uncertainties [23]. So in the simple case, the integral sliding surface $s(x,t)$ and the control signal $u(t)$ are given by:

$$\begin{aligned} &s\left(x(t),t\right) \\ &\quad = G(x)\left(x(t) - x(0)\right) - G(x)\int_{t_0}^{t}\left(F(x) + g(x)u_C(t)\right)d(\tau) \end{aligned} \tag{31}$$

$$u(t) = u_C(t) + u_d(t) \tag{32}$$

where $x(0)$ is the initial value of the states, $G \in \Re^{m\times n}$ satisfies that GB is nonsingular, $u_C(t)$ is the nominal control signal and $u_d(t)$ is a discontinuous control signal given by

$$\begin{aligned} &u_C(t) = -K_C x(t) \\ &u_d(t) = -K_d f_s(s) \end{aligned} \tag{33}$$

K_C is the gain of continues part, $f_s(s)$ is the switching function and K_d is an appropriate positive constant. Then the problem of integral sliding mode control is to find control signal $u(t)$ in equation (30), and matrix $G(x)$ such that the sliding surface given by equation (29) and its derivative remain zero for all time $t > 0$. In sliding mode, we have $s = 0, \dot{s} = 0$. We can obtain the equivalent control law u_{eq} as follows:

$$\begin{aligned}\dot{s} &= G(x)\big(\dot{x} - \big(F(x) + g(x)u_C(t)\big)\big)\\ &= G(x)\big(F(x) + g(x)u(t) + g(x)d(x,t) - \big(F(x) + g(x)u_C(t)\big)\big)\\ &= G(x)\big(g(x)u(t) + g(x)d(x,t) - g(x)u_C(t)\big) = 0\end{aligned}$$

$$u_{eq} = -\big[G(x)g(x)\big]^{-1}\big[G(x)g(x)d(x,t) - G(x)g(x)u_C(t)\big] \tag{34}$$

Substituting (33) and (34), the equivalent controller becomes:

$$u_{eq} = -\big[G(x)g(x)\big]^{-1}\big[G(x)g(x)d(x,t) + G(x)g(x)K_C x(t)\big] \tag{35}$$

According to above equations, the sliding surface and the controlling input of landing gear system can be rewritten as:

$$\dot{s} = G(x)\left[\begin{bmatrix}0\\1\\0\end{bmatrix}u(t) + \begin{bmatrix}0\\1\\0\end{bmatrix}d(x,t) - \begin{bmatrix}0\\1\\0\end{bmatrix}K_C\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right]$$

$$\begin{aligned}\dot{s} = 0 \Rightarrow u_{eq} &= -\frac{1}{G_{12}(x)}\left(G_{12}(x)d(x,t) + G_{12}(x)K_C\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right)\\ &= -d(x,t) - K_C\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\end{aligned} \tag{36}$$

If the nominal controller is chosen as $u_C(t)$, a simple solution to get the sliding condition when the dynamic parameters have uncertainty or with the advent of disturbance is the switching control law [23]:

$$u = u_C(t) - K_d\,\mathrm{sgn}(s) \tag{37}$$

where the switching function sgn(s) is defined as:

$$\mathrm{sgn}(s) = \begin{cases}1 & s > 0\\ -1 & s < 0\\ 0 & s = 0\end{cases} \tag{38}$$

To eliminate the chattering phenomenon $\tanh(s)$ is used instead of $\mathrm{sgn}(s)$ and so the control input can be improved as follows:

$$u = -K_C\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} - K_d\tanh(s) \tag{39}$$

3.5. Design of robust optimal sliding-mode controller (ROSMC)

In this section, the problem of robustifying the SDRE for a class of uncertain nonlinear systems is considered. An optimal controller is designed for the nominal system and an integral sliding surface [24] is constructed. Consider a class of uncertain nonlinear systems as equation (29). If we assume $d(x,t) = 0$, the form of the nonlinear system equation (29) can be described as:

$$\dot{x} = A(x)x + g(x)u(t) \tag{40}$$

According to the optimal control theory, there exists an optimal feedback control law that minimizes the index equation (26). The optimal feedback control law can be then described as equation (28), and the closed-loop system dynamics becomes:

$$\dot{x}(t) = \big(A(x) - B(x)R^{-1}(x)B^T(x)P(x)\big)x(t) \tag{41}$$

According to optimal control theory, the closed-loop system is asymptotically stable. However, if the control law equation (28) is applied for an uncertain system equation (29) or a system with external disturbances as we face for the case of shimmy phenomena, the state trajectory will deviate from the optimal trajectory and even the system may become unstable. To solve this problem, integral sliding mode (ISM) control technique is employed here to increase the robustness of the optimal control law. As a result, the state trajectory of the uncertain system of equation (29) is the same as that of the optimal trajectory of the nominal system of equation (40) while the uncertainty and external disturbances are neutralized at the same time.

For designing the robust optimal sliding mode controller, considering the uncertainty of equation (29), a new integral sliding surface is defined in the form of:

$$\begin{aligned}s\big(x(t),t\big) &= G(x)\big(x(t) - x(0)\big)\\ &\quad - G(x)\int_{t_0}^{t}\big(A(x) - B(x)R^{-1}(x)B^T(x)P(x)\big)x(\tau)d(\tau)\end{aligned} \tag{42}$$

Differentiating equation (42) with respect to t and considering equation (29), we obtain:

$$\begin{aligned}\dot{s}\big(x(t),t\big) &= G(x)\dot{x}(t) - G(x)\big[A(x) - B(x)R^{-1}(x)B^T(x)P(x)\big]x(t)\\ &= G(x)\big(A(x)x + B(x)u(t) + g(x)d(x,t)\big)\\ &\quad - G(x)\big[A(x)x(t) - B(x)R^{-1}(x)B^T(x)P(x)x(t)\big]\\ &= G(x)\big[B(x)u(t) + g(x)d(x,t)\\ &\quad + B(x)R^{-1}(x)B^T(x)P(x)x(t)\big]\end{aligned} \tag{43}$$

For landing gear system this surface can be written as:

$$\begin{aligned}\dot{s}\big(x(t),t\big) = G(x)&\left[\begin{bmatrix}0\\1\\0\end{bmatrix}u(t) + \begin{bmatrix}0\\1\\0\end{bmatrix}d(x,t)\right.\\ &\left.- \begin{bmatrix}0\\1\\0\end{bmatrix}\big(P_{21}(x)\psi + P_{22}(x)\dot{\psi} + P_{23}(x)y_1\big)\right]\end{aligned} \tag{44}$$

Considering the system of equation (43), we can obtain the equivalent control law u_{eq} as:

$$\begin{aligned}&\dot{s}\big(x(t),t\big) = 0\\ &\Rightarrow u_{eq} = -\big[G(x)B(x)\big]^{-1}\big[G(x)B(x)R^{-1}(x)B^T(x)P(x)x(t)\\ &\qquad + G(x)g(x)d(x,t)\big]\end{aligned} \tag{45}$$

