Quantum paradoxes in electronic counting statistics

Adam Bednorz
Institute of Theoretical Physics
University of Warsaw
Warsaw, Poland
Email: Adam.Bednorz@fuw.edu.pl

Kurt Franke and Wolfgang Belzig
Department of Physics
University of Konstanz
D-78457 Konstanz, Germany
Email: Wolfgang.Belzig@uni-konstanz.de

***Abstract*—The impossibility of measuring non-commuting quantum mechanical observables is one of the most fascinating consequences of the quantum mechanical postulates. Hence, to date the investigation of quantum measurement and projection is a fundamentally interesting topic. We propose to test the concept of weak measurement of non-commuting observables in mesoscopic transport experiments, using a quasiprobabilistic description. We derive an inequality for current correlators, which is satisfied by every classical probability but violated by high-frequency fourth-order cumulants in the quantum regime for experimentally feasible parameters. We further address the creation and detection of entanglement in solid-state electronics, which is of fundamental importance for quantum information processing. We propose a general test of entanglement based on the violation of a classically satisfied inequality for continuous variables by 4th or higher order quantum correlation functions. Our scheme provides a way to prove the existence of entanglement in a mesoscopic transport setup by measuring higher order cumulants without requiring the additional assumption of single charge detection.**

I. Introduction

The standard von Neumann definition of measurement [1] works only for perfect detectors with instant measurements. For a general, i.e. time-resolved, measurements one must use positive operator-valued measures [2]. In particular, this allows measuring noncommuting observables. The major breakthrough in the problem of measuring noncommuting observables was achieved by Aharonov, Albert and Vaidman by defining weak measurements [3]. The price paid is the large detection noise [9], which can be deconvolved from the final probability distribution to obtain a quasiprobability. This approach is consistent with projective measurements, the definition of symmetrized noise [4] and full counting statistics [5]. Experimental measurements of current fluctuations in mesoscopic junctions [6], [7], [8] have to be interpreted using weak measurements, although there are various additional effects — like backaction — that make the interpretation complicated [10] The quasiprobability is sometimes negative, for example for current fluctuations in tunnel junctions [11]. However, the nonclassicality of the quasiprobability requires at least fourth order moments. Second order correlations can be always interpreted classically [12]. Finally, the quasiprobability can be used to observe violations of a Bell-type inequality which requires fourth order correlators. All proposed Bell tests based on only second order correlators have to assume dichotomic outcome, which implicitly requires knowledge of higher moments.

This paper is organized as follows. Firstly, we derive the correspondence between weak measurement [3] and the quasiprobability. Next, we show the *weak positivity* – classicality – of second order correlations. We present the inequality for current noise fluctuations, satisfied classically but violated in quantum weak measurement [11]. Finally, we show the Bell-type inequality [13], [14], which can be violated in mesoscopic junctions, requiring only 4$^{\text{th}}$ order correlations [12], while the existing one required 20$^{\text{th}}$ order [15].

II. Weak measurement and quasiprobability

Aharonov, Albert and Vaidman [3] considered a special sequence of measurements of a spin 1/2 particle. The spin is prepared in some initial state $|\psi_i\rangle$, then $\hat{\sigma}_z$ is measured *weakly* by introducing an interaction term in the Hamiltonian, containing $\hat{\sigma}_z$. Finally, one projects the final state on $|\psi\rangle_f$ and takes into account only events that passed this postselection. The (real part of) average ${}_f\langle\hat{\sigma}_z\rangle_i = \langle\psi_f|\hat{\sigma}_z|\psi_i\rangle / \langle\psi_f|\psi_i\rangle$ has been called the *weak value* and it can take arbitrarily large values, much exceeding ± 1 – eigenvalues of $\hat{\sigma}_z$. Here we show that this unusual result can be alternatively interpreted by means of quasiprobability – the possible outcomes do not exceed ± 1 but the "probability" – although normalized to 1 – sometimes takes negative values.