By substituting equations (44), (45) for the shimmy case, the equivalent controller and the control law for landing gear system can be extracted as:

$$\begin{aligned}&\dot{s}\big(x(t),t\big) = 0\\ &\Rightarrow u_{eq} = -G_{12}(x)\big(d(x,t) - \big(P_{21}(x)\psi + P_{22}(x)\dot{\psi} + P_{23}(x)y_1\big)\big)\end{aligned} \tag{46}$$

Substituting equation (45) into equation (29), the sliding mode dynamics becomes:

$$\begin{aligned}\dot{x}(t) &= A(x)x(t) + B(x)u_{eq}(t) + d(x,t)\\ &= A(x)x(t) - B(x)\big(\big[G(x)B(x)\big]^{-1}\\ &\quad \times \big[G(x)B(x)R^{-1}(x)B^T(x)P(x)x(t)\\ &\quad + G(x)g(x)d(x,t)\big]\big) + g(x)d(x,t)\\ &= \big(A(x) - B(x)R^{-1}(x)B^T(x)P(x)\big)x(t)\\ &= A(x)x(t) + B(x)u_{opt}(t)\end{aligned} \tag{47}$$

Comparing equation (47) with equation (41), we can see that the sliding mode of uncertain nonlinear system of equation (29) is the same as optimal dynamics of equation (41); therefore, the sliding mode is also asymptotically stable, and the sliding motion guarantees the closed loop system to be globally robust against the uncertainties which satisfies the matching condition. For the uncertain system of equation (29), the following control law is proposed:

$$\begin{aligned} u(t) &= u_{eq}(t) + u_{dis}(t) \\ u_{eq}(t) &= -R^{-1}(x)B^T(x)P(x)x(t) \\ u_{dis}(t) &= -\big[G(x)B(x)\big]^{-1} \times \big(\gamma \tanh(s)\big) \end{aligned} \tag{48}$$

where $\tanh(s) = [\tanh(s_1), \dots, \tanh(s_m)]^T$ and γ is appropriate positive constant. Therefore, the control law for the landing gear system can be presented as:

$$\begin{aligned} u(t) &= u_{eq}(t) + u_{dis}(t) \\ &= -G_{12}(x)\big(d(x,t) - \big(P_{21}(x)\psi + P_{22}(x)\dot{\psi} + P_{23}(x)y_1\big)\big) \\ &\quad - \frac{\gamma \tanh(s)}{G_{12}(x)} \end{aligned} \tag{49}$$

Here $u_{\mathrm{eq}}(t)$ which is used to stabilize and optimize the nominal system is the continuous part of the control law while $u_{dis}(t)$ which is the discontinuous part provides complete compensation for uncertainties of the system.

From equation (42), we have $s(0) = 0$; which is the initial condition on the sliding surface. According to [24], uncertain system of equation (29) achieves global sliding mode with the integral sliding surface of equation (42) and the control law of equation (48). So the designed system is globally robust and optimal.

The efficiency and applicability of the proposed robust optimal control for the shimmy vibration of aircraft nose landing gear is shown in the simulation results.

4. Nonlinear observer design based on the nonlinear system

State Variable Feedback (SVFB) is straightforward, but it needs online feedback of all of the states while in reality all of the states are seldom available for measurements specially for vibrating systems like shimmy. Given only measurements of some specified outputs of a dynamic system, all the states can be reconstructed using an observer if the system satisfies a property known as observability. Here we would like to design an observer that can estimate the internal state $x(t)$ given knowledge of the control inputs $u(t)$ and the outputs $y(t)$.

In this section a nonlinear continuous-time observer is presented based on the (SDRE) filter with guaranteed exponential stability. For a nonlinear state space, let us introduce an observer as follows [25]:

$$\dot{\hat{x}} = A\big(\hat{x}(t)\big)\hat{x}(t) + B\big(\hat{x}(t)\big)u(t) + L\big(x(t)\big)\big(y(t) - C\big(\hat{x}(t)\big)\hat{x}(t)\big) \tag{50}$$

where the $n \times p$ matrix $L(x(t))$ is time varying and is called the observer gain. The observer has n internal states $\hat{x}(t)$ and two inputs, where $\hat{x}(t)$ provides an estimate of the full state $x(t)$ if $L(x(t))$ is correctly chosen. We define the observer gain by:

$$L\big(x(t)\big) = \overline{P}\big(x(t)\big)C^T\big(\hat{x}(t)\big)R^{-1} \tag{51}$$

where $\overline{P}(x(t))$ is a steady state solution of the difference Riccati equation, obtained by solving equation (52):

$$A^T(x)\overline{P}(x) + \overline{P}(x)A(x) - \overline{P}(x)C^T(x)R^{-1}(x)C(x)\overline{P}(x) + Q(x) = 0 \tag{52}$$

For the landing gear system it can be written as:

$$\begin{aligned} L\big(x(t)\big) &= \overline{P}\big(x(t)\big)C^T\big(\hat{x}(t)\big)R^{-1} \\ &= [\,\overline{P}_{11}(x(t)) \quad \overline{P}_{21}(x(t)) \quad \overline{P}_{31}(x(t))\,]^T \end{aligned}$$

This is an n-th order dynamic system, with initial state $\hat{x}(0)$ equal to the initial estimate of the states. The observer gain matrix $L(x(t))$ must be selected so that, even though the initial estimate $\hat{x}(0)$ is not equal to the actual initial state $x(0)$, as time passes the state estimate $\hat{x}(t)$ converges to the actual state $x(t)$. The quantity $\tilde{y}(t) = y(t) - \hat{y}(t)$ is called the output estimation error. To choose $L(x(t))$, the state estimation error is defined as $\tilde{x}(t) = x(t) - \hat{x}(t)$, and its dynamics can be written as:

$$\begin{aligned} \dot{\tilde{x}}(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= A\big(x(t)\big)x + B\big(x(t)\big)u(t) - A\big(\hat{x}(t)\big)\hat{x}(t) - B\big(\hat{x}(t)\big)u(t) \\ &\quad - L(t)\big[y(t) - C\big(\hat{x}(t)\big)\hat{x}(t)\big] \end{aligned} \tag{53}$$

Adding and subtracting $A(\hat{x}(t))x(t)$ to the whole equation together with adding and subtracting $C(\hat{x}(t))x(t)$ into the bracket lead to:

$$\begin{aligned} \dot{\tilde{x}}(t) &= A\big(\hat{x}(t)\big)x(t) - A\big(\hat{x}(t)\big)x(t) + A\big(x(t)\big)x + B\big(x(t)\big)u(t) \\ &\quad - A\big(\hat{x}(t)\big)\hat{x}(t) - B\big(\hat{x}(t)\big)u(t) - L(t)\big[C\big(\hat{x}(t)\big)x(t) \\ &\quad - C\big(\hat{x}(t)\big)\hat{x}(t) + C\big(x(t)\big)x(t) - C\big(\hat{x}(t)\big)x(t)\big] \end{aligned} \tag{54}$$

So the error dynamics are given by:

$$\begin{aligned} \dot{\tilde{x}}(t) &= \big[A\big(\hat{x}(t)\big) - L(t)C\big(\hat{x}(t)\big)\big]\tilde{x} + \Psi\big(x(t), \hat{x}(t), u(t)\big) \\ &\quad - L(t)\Pi\big(x(t), \hat{x}(t)\big) \end{aligned} \tag{55}$$

where

$$\begin{aligned} &\Psi\big(x(t), \hat{x}(t), u(t)\big) \\ &\quad = \big[A\big(x(t)\big) - A\big(\hat{x}(t)\big)\big]x(t) - \big[B\big(x(t)\big) - B\big(\hat{x}(t)\big)\big]u(t) \\ &\Pi\big(x(t)\big) = \big[C\big(x(t)\big) - C\big(\hat{x}(t)\big)\big]x(t) \end{aligned} \tag{56}$$

For the landing gear system it can be rewritten in this way:

$$\begin{aligned} &\Psi\big(x(t), \hat{x}(t), u(t)\big) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{M_3}{I_z}(\frac{1}{x_3} - \frac{1}{\hat{x}_3}) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \\ &\Pi\big(x(t)\big) = [0] \end{aligned} \tag{57}$$

Thus we have:

$$\begin{aligned} \dot{\tilde{x}}(t) &= \begin{bmatrix} -\overline{P}_{11}(x(t)) & 1 & 0 \\ A_{21} - \overline{P}_{21}(x(t)) & A_{22} & \frac{M_3}{I_z x_3} \\ A_{31} - \overline{P}_{31}(x(t)) & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 - \hat{x}_1 \\ x_2 - \hat{x}_2 \\ x_3 - \hat{x}_3 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{M_3}{I_z}(\frac{1}{x_3} - \frac{1}{\hat{x}_3}) \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &= \begin{bmatrix} -\overline{P}_{11}(x(t)) & 1 & 0 \\ A_{21} - \overline{P}_{21}(x(t)) & A_{22} & \frac{M_3}{I_z}(\frac{2}{x_3} - \frac{1}{\hat{x}_3}) \\ A_{31} - \overline{P}_{31}(x(t)) & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &\quad - \begin{bmatrix} -\overline{P}_{11}(x(t)) & 1 & 0 \\ A_{21} - \overline{P}_{21}(x(t)) & A_{22} & \frac{M_3}{I_z x_3} \\ A_{31} - \overline{P}_{31}(x(t)) & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} \end{aligned} \tag{58}$$

This equation is known as the error dynamics. Integrating the dynamics of the main nonlinear system together with the error dynamics of the mentioned nonlinear observer and defining the

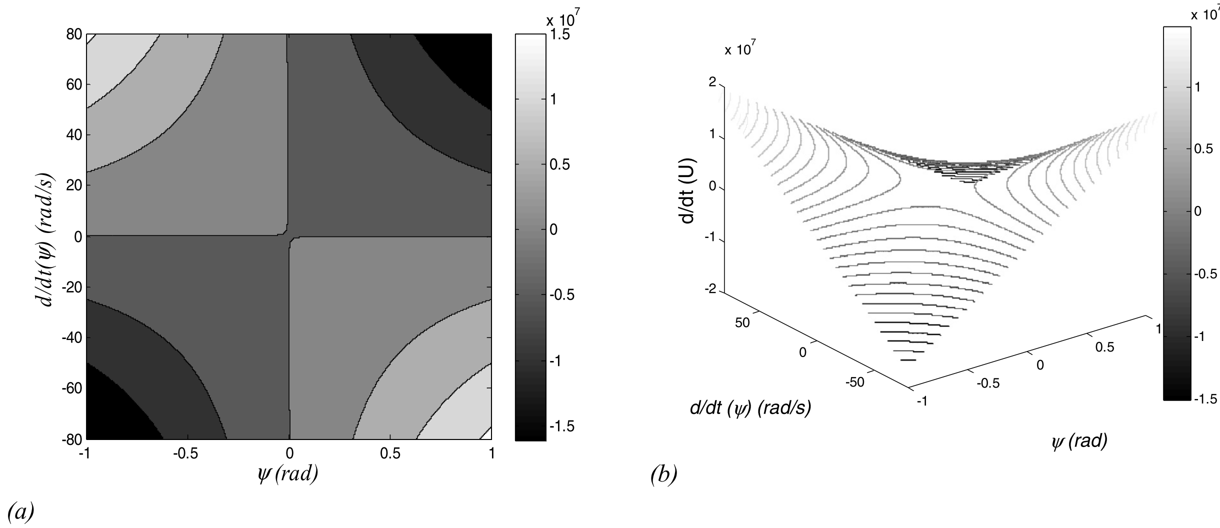

Fig. 2. Stability zones.

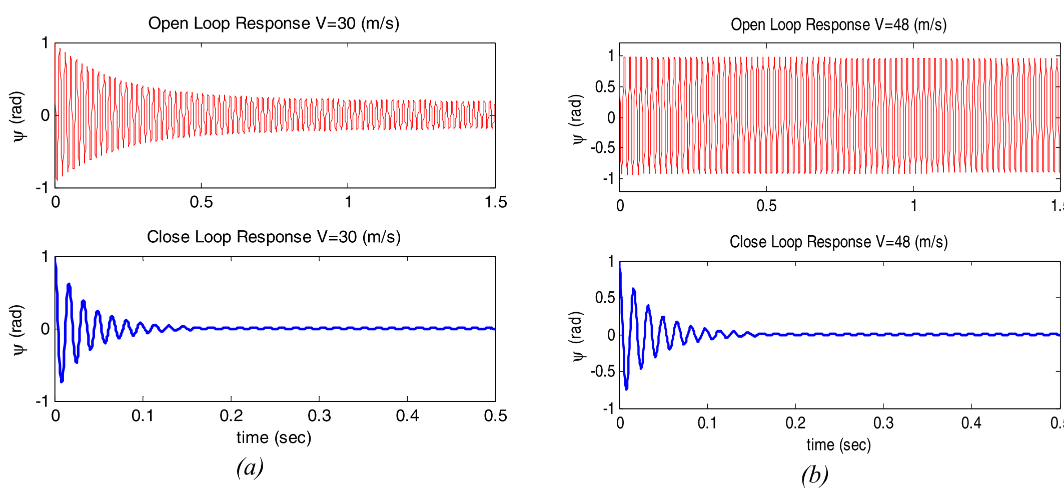

Fig. 3. (a) Yaw-angle time response for $V = 30$ m/s and (b) Yaw-angle time response for $V = 48$ m/s.

augmented system state as $[x^T \tilde{x}^T]^T$, which has $2n$ components we have the following overall closed loop state space:

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} \begin{matrix} 0 & 1 & 0 \\ A_{21} - K_{11}(x) & A_{22} - K_{12}(x) & A_{23}(x) - K_{13}(x) \\ A_{31} & A_{32} & A_{33} \end{matrix} & 0 \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & \frac{M_3}{I_z}\left(\frac{1}{x_3} - \frac{1}{\hat{x}_3}\right) \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} -\overline{P}_{11}(x(t)) & 1 & 0 \\ A_{21} - \overline{P}_{21}(x(t)) & A_{22} & \frac{M_3}{I_z x_3} \\ A_{31} - \overline{P}_{31}(x(t)) & A_{32} & A_{33} \end{matrix} \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix},$$

$$x = \begin{bmatrix} \psi \\ \dot{\psi} \\ y \end{bmatrix}, \qquad \tilde{x} = \begin{bmatrix} \tilde{x}_\psi \\ \tilde{x}_{\dot{\psi}} \\ \tilde{x}_y \end{bmatrix} \tag{59}$$

5. Computer simulations

In this section, a numerical simulation is presented to investigate the superiority of the proposed controlling method. The employed values of the plant parameters and their related physical meaning are listed in Table 1. Initial states are selected as $\psi_0 = 0.1$, $\dot{\psi}_0 = 0$, $y_0 = 0.1$, and the taxiing speed is considered $V = 30, 48, 70$ m/s.

First of all in order to check the stability of the system, Lyapunov function of shimmy vibration is provided. According to equation (12), the stability zones are shown for the phase diagram in Fig. 2. Dark colors area in which the derivative of energy is negative are stable parts as shown.

As it can be seen there are some zones in the phase diagram in which the vibration of the open loop shimmy is unstable. However using the proposed controlling strategies, the stability of the system throughout its dynamic workspace is guaranteed. Response of the open loop system is compared with the response of the simple closed loop controller in Fig. 3. It can be seen that the passive system is stable in speed 30 m/s while it is unstable in 70 m/s and its limit cycle zone is related to speed 48 m/s. Employing, the closed loop control not only the system stability is provided but also the shimmy vibration of the aircraft landing gear is considerably suppressed. Here, the control gain is chosen as $K_c = 10^3\,[7 \quad 0.1 \quad -20\,]$. Fig. 4 compared response of the open loop and close loop system in speed 70 m/s and the control input of the closed loop system.

It can be seen that yaw angle is damped to zero before 0.15 s. Moreover, the system responses and control input for LQR and the simple closed loop control is also depicted in Fig. 5 to show the superiority of using LQR in optimizing the shimmy.

Fig. 6 compares the system responses and control input of LQR and SDRE. It is obvious that due to the possible flexibility of choosing the state depended matrices of $R(x)$ and $Q(x)$ in SDRE, the sys-

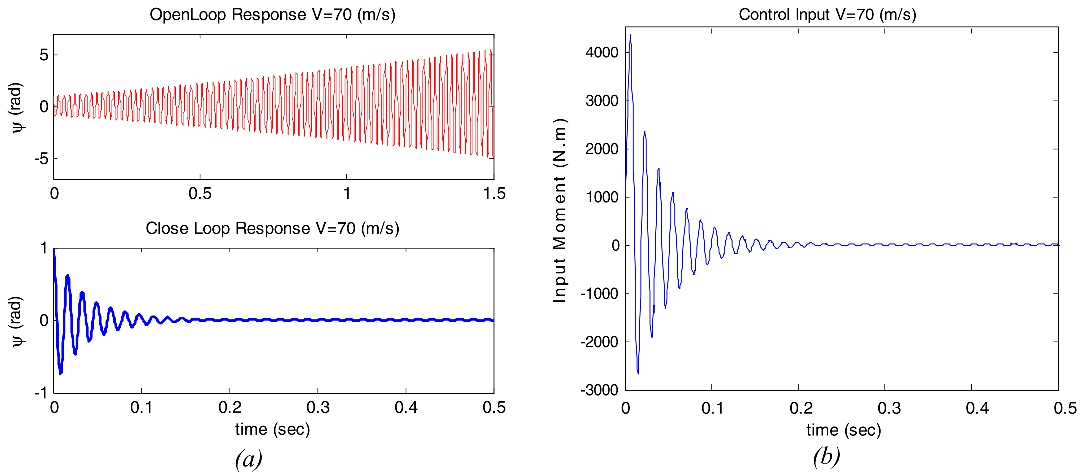

Fig. 4. (a) Yaw-angle time response and (b) input of close loop control for $V = 70$ m/s.

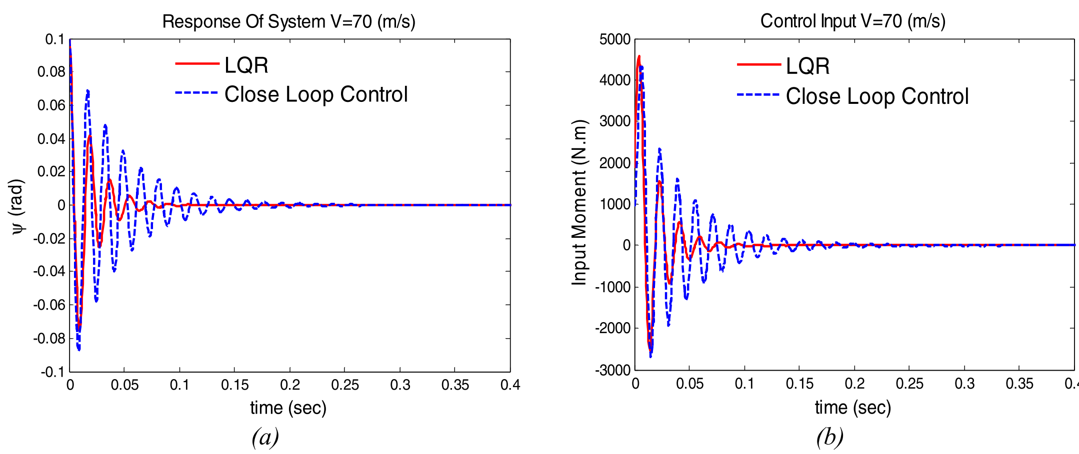

Fig. 5. (a) Comparison of yaw-angle time response between LQR and simple control and (b) comparisons of their control inputs.