Consider the original weak value measurement of $\hat{\sigma}_z$ of a spin 1/2 particle with initial state $|\psi_i\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow\rangle + \sin\left(\frac{\theta}{2}\right)|\downarrow\rangle$ and final state $|\psi_f\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ defining a pre- and post-selected ensemble. The measurement interaction entangles the spin with an ancillary continuous system whose initial state is taken to be Gaussian with unit variance and zero mean. The measurement interaction is $H_i(t) = g(t)\hat{p}\hat{\sigma}_z$ where $g(t)$ turns the interaction on and off quickly in comparison to other time dynamics which are ignored for simplicity, $g := \int \mathrm{d}t g(t)$ is the measurement strength, and $\hat{p}$ the momentum of the ancilla. After the interaction and post-selection, the position x of the ancilla is projectively measured. Defining the measurement variable $\sigma_z = x/g$ to give the correct behavior in the $g \to \infty$ limit, the probability density of σ_z can be written as the convolution $P(\sigma_z) = \int \mathrm{d}s\phi(\sigma_z - s)\varrho(s)$ where $\Phi(x) = g/\sqrt{2\pi}\, e^{-\frac{1}{2}g^2x^2}$ is the scaled probability distribution of the original state of the ancilla, and $\varrho(s)$ is

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the quasiprobability distribution

$$\varrho(s) = \frac{\cos^2(\frac{\theta}{2})\delta(s-1) + \sin^2(\frac{\theta}{2})\delta(s+1) + \sin(\theta)e^{-\frac{g^2}{2}}\delta(s)}{1+\sin(\theta)e^{-\frac{g^2}{2}}}. \tag{1}$$

In the strong, projective limit, (1) reduces to projective probabilities at the eigenvalues ± 1 of $\hat{\sigma}_z$. In the weak limit, the term at 0 contributes with either positive or negative quasiprobability depending on the value of θ. Furthermore, in the weak limit the mean value of $\varrho(s)$ is equal to the real part of the weak value, $\langle\psi_f|\hat{\sigma}_z|\psi_i\rangle / \langle\psi_f|\psi_i\rangle$, which may lie outside of the range of eigenvalues.

These ideas apply to a series of weak measurements without post-selection. Each weak measurement introduces an ancilla system and creates entanglement via a von Neumann interaction. The density matrix after the n$^{\text{th}}$ measurement is

$$\hat{\rho}_n = e^{-ig_n\hat{p}_n\hat{A}_n}\left(\hat{\rho}_{n-1}\otimes|\phi_n\rangle\langle\phi_n|\right)e^{-ig_n\hat{p}_n\hat{A}_n} \tag{2}$$

where $|\phi_n\rangle$ is the initial prepared state of detector n. By inserting identity operations $\int \mathscr{D}A\,|A\rangle\langle A| = \hat{1}$, the measurement interaction can be expressed as shifts of the ancilla wavefunction.

$$\hat{\rho}_n = \int \mathscr{D}A'_n\,\mathscr{D}A''_n\left(|\phi_n(x_n - g_nA'_n)\rangle\langle\phi_n(x_n - g_nA''_n|\right) \times \left(|A'_n\rangle\langle A'_n|\,\hat{\rho}_{n-1}\,|A''_n\rangle\langle A''_n|\right) \tag{3}$$

The joint probability $P(A_1,\ldots,A_n) =: P(\boldsymbol{A})$ is the probability of measuring $x_k = A_k/g_k$ in the ancillas.

$$P(\boldsymbol{A}) = \mathrm{Tr}\left\{\hat{\rho}_n\prod_k\frac{1}{g_k}\left|\frac{A_k}{g_k}\right\rangle\left\langle\frac{A_k}{g_k}\right|\right\} \tag{4}$$

$$= \int \mathscr{D}\boldsymbol{A'}\,\mathscr{D}\boldsymbol{A''}\,\mathrm{Tr}\left\{\tilde{\rho}_n\left(\boldsymbol{A'},\boldsymbol{A''}\right)\right\} \times\prod_k\frac{1}{g_k}\phi(g_k(A_k - A'_k))\,\phi^*(g_k(A_k - A''_k)) \tag{5}$$

In (5), $\tilde{\rho}_n$ is defined recursively as

$$\tilde{\rho}_n = |A'_n\rangle\langle A'_n|\,\tilde{\rho}_{n-1}\,|A''_n\rangle\langle A''_n|\,. \tag{6}$$