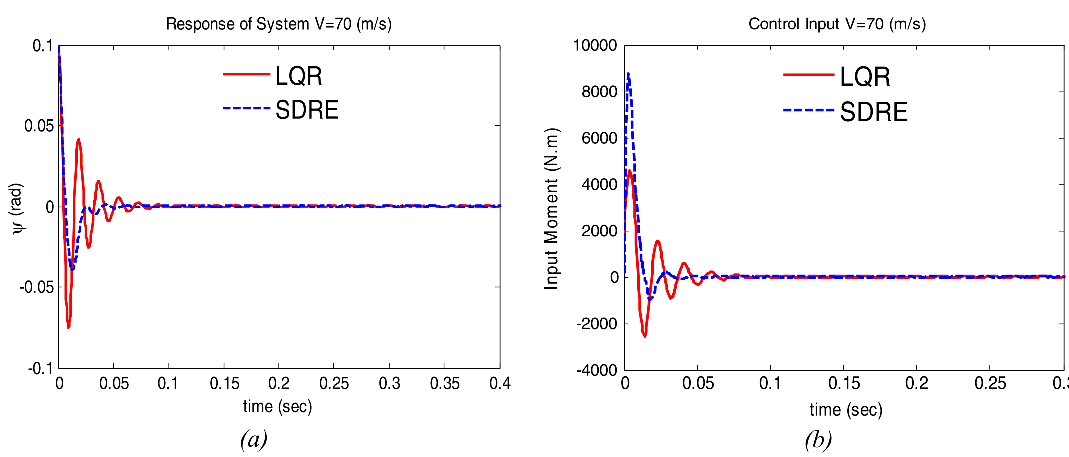

Fig. 6. (a) Comparison of yaw-angle time response between LQR and SDRE and (b) comparison of their control inputs.

tem can be controlled more accurately by the aid of SDRE method. Also the significant limitation of LQR which has an acceptable performance just around the chosen operating point is canceled here and the controller is totally applicable for the whole of the plant workspace. It is shown in Fig. 6 that the performance of the LQR is strengthened even more for the case of SDRE while the required input is smoother too.

In order to show the correctness and superiority of the proposed controllers over the traditional employed systems the results of LQR and SDRE are compared with the results of Refs. [13,14] in Fig. 7. It can be seen that employing the LQR results in state convergence which is almost the same as that in [13,14] with smaller overshoot when dealing with taxiing velocity. However, for the case of SDRE the improvement of the accuracy and stability for SDRE is more observable. The responses of the LQR controller are also compared with semi-active controller in [26]. It was seen in this reference that active controller provides a better response and

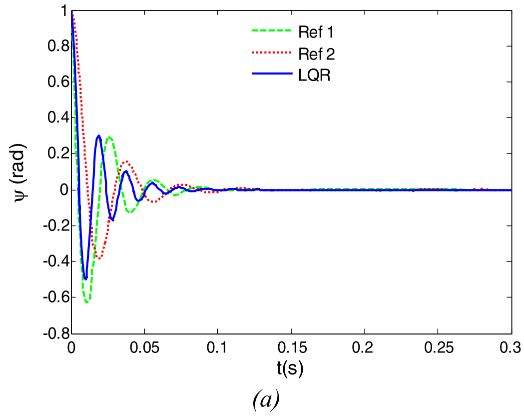

Fig. 7. (a) Comparisons of yaw-angle time response between references and LQR and (b) comparisons of yaw-angle time response between references and SDRE.

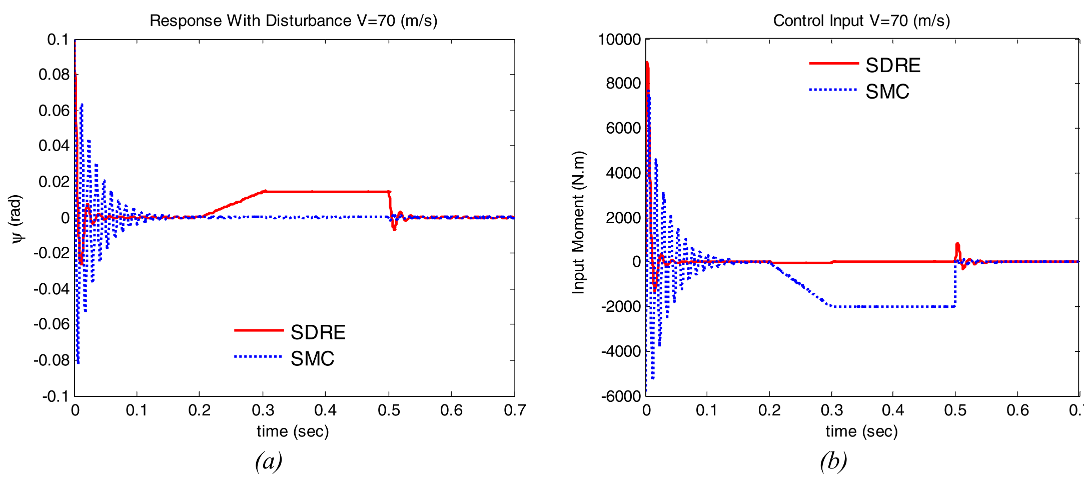

Fig. 8. (a) Comparison of yaw-angle time response between SMC and SDRE and (b) comparison of their control inputs.

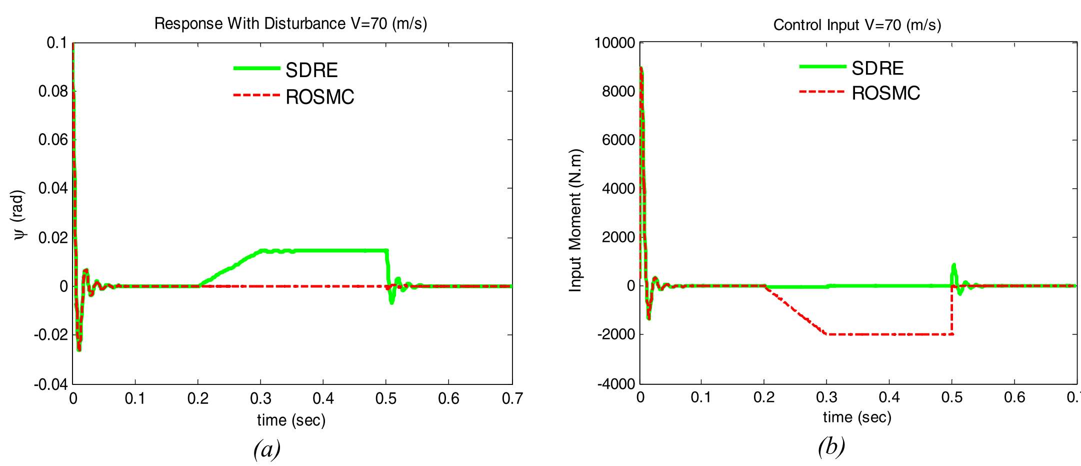

Fig. 9. (a) Comparison of yaw-angle time response with disturbance between SDRE and ROMC and (b) comparison of their control inputs.

sooner damping rather than semi-active and that's why full active controllers are discussed in this paper.