Using Gaussian wavefunctions $\phi(x) = (2\pi)^{-1/4}e^{-x^2/4}$, a change of variables to $\bar{\boldsymbol{A}} = (\boldsymbol{A'}+\boldsymbol{A''})/2$ and $\boldsymbol{\delta A} = \boldsymbol{A'} - \boldsymbol{A''}$ separates the joint probability density into signal and detector noise.

$$P(\boldsymbol{A}) = \int \mathscr{D}\bar{\boldsymbol{A}}\;\Phi(\boldsymbol{A} - \bar{\boldsymbol{A}})\;\varrho(\bar{\boldsymbol{A}}) \tag{7}$$

$$\Phi(\boldsymbol{A} - \bar{\boldsymbol{A}}) = \prod_k g_k|\phi(g_k(A_k - \bar{A}_k)|^2 \tag{8}$$

$$\varrho(\bar{\boldsymbol{A}}) = \int \mathscr{D}\boldsymbol{\delta A}\;e^{-(\boldsymbol{g}\cdot\boldsymbol{\delta A})^2/2}\;\mathrm{Tr}\left\{\tilde{\rho}_n(\bar{\boldsymbol{A}},\boldsymbol{\delta A})\right\} \tag{9}$$

The measure $\mathscr{D}\boldsymbol{\delta A}$ depends on $\bar{\boldsymbol{A}}$. Eq. (9) is the joint quasiprobability density for the series of von Neumann measurements.

III. Construction of Quasiprobability

We will construct the general form quasiprobability by a deconvolution from a suitable POVM [2]. Let us begin with the basic properties of a POVM. The Kraus operators $\hat{K}(A)$ for an observable described by $\hat{A}$ with continuous outcome A need only satisfy $\int dA\hat{K}^\dagger(A)\hat{K}(A) = \hat{1}$. The act of measurement on the state defined by the density matrix $\hat{\rho}$ results in the new state $\hat{\rho}(A) = \hat{K}(A)\hat{\rho}\hat{K}^\dagger(A)$. The new state yields a normalized and positive definite probability density $\rho(A) = \mathrm{Tr}\,\hat{\rho}(A)$. The procedure can be repeated recursively for an arbitrary sequence of (not necessarily commuting) operators $\hat{A}_1,\ldots,\hat{A}_n$,

$$\hat{\rho}(A_1,\ldots,A_n) = \hat{K}(A_n)\hat{\rho}(A_1,\ldots,A_{n-1})\hat{K}^\dagger(A_n)\,. \tag{10}$$

The corresponding probability density is given by $\rho(A_1,\ldots,A_n) = \mathrm{Tr}\;\hat{\rho}(A_1,\ldots,A_n)$. We now define a family of Kraus operators, namely $\hat{K}_\lambda(A) = (2\lambda/\pi)^{1/4}\exp(-\lambda(\hat{A}-A)^2)$. It is clear that $\lambda \to \infty$ should correspond to exact, strong, projective measurement, while $\lambda \to 0$ is a weak measurement and gives a large error. We also see that strong projection changes the state (by collapse), while $\lambda \to 0$ gives $\hat{\rho}(A) \sim \hat{\rho}$, and hence this case corresponds to the weak measurement. However, the repetition of the same measurement k times effectively means one measurement with $\lambda \to k\lambda$ so, with $k \to \infty$, even a weak coupling $\lambda \ll 1$ results in a strong measurement. For an arbitrary sequence of measurements, we can write the final density matrix as the convolution

$$\hat{\rho}_{\boldsymbol{\lambda}}(\boldsymbol{A}) = \int DA'\;\hat{\varrho}_{\boldsymbol{\lambda}}(\boldsymbol{A'})\prod_k g_k(A_k - A'_k) \tag{11}$$

with $g_k(A) = e^{-2\lambda_kA^2}\sqrt{2\lambda_k/\pi}$. Here $\boldsymbol{\lambda} = (\lambda_1,\ldots,\lambda_n)$ $\boldsymbol{A} = (A_1,\ldots,A_n)$, and $DA = dA_1\ldots dA_n$. The quasi-density matrix $\hat{\varrho}$ is given recursively by

$$\hat{\varrho}_{\boldsymbol{\lambda}}(\boldsymbol{A}) = \int\frac{d\chi}{2\pi}e^{-i\chi A_n}\int\frac{d\phi}{\sqrt{2\pi\lambda_n}}e^{-\phi^2/2\lambda_n}\times e^{i(\chi/2+\phi)\hat{A}_n}\hat{\varrho}_{\boldsymbol{\lambda}}(A_1,\ldots,A_{n-1})e^{i(\chi/2-\phi)\hat{A}_n} \tag{12}$$

with the initial density matrix $\hat{\varrho} = \hat{\rho}$ for $n = 0$. We can interpret g in (11) as some internal noise of the detectors which, in the limit $\lambda \to 0$, should not influence the system. We *define* the quasiprobability $\varrho_{\boldsymbol{\lambda}} = \mathrm{Tr}\;\hat{\varrho}_{\boldsymbol{\lambda}}$ and abbreviate $\varrho \equiv \varrho_{\boldsymbol{0}}$. In this limit (12) reduces to

$$\hat{\varrho}(\boldsymbol{A}) = \int\frac{d\chi}{2\pi}e^{-i\chi A_n}e^{i\chi\hat{A}_n/2}\hat{\varrho}(A_1,\ldots,A_{n-1})e^{i\chi\hat{A}_n/2}\,. \tag{13}$$

Note that $\varrho_{0\ldots0,\lambda} = \varrho$, so the last measurement does not need to be weak (it can be even a projection). The averages with respect to ϱ are easily calculated by means of the generating function (13), e.g. $\langle A\rangle_\varrho = \mathrm{Tr}\,\hat{A}\hat{\rho}$, $\langle AB\rangle_\varrho = \mathrm{Tr}\,\{\hat{A},\hat{B}\}\hat{\rho}/2$, $\langle ABC\rangle_\varrho = \mathrm{Tr}\,\hat{C}\{\hat{B},\{\hat{A},\hat{\rho}\}\}/4$ for $\boldsymbol{A} = (A,B,C)$. In general

$$\langle X_1(t_1)\cdots X_n(t_n)\rangle_\varrho = \mathrm{Tr}\hat{\rho}\{\hat{X}_1(t_1),\{\cdots\{\hat{X}_{n-1}(t_{n-1}),\hat{X}_n(t_n)\}\cdots\}\}/2^{n-1} \tag{14}$$

for time ordered observables, $t_1 \leq t_2 \leq \cdots \leq t_n$.

The quasiprobability from the spin 1/2 example can be interpreted as conditional quasiprobability $\varrho(A|B) = \varrho(A,B)/\varrho(B)$, where $\hat{B} = |\psi_f\rangle\langle\psi_f|$, $\hat{A} = \hat{\sigma}_z$ and $\hat{\rho} = |\psi_i\rangle\langle\psi_i|$ and $g^2 = \lambda \to 0$.

IV. Weak positivity

We shall prove that first and second order correlations functions can be always reproduced classically. To see this, consider a real symmetric correlation matrix $2C_{ij} = 2\langle A_i A_j\rangle_\varrho = \mathrm{Tr}\hat{\rho}\{\hat{A}_i, \hat{A}_j\}$ with $\{\hat{A},\hat{B}\} = \hat{A}\hat{B}+\hat{B}\hat{A}$ for arbitrary, even noncommuting observables $\hat{A}_i$ and density matrix $\hat{\rho}$. This includes all possible first-order averages $\langle A_i\rangle$ by setting one observable to identity or subtracting averages ($A_i \to A_i - \langle A_i\rangle$). Since $\mathrm{Tr}\hat{\rho}\hat{X}^2 \geq 0$ for $\hat{X} = \sum_i \lambda_i \hat{A}_i$ with arbitrary real λ_i, we find that the correlation matrix C is positive definite and any correlation can be simulated by a classical Gaussian distribution $\rho \propto \exp(-\sum_{ij} C^{-1}{}_{ij} A_i A_j/2)$.