In order to show the necessity of robustness of the designed controller, external disturbance or parametric uncertainty should be engaged. Assume that there is an external disturbance in the applied torque of the aircraft landing gear, which can be modeled as:

$$d(x,t) = \begin{cases} 20\,000(t-0.2) & 0.2 \le t \le 0.3 \\ 2000 & 0.3 \le t \le 0.5 \\ 0 & t \le 0.2 \\ 0 & t \ge 0.3 \end{cases} \Rightarrow d_m = 2000\ (\mathrm{N\,m}) \tag{60}$$

Fig. 8 compares the system responses of SDRE and SMC for the system of equation (29) considering the disturbance $d(x,t)$. It can

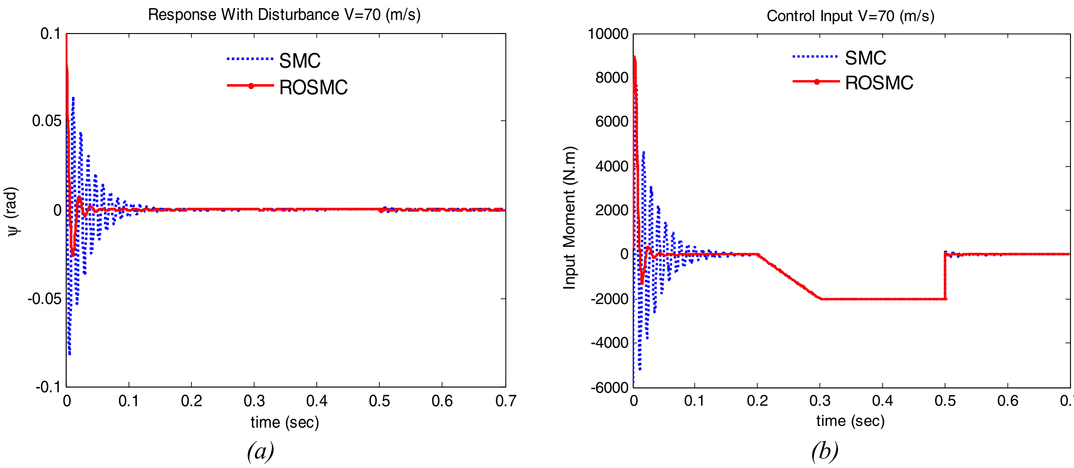

Fig. 10. (a) Comparison of yaw-angle time response with disturbance between SMC and ROMC and (b) comparison of their control inputs.

be seen that although these controllers are optimal but they are not robust enough facing the mentioned disturbances. It is also notable that vibrating response of the system during implementation of the disturbance is limited to its end time in where the change of input is abrupt and during its initial moments the response is smooth since the disturbance is continues here. But again here the pattern and amplitude of the vibration of each method are the same as initial vibration of the system.

That's why another extra upgrading is proposed for the designed system and the optimal controller is equipped by an additive robust sub-controller, i.e. sliding mode (ROSMC). Fig. 9 compares the system response of SDRE and ROSMC for the system of equation (29) considering the disturbance $d(x,t)$. It can be seen from Fig. 9(a) that although both controllers are optimal, but at the instant when the system is subject to uncertainties, the response of the system with SDRE control deviates from the optimal trajectory, while the response of the system with ROSMC keeps the nominal path of the desired setpoint. This correction is provided thanks to applying the required torque in the input of the system as can be seen in Fig. 9(b). So it can be concluded that the proposed ROSMC has a better performance rather than simple SDRE.

Finally it is proved that the proposed ROSMC is even better that simple sliding mode. The comparison of SMC and ROSMC can be seen in Fig. 10. It can be seen that for the case in which an external disturbance is applied to the system, the performance of the designed nonlinear robust optimal controller i.e. ROSMC is considerably better than its equivalent sliding mode controller, i.e. SMC. Both controllers show a good robust reaction however the simple sliding mode is not optimal enough and its accuracy is not acceptable compared to its energy consumption.

Figs. 11 and 12 compare the system responses of SDRE, SMC and ROSMC for the system of equation (29) considering the disturbance $d(x,t)$.

Fig. 13 shows the disturbance and switching function in the optimal sliding mode controller.

In order to have a more general and realistic sense of the mentioned observations and comparison which is investigated through the mentioned methods and also show the efficiency of the proposed method, the net area under the curve of the error and also control input of all of the methods are computed versus time which is in fact a criteria of the optimization objective function and its results are summarized in Table 2 for the shimmy vibration. It can be seen that for the case in which no external disturbance is engaged, the results of SDRE for both error and input are better than LQR since the original system is nonlinear. The same result is also valid for the case that we have external disturbance however the value of error is not acceptable for these two approaches. So

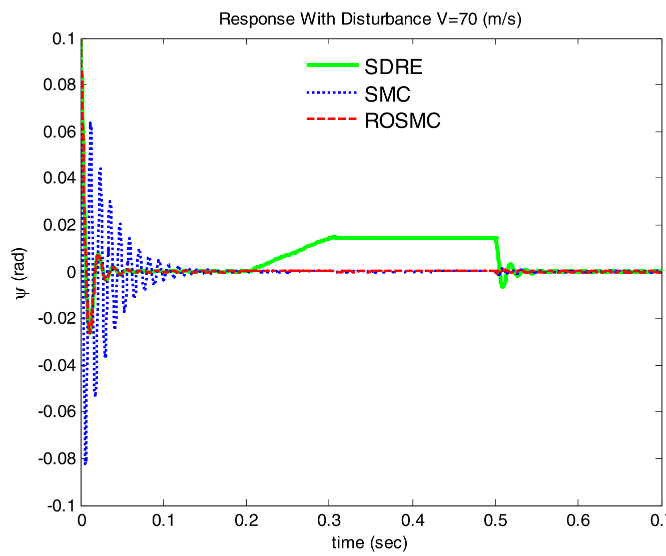

Fig. 11. Yaw-angle time response with disturbance and its comparison between SMC, SDRE and ROSMC.

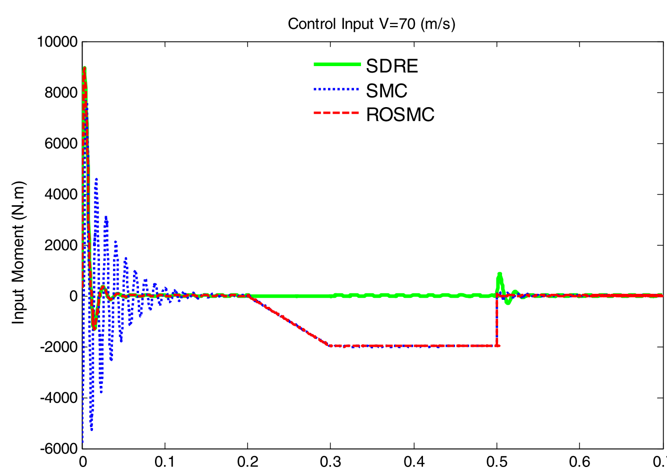

Fig. 12. Comparison of input with disturbance between SMC, SDRE and ROSMC.

we need robust methods. Finally investigating the results of SMC and the proposed ROSMC shows that although the criteria value of input is increased which is unavoidable for compensating the error, but this increase is extremely lower for the proposed method while the error is also considerably better for this case.

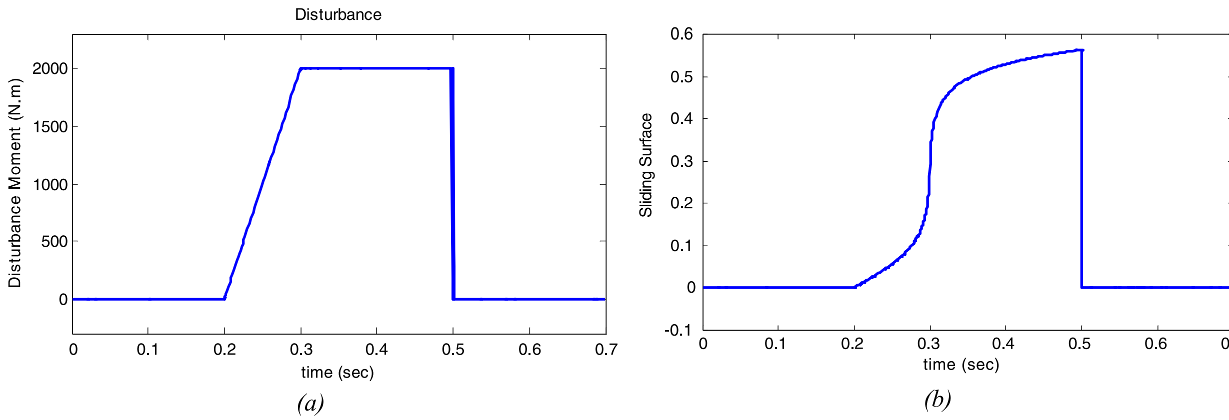

Fig. 13. (a) Disturbance in system, (b) Sliding functions in ROSMC.

Table 2
Net area under the curve of the error and also control input.

Controller	Curve			
	Yaw angle versus time without disturbance	Input moment versus time with disturbance	Yaw angle versus time without disturbance	Yaw angle versus time with disturbance
LQR	0.0014	68.8393	0.0051	79.7248
SDRE	0.0006	68.6196	0.0044	79.0312
SMC	–	–	0.0020	663.4573
ROSMC	–	–	0.0007	571.8559

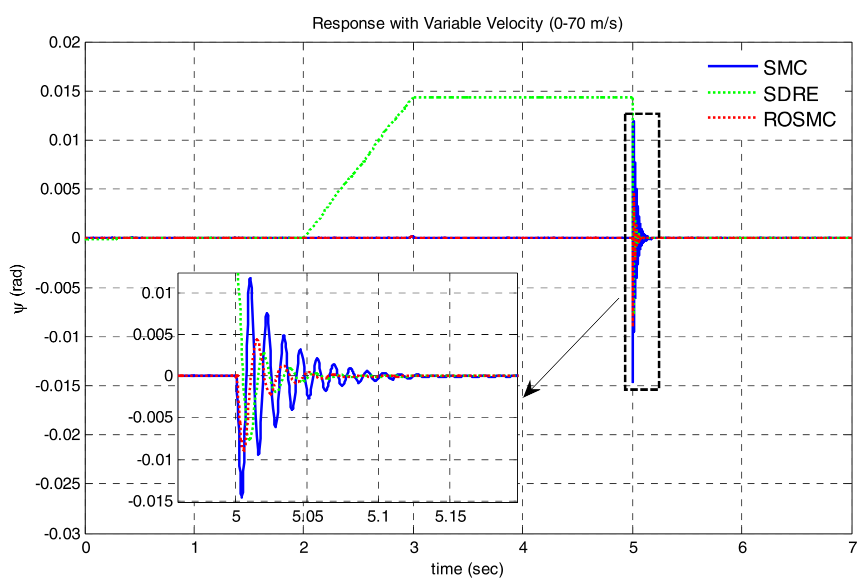

Fig. 14. The response of the landing gear with variable speed of aircraft and its comparison between SMC, SDRE and ROSMC.

Finally the response of the landing gear with variable speed of the aircraft is also obtained and is depicted in Fig. 14 to show the efficiency of the proposed method in the entire possible situations of the aircraft. Here the time dependent setpoint of the aircraft velocity and its engaged external disturbance are considered as Fig. 16 and different controlling strategies are employed in order to investigate their performance. It can be seen that again the best performance is related to ROSMC and using this method the response is stable and the vibrations after each changing of the input or disturbance are immediately damped (lower than 1 s) with the least amount of overshoot. Required input torque can be seen in Fig. 15. Not only the consumption of the energy is optimum but also for each change of setpoint input or disturbance, it changes in a way to damp the vibrations rapidly. It can be seen that though SDRE is optimum but it has unwanted vibrations after each change of velocity or implementing the disturbance since it has no compensation in its input. On the other hand although the SMC doesn't have the mentioned problem but its vibrations after each switching and consequently it's required torque is significantly higher.

As it was mentioned, in order to implement the mentioned controllers it is first required to feedback all of the states. Considering the vibrating nature of the plant, this importance is not possible for all of the states through sensor installation and a nonlinear observer is thus required. For evaluating the gains of the observer pole placement of errors of the observer is performed. To provide an acceptable performance of the system it is required that the error of the observer could be damped faster enough respect to the time constant of the state errors. In order to check this condition, time response of the error of yaw angle, yaw rate $\dot{\psi}$, and

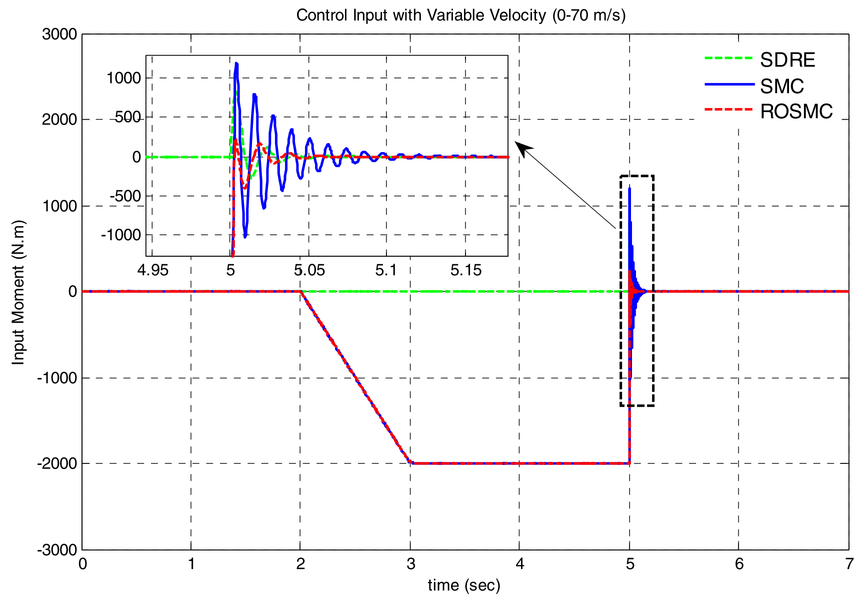

Fig. 15. Control input of the landing gear with variable speed of aircraft and its comparison between SMC, SDRE and ROSMC.