Note that the often used dichotomy $A = \pm 1$ is equivalent to $\langle (A^2-1)^2\rangle = 0$, which requires $\langle A^4\rangle$. Moreover, every classical inequality $\langle (f(\{A_i\})^2\rangle \geq 0$ contains the highest correlator of even order. Hence, to detect nonclassical effects with unbounded observables, we have to consider the fourth moments.

V. Test of negative quasiprobability

We shall apply the above scheme to the measurement of current $I(t)$ through a mesoscopic junction in a stationary state, with $\delta I = I - \langle I\rangle$. It is convenient to define the noise (second cumulant), $\tilde{S}(\alpha,\beta) = 2\pi\delta(\alpha+\beta)S_\alpha = \langle \delta I(\alpha)\delta I(\beta)\rangle$, and the fourth cumulant $\tilde{C}(\alpha,\beta,\gamma,\eta) = 2\pi\delta(\alpha+\beta+\gamma+\eta)C(\alpha,\beta,\gamma,\eta)$ with

$$\begin{aligned}&\tilde{C}(\alpha,\beta,\gamma,\eta) = \langle \delta I(\alpha)\delta I(\beta) I(\gamma)\delta I(\eta)\rangle \\ &-\tilde{S}(\alpha,\beta)\tilde{S}(\gamma,\eta) - \tilde{S}(\alpha,\gamma)\tilde{S}(\beta,\eta) - \tilde{S}(\alpha,\eta)\tilde{S}(\gamma,\beta).\end{aligned} \tag{15}$$

Here and throughout the text we use Latin arguments in time domain and Greek ones in frequency domain, related by $a(\omega) = \int dt\, e^{i\omega t} a(t)$. Note, that the delta function of the frequency sum has a cut-off of the order of the measuring time t_0 (larger than all relevant timescales of the system), which in some following expressions is a simple prefactor and does not enter final conclusions.

Let us define the fluctuating noise spectral density $X_\omega = \int_{\omega_-}^{\omega_+} \delta I(\alpha)\delta I(-\alpha) d\alpha$ with $\omega_\pm = \omega \pm \delta\omega/2$, for which we obtain the average fluctuations

$$\begin{aligned}\langle (\delta X_\omega)^2\rangle/t_0 &= \int_{\omega_-}^{\omega_+} C_{\alpha\beta} d\alpha d\beta + 2\pi \int_{\omega_-}^{\omega_+} S_\alpha^2 d\alpha, \\ \langle \delta X_\omega \delta X_{\omega'}\rangle/t_0 &= \int_{\omega_-}^{\omega_+} d\alpha \int_{\omega'_-}^{\omega'_+} C_{\alpha\beta} d\beta\end{aligned} \tag{16}$$

where $\delta X = X - \langle X\rangle$ and $C_{\alpha\beta} = C(\alpha,-\alpha,\beta,-\beta)$. The intervals $[\omega_-,\omega_+]$ and $[\omega'_-,\omega'_+]$ are nonoverlapping. Considering classical correlators of δX at different frequencies we obtain the Cauchy-Bunyakovsky-Schwarz inequality

$$B = \frac{\langle \delta X_\omega \delta X_{\omega'}\rangle^2}{\langle (\delta X_\omega)^2\rangle \langle (\delta X_{\omega'})^2\rangle} \leq 1. \tag{17}$$

If we choose e.g. $0 \leq \omega'_- < \omega'_+ < \omega_-$, the correlators correspond to a low and high frequency measurement.

To find conditions in which the inequality (17) is violated, let us take the tunneling junction. Using Eq. (14) we obtain $S_\alpha = \hbar G w(\alpha)$ and $C_{\alpha\beta} = \hbar F G e^2 (w(\alpha) + w(\beta))/2$ [4], [16], [10]. Here we denote $w(\omega) = \omega \coth(\hbar\omega/2k_B T)$, conductance G and Fano factor $F \simeq 1$ in the tunneling limit. For $T = 35$mK, $\delta\omega = \delta\omega' = 2\omega' = 2\pi\cdot 200$MHz, $\omega = 2\pi\cdot 6$GHz, $G^{-1} = 500\text{k}\Omega$, we get $B = 1.4$, which contradicts our classical expectation (17) and clearly shows that the quasiprobability ϱ must take negative values.

VI. Bell-type inequality

In the last part use the previous concept to discuss a novel concept of entanglement detection in mesoscopic conductors. Due to weak positivity discussed earlier, second order correlations are not sufficient to detect entanglement using electronic currents.

As usual we introduce two separate observers, Alice and Bob that are free to choose between two observables, (A, A') and (B, B'), respectively. The measurements can give arbitrary outcomes (not just ± 1). As shown in Ref. [12] we end up with our main inequality

$$\begin{aligned}&|\langle AB(A^2+B^2)\rangle + \langle A'B(A'^2+B^2)\rangle \\ &+\langle AB'(A^2+B'^2)\rangle - \langle A'B'(A'^2+B'^2)\rangle|/2 \leq \\ &\left(\langle A^4\rangle + \langle A'^4\rangle + \langle B^4\rangle + \langle B'^4\rangle\right)/2 + \\ &\frac{1}{4}\sum_{C,D,E=\{A,A',B,B'\}}^{D\neq C;E\neq C,D,D'} \sqrt{\sqrt{\langle C^4\rangle}\sqrt{\langle D^4\rangle}\left\langle (D^2-E^2)^2\right\rangle},\end{aligned} \tag{18}$$

where $D' = A(B)$ when $D = A'(B')$.

The inequality contains up to 4^{th} order averages which is a trade-off for relaxing the condition of dichotomy (or trichotomy, considering also 0). It reduces to the standard Bell inequality

$$|\langle AB\rangle + \langle A'B\rangle + \langle AB'\rangle - \langle A'B'\rangle| \leq 2 \tag{19}$$

if we restrict the possible values of A,A',B,B' to ± 1. If all observables are allowed to take the additional value 0 only simultaneously then the inequality still reproduces Bell multiplied by the probability of nonzero outcomes. All correlations in the inequality are measurable also in a Bell-type test, because none of them contains AA' or BB'. Hence, we can say that the degrees of freedom measured by $A^{(\prime)}$ and $B^{(\prime)}$ are entangled if the inequality (18) of their correlators is violated. We emphasize that this requires only the assumption of nonnegative probability distribution $\rho(A,A',B,B') \geq 0$ and provides an unambiguous proof for entanglement.

Returning to quantum mechanics, let us take the standard Bell state [14] $\hat{\rho} = (\hat{1} - \hat{\boldsymbol{\sigma}}_A \cdot \hat{\boldsymbol{\sigma}}_B)/4$, $\boldsymbol{\sigma} = (\sigma_1,\sigma_2,\sigma_3)$, with $\hat{\sigma}_i$ – standard spin Pauli matrices $\{\hat{\sigma}_i,\hat{\sigma}_j\} = 2\delta_{ij}\hat{1}$ acting in Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$, $\hat{A}^{(\prime)} = \boldsymbol{a}^{(\prime)}\cdot\hat{\boldsymbol{\sigma}}_A$, $|\boldsymbol{a}^{(\prime)}| = 1$ ($A \leftrightarrow B$) and averages $\langle A^{(\prime)n}B^{(\prime)m}\rangle = \mathrm{Tr}\hat{\rho}\hat{A}^{(\prime)n}\hat{B}^{(\prime)m}$. In particular $\langle A^{(\prime)4}\rangle = \langle B^{(\prime)4}\rangle = \langle A^{(\prime)2}B^{(\prime)2}\rangle = 1$ and $\langle A^{(\prime)}B^{(\prime)3}\rangle = \langle A^{(\prime)3}B^{(\prime)}\rangle = \boldsymbol{a}^{(\prime)}\cdot\boldsymbol{b}^{(\prime)}$. The inequality (18)