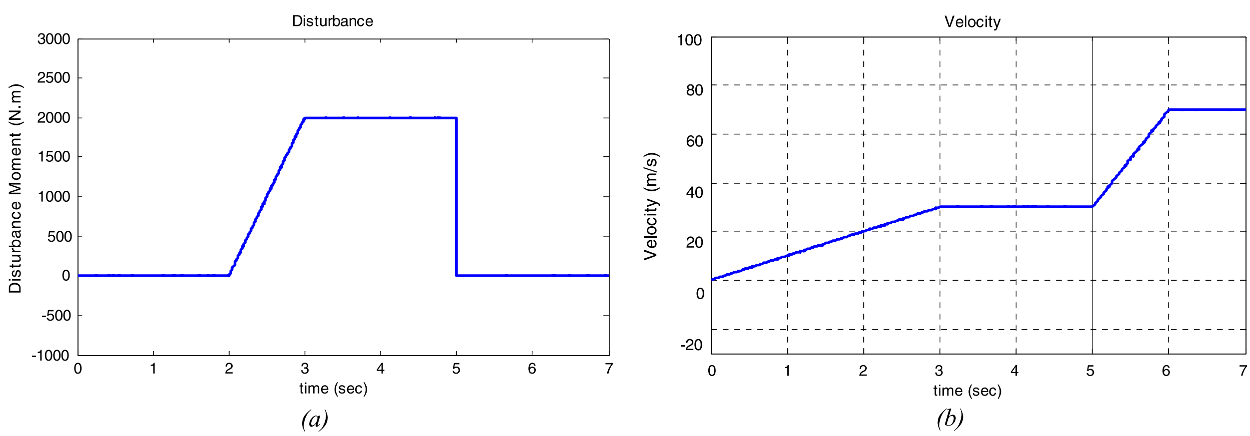

Fig. 16. (a) Disturbance in system, (b) variable velocity of the aircraft.

the lateral shift y_1 are shown in Fig. 17 for both the observer and the controller. Faster damping of the observer states respect to the plant states confirms the acceptable performance of the designed observer since the error of the observer is damped roughly during 0.1 period of settling time of the main states.

6. Conclusion

A robust and optimal nonlinear controller was designed for neutralizing the destructive effect of shimmy phenomenon in the nose landing gear of the aircraft. Both linear and nonlinear closed loop optimization approaches were employed to minimize the unwanted vibrations together with the consumption of energy. LQR was used for linear approach while SDRE was employed for nonlinear optimization. The sliding mode control strategy has been used to design a robust controller for the nonlinear system with uncertainties and disturbances. An integrated controlling strategy was proposed combined of SDRE together with sliding mode in order to cancel the uncertainty and disturbances effects of the vibrating response of the shimmy at the same time with minimizing the consumption of energy. In order to provide the required actual value of all the states of the plant which is not easily measurable because of the high frequency vibrating nature of the shimmy, an optimal nonlinear observer was designed and implemented. The efficiency and robustness of the proposed controllers were verified using simulation in MATLAB and comparative analysis of the results. The results show that all of the mentioned designed controllers are able to stabilize the shimmy vibrations of the landing gear aircraft while the passive system suffers from instability. Afterwards, the response of these optimized closed loop controllers was compared with their equivalent ordinary closed loop controller in which no optimization tool was employed. The results showed the superiority of the proposed optimal controller since a better accuracy was fulfilled using lower amount of controlling input. LQR and SDRE controllers were also compared by which better performance of the nonlinear optimizer tool of SDRE was proved since the original system is itself nonlinear. It was also seen that in presence of external disturbances, the performance of SMC as a robust controller is more acceptable rather than the simple optimal cases while the norm of input is not as optimized as the proposed optimal controllers. Finally in a complete comparative simulation in which the SDRE, sliding mode and the proposed optimal sliding mode systems are compared, it was shown that the proposed ROSMC based on SDRE has the best controlling performance on shimmy plant since the robustness and optimality are satisfied simultaneously and it is able to guarantee the stability and accuracy of the system even during the effect of noises and disturbances. The efficiency of the proposed method was also in-

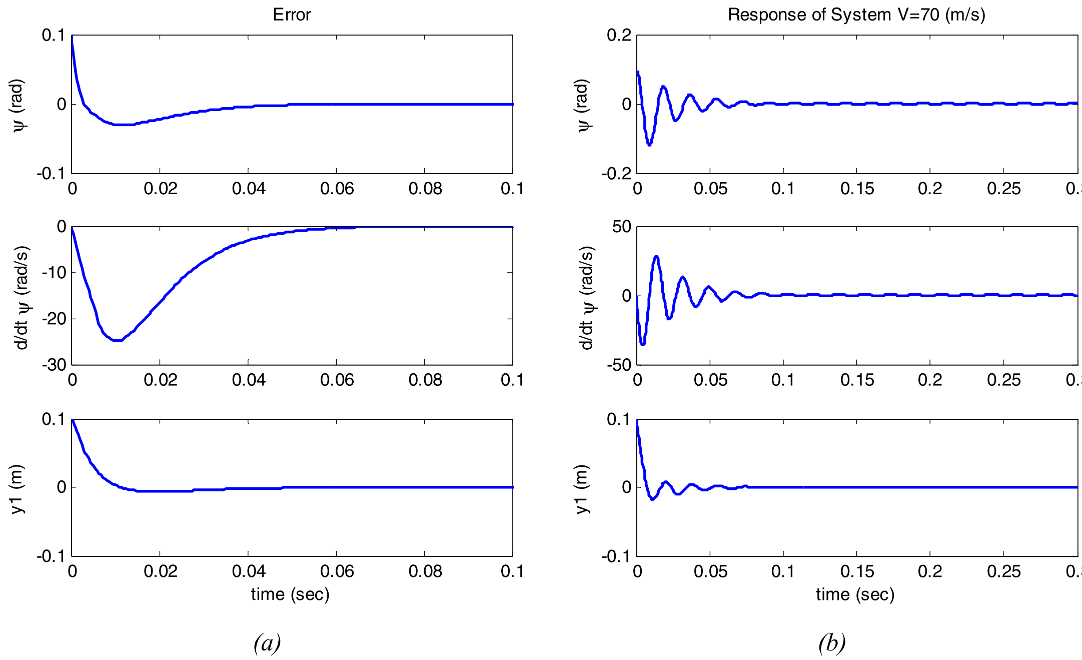

Fig. 17. (a) Error of the observer states, (b) response of the system with observe.

vestigated for a simulation with variable speed of aircraft and also checking and comparing the objective function values for each approach. The correctness, efficiency and superiority of the proposed controlling algorithm for the studied plant were finally investigated and proved by the aid of comparing the results with the results of references with traditional controllers. Considering the mentioned observations it can be concluded that the proposed robust optimal controller of ROSMC based on SDRE has the best overall performance compared to simple controllers and also LQR in order to damp the destructive effect of shimmy in all of environmental situations.

Conflict of interest statement

There is no conflict of interest.

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