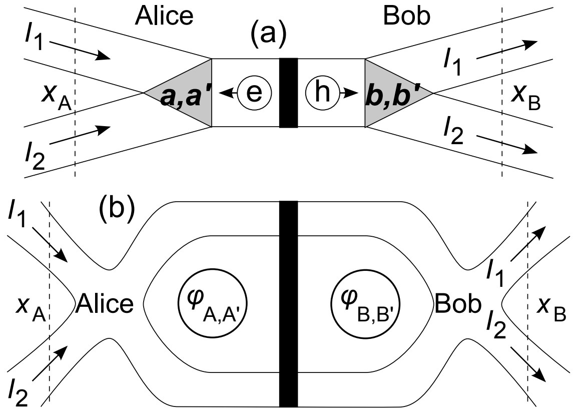

Fig. 1. Proposals of experimental setup for the Bell test. In both cases the black bar represents the scattering barrier, producing entangled electron-hole pairs. The tested observable is the difference of currents, $I_1 - I_2$, at left (Alice) or right (Bob) side. The correlations depend on the spin scattering (a) or magnetic fluxes (b).

is violated as it reads $2\sqrt{2} \leq 2$ for $\boldsymbol{a}'$,$\boldsymbol{b}$,$\boldsymbol{a}$,$\boldsymbol{b}'$ in one plane at angles 0, $\pi/4$, $\pi/2$, $3\pi/4$, respectively.

A. Test on tunnel junction

Now we implement the Bell example in a beam splitting device involving fermions scattered at a tunnel junction at the temperature $k_B T$ and biased with voltage eV. The Bell measurement will be performed by adding spin filters or magnetic flux at both sides of the junctions as shown in Fig. 1. In both cases, the transmission coefficients for the total scattering matrix are $\mathcal{T}_{11} = \mathcal{T}_{22} = \mathcal{T}(1 + \boldsymbol{a}\cdot\boldsymbol{b})/2$ and $\mathcal{T}_{12} = \mathcal{T}_{21} = \mathcal{T}(1 - \boldsymbol{a}\cdot\boldsymbol{b})/2$ where $\boldsymbol{a} = (\cos\phi_A, \sin\phi_A, 0)$ in the case of magnetic fluxes.

As in the previous proposals [17] the tunnel barrier produces electron-hole pairs with entangled spins or orbitals. Alice and Bob can test the inequality (18) by measuring the difference between charge flux in upper and lower arm as shown in Fig. 1. For Alice, the measured observable reads in the Heisenberg picture

$$\hat{A} = \int dt\, f(t)(\hat{I}_1(x_A, t) - \hat{I}_2(x_A, t))/e. \qquad (20)$$

for the filter setting $\boldsymbol{a}$. Here x_A is the point of measurement, satisfying $\max\{|eV|, k_B T\}|x_A/v_n\hbar| \ll \mathcal{T}$ with $f(t)$ slowly changing on the timescale $\hbar/\max\{|eV|, k_B T\}$. One defines analogically A' for $\boldsymbol{a}'$ and B, B' for Bob. In the limit

$$1/N\mathcal{T} \gg t_0 \max\{|eV|, k_B T\}/h \gg t_0/\delta \gg 1 \qquad (21)$$

the averages read $\langle A^{(\prime)4}\rangle_\varrho = \langle B^{(\prime)4}\rangle_\varrho = \langle A^{(\prime)2}B^{(\prime)2}\rangle_\varrho = c$ and $\langle A^{(\prime)}B^{(\prime)3}\rangle_\varrho = \langle A^{(\prime)3}B^{(\prime)}\rangle_\varrho = \boldsymbol{a}^{(\prime)}\cdot\boldsymbol{b}^{(\prime)}c$ where c is a constant. The inequality (18) is violated for the same spin directions an in the standard Bell test.

VII. Conclusions

In the present work we have discussed the concept of a quasiprobability in the context of weak measurement of current cumulants in a mesoscopic conductor. An important observation is, what we call the weak positivity, that all quantum second-order correlation functions can be simulated by a Gaussian distribution and, hence, cannot be used to violate classical inequalities. We show several examples, how the quasiprobabilistic nature of the quantum distribution can be probed. First, to assess the quantum dynamics of a simple tunnel junction, it is possible to violate a Cauchy-Bunyakowski-Schwarz inequality using the 4th-order correlations. The most favourable conditions are found at low temperatures and voltages, but at high frequencies. Second, we have discussed an inequality for non-local entanglement detection using contineous variables. Using this inequality, we can overcome the assumptions underlying the usual Bell inequality for second-order correlators. The experimental violation of our inequality seem in the range of present technology, if suitable high-frequency detection is used.

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